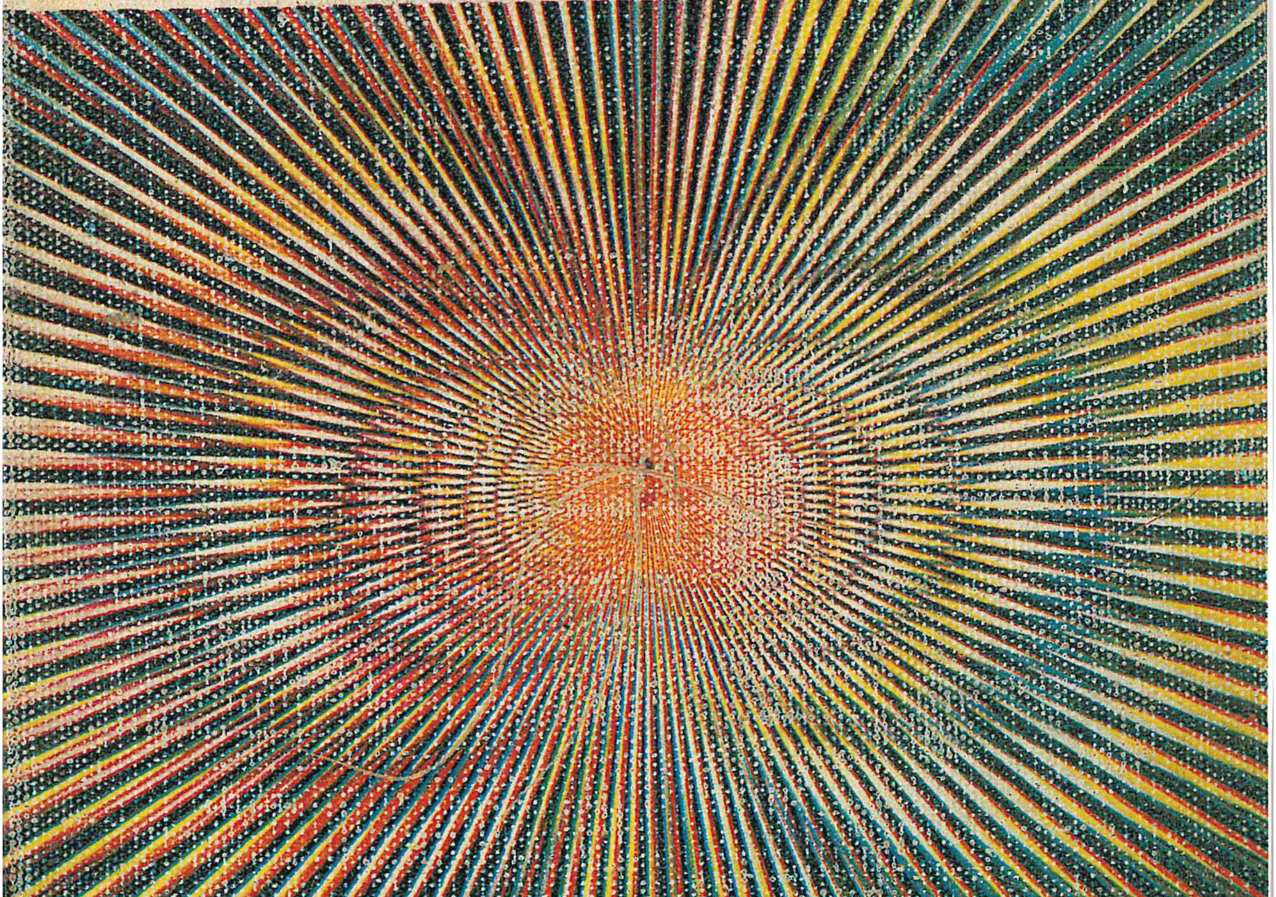
ELEMENTARY MATHEMATICS

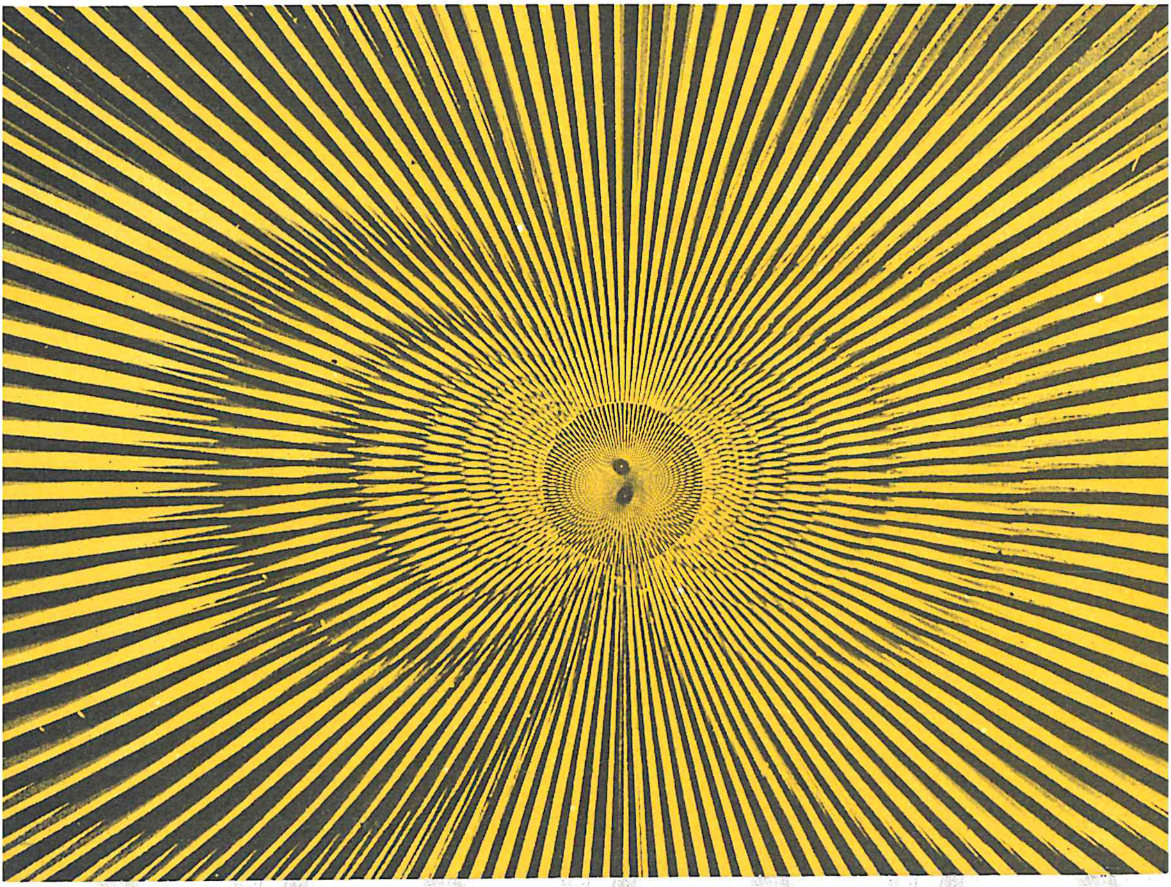
5

SECOND EDITION



Elementary Mathematics

ELEMENTARY



MATHEMATICS 5

SECOND EDITION

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FORWARD
VCAT
VEHICLE COURSE DISPLAY
AFT
0
9
MINUTES AFTER LAUNCHING
000 000 009

CHAPTER 1

Digits

1. The astronauts in the picture at the left have just begun a space flight. As one of the instruments on the panel shows, the flight is 9 minutes old. What numeral appeared in this place just before the launching? What numerals other than 0 and 9 have appeared in this place since the launching?

We call each of these ten numerals a **digit**.

2. Suppose another minute passes and the flight is ten minutes old. Now what digit would appear in the first place at the right of the instrument? What digit would appear in the second place?

3. When the flight is ninety-nine minutes old, what digits will appear in the first two places at the right of the instrument? Another minute passes. Now what digits appear in the first two places? What does the digit 1 in the third place mean?

4. Has the instrument recorded each of the first one hundred minutes using only ten digits? Would any more than ten digits be needed to record another hundred minutes? another thousand minutes? another hundred thousand minutes?

5. Another space flight has been launched from an unknown planet far off in space. The space people who launched this flight have only three "digits" in their numeration system: a nil ○, a wog —, and a toggle ∟.

Before launching, their timing instrument showed a nil, ○, which meant no time units had passed. After launching, a wog, —, appeared to show that one unit of time had passed. Then a toggle showed, which meant two units of time had passed. What digits in this system would appear to show that three units of time had passed?

6. The chart below shows space numerals for a set of numbers in order beginning with zero.

zero	○	three	— ○
one	—	four	— —
two	∟	five	— ∟

Study the chart. Then tell which of these names the number six.

∟○ ∟— ∟∟

7. Copy the chart in Ex. 6 and see if you can write space numerals for the next ten numbers. (Hint: —○○ follows ∟∟, as 100 follows 99.)

Place value

1. On page 1 you saw how a few digits can be used to name many numbers. Our system of numeration uses ten such digits. Write them.

2. Consider just one of these digits — the digit 3. Written alone, the digit 3 stands for 3 ones. But what does the digit 3 stand for in the numeral 30? Does the 3 in 30 have a value ten times the value it has when written alone?

3. What is the value of the digit 3 in the numeral 300? Is its value in 300 one hundred times its value when written alone?

4. Write a numeral in which the digit 3 has a value one thousand times its value when written alone.

5. Does the value of any digit in a numeral in our numeration system depend upon its *place* in the numeral?

We say that our system of numeration is a **place-value system.**

6. How many digits are there in the numeral below? Does each digit in this numeral have the same place value?

7. The first 3 at the right in the numeral below stands for 3 ones, but the second 3 from the right stands for 3 tens, or 30. Is 30 ten times 3? Does the second 3 have a place value ten times the first 3?

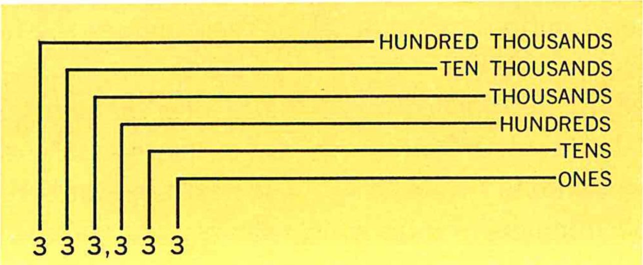
8. What does the third 3 from the right stand for? Is 300 ten times 30? Does the 3 in the hundreds place have a place value ten times the place value of the 3 in the tens place?

9. What is the place value of the fourth 3 from the right? Is 3,000 ten times 300? Then is the place value of this 3 ten times the place value of the 3 just to the right of it?

10. Each 3 in 333,333 has a value _?_ times the value of the 3 just to the right of it.

11. The value of any place in a numeral in our numeration system is _?_ times the value of the place to the right of it. In Latin, *decem* means ten. We call our system of numeration a **decimal system.**

12. The decimal numeral 7,000 names a number that is _?_ times the number named by the numeral 700.



Naming numbers in different ways

1. In the numeral 376, what digit is in the hundreds place? Does the 3 in 376 mean 3 × 100, or 300? What does the 7 mean? the 6? Does the following expression name the same number as 376?

$$(3 \times 100) + (7 \times 10) + (6 \times 1)$$

Does the expression below also name the same number as 376?

$$300 + 70 + 6$$

— Write a 3-digit numeral for each number named below.

	a	b
2.	$(5 \times 100) + (8 \times 10) + (2 \times 1)$	$(7 \times 100) + (4 \times 10) + (4 \times 1)$
3.	$(9 \times 100) + (4 \times 10) + (7 \times 1)$	$(6 \times 100) + (1 \times 10) + (3 \times 1)$

4. In the numeral 4,082, what does the digit 4 mean? To show the value of this digit, can we write 4 × 1,000? In the numeral 4,082, what digit is in the hundreds place? Does this mean that there are zero hundreds in the hundreds place? Does the following name the same number as 4,082?

$$(4 \times 1{,}000) + (0 \times 100) + (8 \times 10) + (2 \times 1)$$

5. Express each number named below in the way shown in Ex. 4.

3,462 5,871 9,027 2,105 7,200 4,340

6. What number is named by 3 × 100? by 3 × 10 × 10? Are 3 × 100 and 3 × 10 × 10 different names for the same number? In the same way, write two names for 400; for 800.

7. Do 10 × 100 and 10 × 10 × 10 name the same number? Write a 4-digit numeral for the number named by 4 × 10 × 10 × 10; by 8 × 10 × 10 × 10.

— Write a 4-digit numeral for each number named below.

8. $(3 \times 10 \times 10 \times 10) + (2 \times 10 \times 10) + (5 \times 10) + (8 \times 1)$

9. $(9 \times 10 \times 10 \times 10) + (6 \times 10 \times 10) + (1 \times 10) + (3 \times 1)$

10. $(2 \times 10 \times 10 \times 10) + (0 \times 10 \times 10) + (4 \times 10) + (9 \times 1)$

11. Can you express the following as a 6-digit numeral? Try it.

$$(7 \times 10 \times 10 \times 10 \times 10 \times 10) + (3 \times 10 \times 10 \times 10 \times 10)$$
$$+ (8 \times 10 \times 10 \times 10) + (2 \times 10 \times 10) + (4 \times 10) + (9 \times 1)$$

Understanding the system to billions

NUMERAL	THE DIGIT 5 STANDS FOR	10 IS USED AS A FACTOR
5	5 × 1	0 times
50	5 × 10	1 time
500	5 × 10 × 10	2 times
5,000	5 × 10 × 10 × 10	? times
50,000	?	? times
500,000	?	? times
5,000,000	?	? times

1. Look at the numerals in the left column of the chart above. Does each numeral contain the digit 5? Does the digit 5 have the same place value in each numeral?

2. In the first numeral, the 5 means 5 ones; but in the second numeral, the 5 means 5 × 10, or 5 tens.

As you know, 50 is the **product** of 5 and 10. The 5 and the 10 are called **factors** of 50. The 10 is used as a factor one time to get 50. How many 0's are there in the numeral for the product of 5 and 10?

3. The 5 in 500 means 5 × 10 × 10, or 5 hundreds. How many times is 10 used as a factor to get 500? How many 0's are there in 500?

4. In 5,000, does the digit 5 mean 5 × 10 × 10 × 10, or 5 thousands? How many times is 10 used as a factor to get 5,000? How many 0's are there in 5,000?

5. How would you write 50,000 as the product of 5 and some 10's? How many times is 10 used as a factor to get 50,000? How many 0's are there in 50,000?

6. On your paper, copy and complete the chart shown at the top of the page.

7. We read the numeral 5,000,000 as "five million." How many times is 10 used as a factor to get 5,000,000? How many 0's are there in 5,000,000? Write the numeral for four million; for twelve million; for two hundred million.

8. Write the numeral for one thousand million. How many 0's are there in this numeral? How many times is 10 used as a factor to get 1,000,000,000? We read the numeral 1,000,000,000 as "one **billion**." Write the numeral for five billion; for ten billion; for one hundred billion.

The meaning of a billion

1. Count by 100,000's to 1 million; by 100,000,000's to 1 billion.

10 one hundred millions = $\underline{?}$

2. One million is 10 times $\underline{?}$.
Ten million is 10 times $\underline{?}$.
One hundred million is 10 times $\underline{?}$.
One billion is 10 times $\underline{?}$.

3. Read these numerals.

- 1,485,629,743
- 7,328,000,000
- 12,722,000,405
- 620,042,000

4. Write numerals using digits for the numbers named below.

- one million
- ten billion ten
- two billion eight hundred million
- seven billion six hundred sixty-five.

5. In the numeral 1,485,629,731, what number does the 1 on the right stand for? What number does the 3 stand for? the 7? What number does each of the other digits stand for?

- If the 4 in 1,485,629,731 is changed to a 7, how is the value of he number changed?
- If the 1 on the left is changed to 3, how is the value of the number hanged?

6. Write a numeral for the greatest umber you can, using only 6 digits; digits; 10 digits. Read each numeral you wrote.

7. Write the numeral that names 1 greater than each number you named in Ex. 6; 100 greater; 1,000,000 greater.

8. Write the numeral that means 10 times each number you named in Ex. 6.

9. Bob has been collecting stamps for a year. He has 1,000 stamps. If he continues collecting the same number of stamps each year, he will have 10,000 stamps in $\underline{?}$ years.

$\underline{?} \times 1{,}000 = 10{,}000$

- How many years would it take Bob to collect 1 million stamps?

$\underline{?} \times 1{,}000 = 1{,}000{,}000$

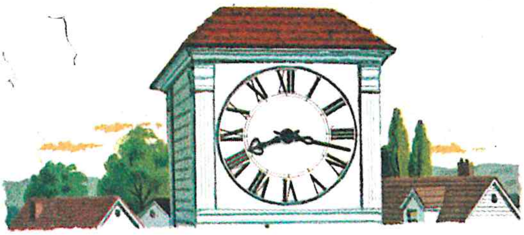
- How many years would it take Bob to collect 1 billion stamps?

$\underline{?} \times 1{,}000 = 1{,}000{,}000{,}000$

10. If a man could spend a dollar a minute without stopping, it would take him about 19 years to spend $10,000,000. About how long would it take him to spend a hundred million dollars? 1 billion dollars? 10 billion dollars?

11. If you said one number name each second without stopping, it would take you almost 32 years to count to 1 billion by ones. About how long would it take you to count to 2 billion by ones? to 10 billion? to 100 billion?

Roman numerals



Numerals are symbols or names for numbers. The numerals we use in our decimal system are called **Hindu-Arabic numerals.** They were developed many years ago by the Hindu people of India and introduced to the Western world by the Arabs of Arabia. As you know, this system of numeration is based on sets of ten and place value.

1. On the clockface pictured above, you can see a set of numerals developed by the ancient Romans. What number is named by I? III? V? X? XII?

2. The Romans wrote III as a name for three. Does the Roman numeral III name the same number as the Hindu-Arabic numeral 111? Explain.

3. What number does VII name? Does it name the same number as the Hindu-Arabic numeral 511? Explain.

4. Find the Roman numeral for four on the clockface. Does IV name the same number as the Hindu-Arabic numeral 15?

5. Does the decimal numeral 11 name the same number as 10 + 1? Does the Roman numeral II name the same number as 1 + 1? What Roman numeral names the same number as 10 + 1?

6. Does each 1 in 11 name the same number? Does each I in II name the same number? Is the Roman system a place-value system?

7. The Romans thought of the number 2 as the sum of 1 and 1. What numeral did they write as a name for 2?

They thought of 20 as 10 + 10. What numeral do you think they wrote for 20? What numeral did they write for 10 + 10 + 10, or 30?

8. The Romans thought of 6 as 5 + 1, and wrote the numeral VI. Is the I at the left or at the right of the V?

They thought of 7 as 5 + 1 + 1. What numeral did they write for 7?

XX names 20, so XXII names 22. Write the Roman numeral for 33.

9. XV names 10 + 5, or 15. What number does XXV name? XXVII?

10. If a symbol such as I, V, or X follows another symbol of equal or greater value, do you add the values or find the difference between them?

11. The Roman numeral for 50 is L. Which numeral, XL or LX, is the numeral for 50 + 10, or 60?

12. The Romans thought of 4 as 1 from 5 (5 minus 1) and wrote IV. Is the I at the left or at the right of the V? What number is named by VI?

They thought of 9 as 1 from 10. What numeral did they write for 9?

13. Fourteen is named by the Roman numeral XIV. The Romans thought of 14 as 10 + (5 − 1). They thought of 19 as 10 + (10 − 1). Write the Roman numeral for 19.

What number is named by XXIV? by XXIX? by XXXIV?

14. L names 50, so XL names 10 from 50, or ?. What number is named by XLV? XLVII? XLIX?

15. If a symbol such as I or X stands before another symbol of greater value, do you add the values or find the difference between them?

16. What do you think it means to ay the Roman numeration system s an additive-subtractive system?

1	5	10	50	100	500	1,000
I	V	X	L	C	D	M

– The chart above shows seven ymbols used in the Roman numera- on system. Use it to write the oman numeral that names each ımber shown below.

	a	b	c	d
'.	200	505	3,000	400
.	102	95	54	329

— The Romans thought of IV as 5 minus 1 and of VI as 5 plus 1. Tell how they thought of each number named below.

	a	b	c	d
19.	CX	XXVI	LXXIV	XVIII
20.	MC	MCCI	MCM	XLIV

21. What numbers do these Roman numerals name?

{X, XX, XXX, XL, L, . . .}

The three dots at the right of the last numeral mean that the pattern of numbers continues without end. Would the next number named in this set be 60? Write the Roman numeral for 60.

— Continue to write Roman numerals for the numbers named in the set shown above until you reach the numeral for 100.

— Study each set of numbers shown below to see if you can discover the pattern. Then write Roman numerals for the next ten numbers in each set.

22. {V, X, XV, XX, XXV, . . .}

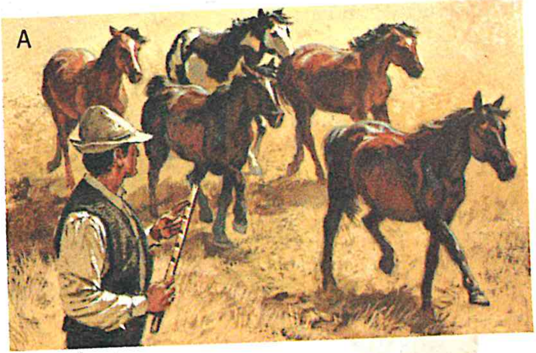
23. {II, IV, VI, VIII, X, . . .}

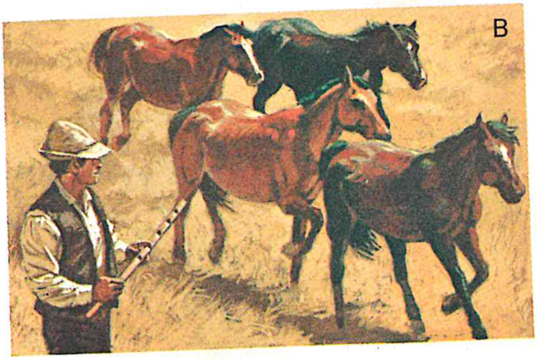
24. {III, VI, IX, XII, XV, . . .}

25. {XX, XL, LX, LXXX, C, . . .}

26. The cornerstone of a building was laid in MCMXXXI. In what year was it laid?

Sets and numbers





1. The man in the picture above has never learned how to count. He cut a notch in the stick for each horse as it went out to the fields in the morning. Now the horses are returning. In picture A, are all the horses coming back? How does he know?

2. Picture B shows the same man as in picture A. It is a few days later, and his horses are again returning from the fields. Are all the horses coming back? How does he know?

3. Suppose this man who owns five horses trades some of his blankets for another horse. After he makes the trade, what must he do to the stick to make sure he will know if all the horses return from the field each night? Explain.

4. In picture B, which set has the greater number of members, the set of notches or the set of horses? Explain how the man who cannot count can tell this.

5. What is the number of the set of notches in picture B? of the set of horses in B?

To write "5 is greater than 4," we can use this symbol, >, and write 5 > 4. Is the symbol open toward the numeral for the greater number or the smaller number?

6. This sentence, 4 < 5, is read "4 is less than 5." Is the symbol for "is less than" open toward the numeral for the greater number or the smaller number?

— Copy each of the following and replace the ● with >, <, or = to make a true sentence.

500

10

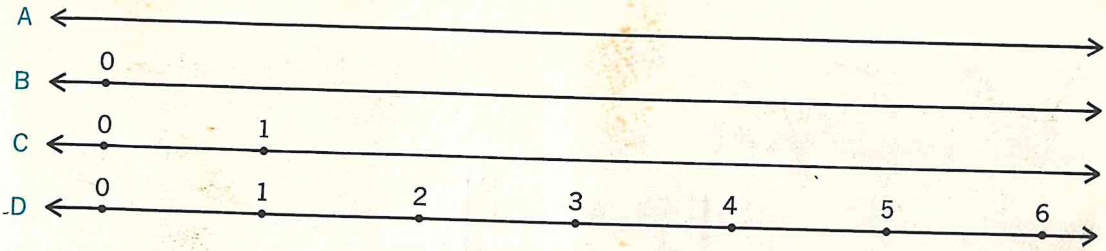
	a	b	c
7.	9 ● 3	8 ● 7	4 ● 6
8.	15 ● 25	32 ● 14	0 ● 1
9.	20 ● 2	48 ● 84	1 ● 0

10. 5 × 10 × 10 × 10 ● 5,000

11. XXIX ● 30

12. 1 billion ● 1,000 million

The number line



The exercises below will help you understand the number line.

1. The drawing in A represents a line. A line goes on and on in both directions. How does the drawing show that the line goes on and on?

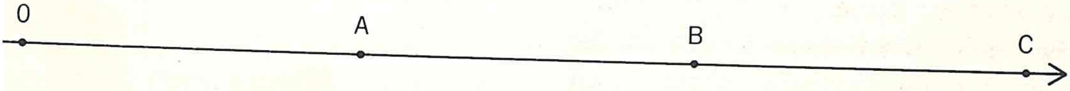
2. In B, any point on the line is chosen and matched with the number 0. In C you see that a point to the right of 0 is chosen to match the number 1. Could you choose any point you wish to match 1?

3. The segment from 0 to 1 is used o find the point that matches 2. Iow does the distance from 0 to 1 ompare with the distance from 1 to ? from 2 to 3? 3 to 4? 4 to 5? to 6? Is the distance between any wo neighboring whole numbers the ame?

When you match points on a line with numbers, you have a **number line.**

4. Move your eyes from left to right along number line D shown above. Do the numbers become greater or do they become smaller? Are the numbers named in order? As your eye moves from right to left along the line, do the numbers become greater or do they become smaller?

5. What is the greatest number named on line D above? Can we think of this line as going on and on to the right? Then can even greater numbers be matched with points on this line?

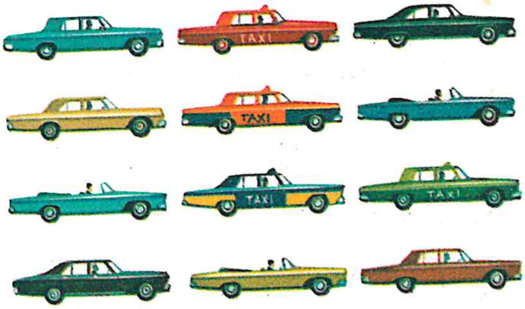


. The number line shown above as four points marked on it. The rst point at the left is matched with e number 0. Should the point arked *A* be matched with a numr greater than 0 or a number less an 0?

7. Should point *B* on the number line above be matched with a number greater than or less than the one matched with *A*?

8. Which point should match the greatest number, *A*, *B*, or *C*?

Sets and subsets



1. What is the number of members in the set of automobiles shown in the picture above? Are all the automobiles alike? Are some of them convertibles? Are some of them taxicabs?

2. What is the number of members in the set of convertibles in the picture? Is a convertible a special kind of automobile?

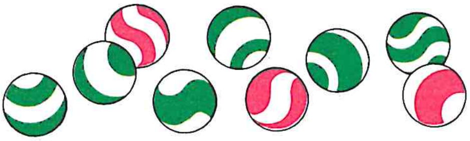
Then is each member of the set of convertibles also a member of the set of automobiles?

We say the set of convertibles is a **subset** of the set of automobiles.

3. What is the number of the set of taxicabs in the picture above? Is each member of the set of taxicabs also a member of the set of automobiles? Is the set of taxicabs a subset of the set of automobiles?

4. Is the set of taxicabs a subset of the set of convertibles? Is the set of convertibles a subset of the set of taxicabs?

5. Think of a set of ten yellow pencils. Eight of the pencils have sharp points. Two of the pencils have broken points. Is the set of pencils with sharp points a subset of the set of yellow pencils? Name one other subset of the set of yellow pencils.



6. Are some of the marbles in the set shown above green? Are all of the marbles green? What is the number of the set of green marbles? Is the set of green marbles a subset of the set of marbles? Name one other subset of the set of marbles.

7. How many marbles shown in the picture for Ex. 6 are blue? Are there any members in the set of blue marbles in the picture?

A set that has no members is called **the empty set**. The number of members in the empty set is 0 (zero).

8. Which of the following are examples of the empty set?

- the set of trucks in the picture at the top of the page
- the set of girls in your class
- the set of boys in your class
- the set of pupils in your clas who are five years old

Sets and subsets of numbers

1. Suppose you are counting the cars that pass by a corner. What number do you assign to the first car? the second? the third? Is there any end to the numbers you can use to count the cars? This set of numbers is the set of **counting numbers.** We show it like this.

{1, 2, 3, 4, 5, · · ·}

What do the three dots written after the 5 mean?

2. Look at this set of numbers.

{0, 1, 2, 3, 4, 5, · · ·}

What is the next member in this set? the next? Is there any end to the numbers in this set? This set is called the set of **whole numbers.**

If you are told a whole number, what can you do to name the next greater whole number?

What member of the set of whole numbers is not a member of the set of counting numbers?

3. Does the set of numbers shown below continue without end? How do you know?

{0, 2, 4, 6, 8, · · ·}

Is 0 the product of 0 and 2? Is 2 the product of 1 and 2? Four is the product of 2 and ?. Six is 3 × ?. Is each number in {0, 2, 4, 6, 8, · · ·} the product of 2 and some whole number? This set of numbers is called the set of **even numbers.**

4. If you are told an even number, can you tell the next greater even number? Explain how you can do it.

5. Is the set of even numbers a subset of the set of whole numbers?

— Which of the following are subsets of the set of even numbers?

	a	**b**
6.	{2, 3, 4, 5}	{2, 4, 6, 8}
7.	{0, 6, 12, 18}	{5, 10, 15, 20}

8. Study this set of numbers.

{1, 3, 5, 7, 9, · · ·}

Is there any end to this set of numbers? Is each member of this set a whole number?

In {1, 3, 5, 7, 9, · · ·}, is the first member an even number? Is the second member? Is any member? This set is called the set of **odd numbers.** Any whole number that is not an even number is an odd number.

9. If you are given the name of an odd number, how can you find the next greater odd number?

— Each set below is a subset of the set of whole numbers. Which are also subsets of the set of even numbers? Which are subsets of the set of odd numbers?

10.	{16, 22, 48}	{5, 15, 25}
11.	{0, 2, 4, 5}	{3, 9, 15}
12.	{88, 44, 22}	{3, 6, 9, 12}

Patterns in sets of numbers

1. Think of an even number. Add 1 to it. Is the sum an even number or an odd number? Think of another even number. Add 1 to it. Is the sum even or is it odd?

If you add 1 to any even number, do you think the sum will always be an odd number?

2. Think of an odd number. Add 1 to it. Is the sum an even number or an odd number?

When you add 1 to any odd number, is the sum ever an odd number?

3. Add 2 to any even number. Is the sum even or is it odd?

Will the sum of 2 and an even number always be an even number?

4. Add 2 and any odd number. Is the sum an even number or an odd number?

Will the sum of 2 and an odd number always be an odd number?

5. Add two even numbers. Is the sum even or odd? Add two other even numbers. Is the sum even or odd?

Do you think the sum of two even numbers will always be even?

6. Before you add two odd numbers, guess whether the sum will be even or odd. Now add two odd numbers. Is the sum even or is it odd?

Will the sum of two odd numbers ever be an odd number?

7. Now add an even number and an odd number. Add another even number and another odd number.

Do you think that the sum of an even number and an odd number will always be even or will it always be odd?

8. When any even number is multiplied by 2, is the result an even number or is it an odd number? When any odd number is multiplied by 2, is the result an even number or is it an odd number? If any whole number is multiplied by 2, is the result even or is it odd?

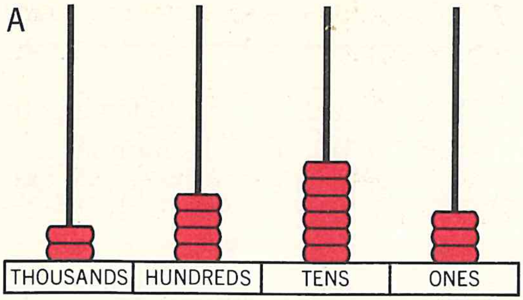
9. If any whole number is multiplied by an even number, is the result even or is it odd?

10. If any odd number is multiplied by another odd number, is the result an even number or is it an odd number?

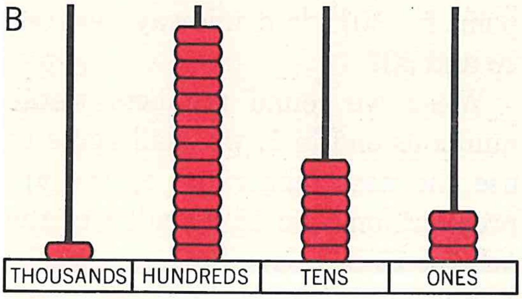
— Tell whether each number named below is an even number or an odd number.

	a	b
11.	527 + 163	27 × 59
12.	452 × 697	413 + 3,274
13.	718 + 846	904 × 38
14.	245 × 314	698 + 711
15.	339 + 912	736 × 245

Renaming numbers



1. The number shown on abacus A above is 2,463. Explain how the beads on the abacus show that we can name 2,463 by saying 2 thousands 4 hundreds 6 tens 3 ones.



2. Compare abacus B with abacus A. Does 1 thousand 14 hundreds name the same number as 2 thousands 4 hundreds? Then does abacus B show the same number as abacus A?

Is 1 thousand 14 hundreds 6 tens 3 ones another way to name 2,463?

3. Draw an abacus and mark beads on the wires to show that still another name for 2,463 is 1 thousand 13 hundreds 16 tens 3 ones.

4. Which of the following decimal numerals names the same number as 4 hundreds 18 tens 6 ones?

486 4,186 586

— Write a decimal numeral for each number named below.

5. 15 tens 4 ones

6. 8 tens 16 ones

7. 7 hundreds 16 tens 19 ones

8. How many pennies are equal in value to one dime? How many dimes are equal in value to one dollar? Does 2 dollars 8 dimes 4 pennies have the same value as 1 dollar 18 dimes 4 pennies? as 2 dollars 7 dimes 14 pennies?

9. One name for 284 is 2 hundreds 8 tens 4 ones. Some other names are 1 hundred ? tens 4 ones, and 2 hundreds 7 tens ? ones, and ? tens 4 ones.

— Copy and complete.

10. 32 = ? tens 12 ones

11. 58 = 4 tens ? ones

12. 275 = 2 hundreds 6 tens ? ones

13. 275 = 1 hundred ? tens 15 ones

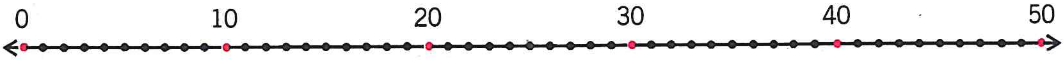
14. 275 = ? tens 25 ones

15. Three names for one number are written below. Can you think of at least 10 other names for that number?

234 234 ones

2 hundreds 3 tens 4 ones

Approximations for numbers



1. Jim had 80 cents. He wanted to buy a book for 49 cents and a pen for 19 cents. To find out if he had enough money, he thought: "49 cents is almost 50 cents, and 19 cents is almost 20 cents. So I need about 50 + 20, or ?, cents."

Did Jim have enough money to buy the book and the pen?

To see if he had enough money, Jim **approximated** the costs of the two items. Using approximations for numbers is often called **rounding** numbers.

2. On the number line shown at the top of the page, find the point that represents 17. Is the point nearer to the point for 10 or to the point for 20? Is 10 or is 20 a better approximation for 17?

3. Find the point on the number line that represents 12. Is this point nearer to the point for 10 or to the point for 20? Is 10 or is 20 a better approximation for 12?

4. When we use some number of tens as an approximation for a number, we say we are rounding the number to the nearest ten. Is 38 nearer to 3 tens or to 4 tens? Then 38 rounded to the nearest 10 is 40.

— Round each of the following to the nearest 10. Use the number line at the top of the page if necessary.

	a	b	c	d	e
5.	19	21	28	32	44
6.	27	13	41	39	16
7.	36	54	8	92	58

8. Find the point on the number line for 25. Is this point nearer to the point for 20 than it is to the point for 30? Is it midway between 20 and 30?

When we round numbers whose numerals end in 5, we shall agree to use the next higher ten as the approximation. So 25 rounded to the nearest 10 is 3 tens, or ?.

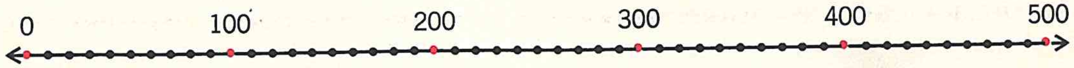
— Round each of the following to the nearest 10.

9.	45	38	64	85	31
10.	72	94	15	132	165
11.	59	97	5	128	315

— Round each of the following to the nearest 10¢.

12.	9¢	12¢	25¢	18¢	21¢
13.	34¢	46¢	41¢	49¢	35¢
14.	62¢	54¢	75¢	83¢	98¢

More approximations



1. Each point marked on the number line above represents some number of tens. Find the point for 120. Is this point nearer to the point for 100 or to the point for 200? Then is 100 or is 200 a better approximation for 120?

120 rounded to the nearest hundred is ? .

2. Is the point for 380 nearer to the point for 300 or to the point for 400? Then which is a better approximation for 380: 300 or 400?

380 rounded to the nearest hundred is ? .

3. Is the point for 250 midway between the point for 200 and the point for 300? The next higher hundred shall be used as the approximation for a number whose numeral ends in 50.

250 rounded to the nearest hundred is 300. 450 rounded to the nearest hundred is ? .

— Round each of the following to the nearest hundred.

	a	b	c	d	e
4.	180	240	450	130	380
5.	225	372	198	417	583
6.	720	651	77	849	919

7. Copy the number line shown above, but in place of 100, 200, 300, 400, and 500, write 1,000, 2,000, 3,000, 4,000, and 5,000.

8. Is the point for 1,400 nearer to 1,000 than it is to 2,000? Then 1,400 rounded to the nearest thousand is ? .

9. Use your drawing of the number line to tell whether 2,600 rounded to the nearest thousand is 2,000 or 3,000.

10. The next higher thousand shall be used as the approximation for a number whose numeral ends in 500.

3,500 rounded to the nearest thousand is 4,000. What is 4,500 rounded to the nearest thousand?

— Round to the nearest thousand.

	a	b	c
11.	700	1,750	2,410
12.	6,800	8,500	9,200
13.	1,500	5,240	2,449

— Round to the nearest ten dollars; nearest hundred dollars; nearest thousand dollars.

14.	$1,483	$964	$1,850
15.	$6,359	$3,149	$4,500
16.	$5,200	$7,506	$7,466

Using approximations

1. Donald read that the distance to the moon is 238,857 miles. This distance rounded to the nearest ten thousand miles is ? miles.

Which is easier for you to remember: 238,857 miles or the approximation to the nearest ten thousand miles?

2. The distance to the sun is 92,897,416 miles. What is an approximation for this distance to the nearest million miles?

Which is easier to remember, the number of miles to the sun, or the approximation for that number of miles?

3. The distance around the earth at the equator is 24,902 miles. What is this distance rounded to the nearest thousand miles?

4. Round each of the following to the nearest ten cents; to the nearest dollar; to the nearest ten dollars.

$65.82 $34.43 $47.95 $23.50

5. Jim needs batteries for his new model rocket motor. He has a dollar to pay for 5 batteries at 19¢ each.

He estimates to find if he has enough money for the batteries: "19¢ is about 20¢. 5 × 20¢ is 100¢." Does Jim have enough money to buy the batteries?

Are approximations useful in estimating answers quickly?

6. Mark has $5.00. Estimate to tell quickly which of the following he can buy.

- a slide viewer for $2.98 and a set of slides for $1.49
- two model airplane kits at $2.98 each
- a canteen for $1.89 and a penknife for $3.95

7. Would an estimate of the amount of money be good enough if you were

- sending money to cover the postage and cost of 3 sets of dress patterns ordered by mail?
- in a store wondering how much you would have left from $5.00 if you bought a slide viewer for $3.95?

8. One year there were 22,000,000 television sets in the United States. Later there were 42,000,000 sets. Do you think these numbers are approximations?

Were there nearly twice as many or nearly three times as many sets in the later year?

Do approximations make the comparison of large numbers easy?

9. One baseball park seats 29,980 fans. Another seats 59,970. Use approximations to tell whether the second park seats about twice as many or about three times as many fans.

Number sentences

1. Read the following sentences.

a. September has thirty days.
b. June has thirty-one days.
c. That month has thirty days.

- Is sentence **a** true or is it false? Is sentence **b** a true sentence or a false sentence?
- Is sentence **c** a true sentence? Is it a false sentence? What must you know before you can tell whether it is true or false?

2. Suppose we replace "That month" in sentence **c**, Ex. 1, with "April." Is the new sentence true? Would sentence **c** be true if we replaced "That month" with "July"? What other names of months could we use as replacements to make sentence **c** a true sentence?

Sentences like **c** are called **open sentences.** Sentence **c** is neither true nor false until we know what "that month" represents.

As you know, sentences that are used in work with numbers are called **number sentences.** Like other sentences, number sentences can be true, they can be false, or they can be open — that is, neither true nor false.

3. The symbol $\neq$ means "is not equal to."

What does the symbol $=$ mean? the symbol $>$? the symbol $<$?

— Which of the following number sentences are true? Which are false? Which are open sentences?

	a	b
4.	$3 + 2 = 5$	$6 + 8 > 10$
5.	$7 - 4 = 8$	$9 - 5 < 3$
6.	$5 + \square = 9$	$2 + \square \neq 4$

Number sentences such as those in column **a** are called **equations.** Number sentences such as those in column **b** are called **inequalities.**

— Which sentences below are equations? Which are inequalities?

7. $12 + 17 = 29$ $15 + 28 < 45$
8. $32 - n = 14$ $64 \div \square > 4$

In mathematics, symbols such as $\square$, $\triangle$, and n are used to stand for some number. Each symbol can be read as "some number." When the same symbol is used more than once in a number sentence, it stands for the same number each time.

— Read each sentence below. Then tell one whole number that will make the sentence true.

9. $3 + 7 = \square$ $3 \times 9 < n$
10. $n + n = 12$ $4 - 3 > \triangle$
11. $\triangle + \triangle + \triangle = 6$ $\square \times 6 \neq 18$

12. Which of the number sentences in Ex. 9–11 are equations? Which are inequalities? Did you find that sometimes more than one number will make an inequality true?

Open sentences and solution sets

1. Is this number sentence true, false, or open? $8 - 4 > \square$ Will this number sentence be true if the $\square$ is replaced with 0? 1? 2? 3? 4? 5? Is this number sentence true for only one whole number? for more than one whole number? Is

$$\{0, 1, 2, 3\}$$

the set of whole numbers that make this sentence true?

2. Is this number sentence true for only one whole number?

$$2 + 3 > \triangle$$

How many whole numbers will make this number sentence true?

Which of the following is the set of whole numbers that make this sentence true? $2 + 3 > \triangle$

$\{1, 2, 3, 4\}$ $\quad$ $\{1, 2, 3, 4, 5\}$
$\{6, 7, 8, 9\}$ $\quad$ $\{0, 1, 2, 3, 4\}$

3. How many whole numbers will make this sentence true?

$$3 + 6 = n$$

Then how many members are there in the set of whole numbers that make this sentence true?

When you find the set of numbers that will make an open sentence true, you solve the sentence. The set of numbers that make the sentence true is called the solution set of the sentence.

— Solve each of the following number sentences — that is, find the solution set for each.

	a	b
4.	$12 - 7 > n$	$\square + 4 < 9$
5.	$15 + 5 = \triangle$	$3 - n > 2$
6.	$16 \div 8 > \square$	$18 - 12 = \triangle$

7. Is there any whole number that will make this number sentence true? $4 + n < 2$ Then is the solution set for this sentence the empty set? We write the symbol $\{\ \}$ to show the empty set.

8. Study this inequality.

$$3 + 2 < n$$

Will 6 make this sentence true? Will 7? 8? 9? 10? Is there any end to the members of the solution set? We can express the solution set for this sentence by writing $\{6, 7, 8, \cdots\}$.

— What is the solution set for each of the following number sentences? Write your answers.

9.	$14 - 7 = n$	$12 \div 6 > \square$
10.	$15 - 27 = \triangle$	$4 - n > 2$
11.	$16 + \square = 20$	$5 + 7 > n$
12.	$18 - 14 < \triangle$	$2 + \square < 12$
13.	$21 - \triangle = 14$	$3 - 8 = n$
14.	$12 + 8 > \square$	$5 \times 2 < \triangle$

800
30
830
4

IT'S CHECK–UP TIME

1. Each place in a decimal numeral has a value $\underline{?}$ times the value of the place at its right.

2. In the numeral 421,685, what does the 8 mean? the 6? the 2? the 4? the 1?

3. Write a 3-digit numeral for each number named below.

- $(8 \times 100) + (3 \times 10) + (4 \times 1)$
- $(7 \times 100) + (0 \times 10) + (8 \times 1)$

4. How many times is 10 used as a factor to get 1,000,000,000?

5. Fred's father paid $1,000 in taxes last year. How many people would have to pay $1,000 in taxes to make a total of one billion dollars?

6. Write a Hindu-Arabic numeral for each number named below.

VII   IX   XXVI   XL   DCL

7. Write a Roman numeral for each umber named below.

28   34   52   110   549

— Write the names of the next two embers in each set of numbers hown below.

8. $\{2, 4, 6, 8, \cdots\}$

9. $\{5, 10, 15, \cdots\}$

0. $\{4, 8, 12, 16, \cdots\}$

1. On a number line, point *A* is a reater distance to the right of zero han point *B*. Which point, *A* or *B*, epresents the greater number?

12. What do we call the set that has no members?

13. Look at this set of numbers.

$\{0, 1, 2, 3, 4, 5\}$

Which of the following sets are subsets of this set of numbers?

$\{0, 2, 4\}$ $\{1, 3, 5, 7\}$
$\{2, 4, 6\}$ $\{1, 2, 3, 4\}$

— Write the missing numerals.

14. $47 = \underline{?}$ tens 17 ones

15. $52 = 4$ tens $\underline{?}$ ones

16. $185 = 1$ hundred $\underline{?}$ tens 15 ones

17. $342 = 2$ hundreds $\underline{?}$ tens 2 ones

18. Round each of the following to the nearest thousand.

3,419   4,500   2,641   8,062   1,936

— From the set of whole numbers, list the solution set for each of the following open sentences.

	a	b
19.	$3 \times \square = 12$	$n + 5 < 10$
20.	$\triangle - 2 > 8$	$36 \div n = 6$

If you have time, try these —

— If $\triangle$ and $\square$ represent whole numbers, will the solution set for each sentence shown below be a subset of the set of even numbers or will it be a subset of the set of odd numbers?

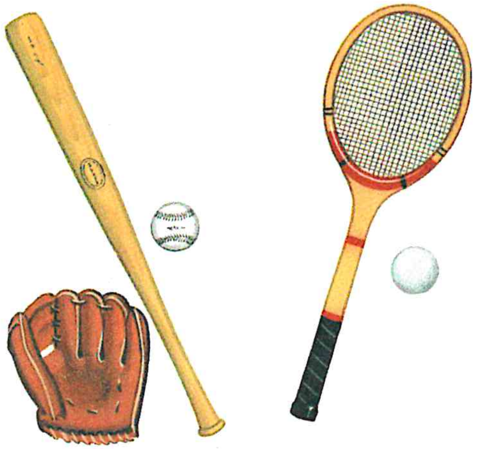
21. $483 \times \triangle = 102{,}879$

22. $483 + \square = 102{,}879$

2
102879
483

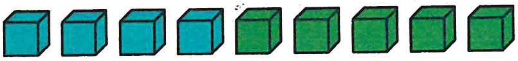
CHAPTER 2

Addition of whole numbers



1. Each set shown above is made up of pieces of sports equipment. What is the number of members in the set at the left? in the set at the right?

If we join or unite the two sets into one set, how many pieces of equipment will we have in the one set? What whole number makes this sentence true? $3 + 2 = n$



2. What is the number of the set of blue blocks shown above? What is the number of the set of green blocks? If we unite these two sets into one set, what is the total number in the union of the two sets? $4 + 5 = \underline{?}$

3. Here are two ways to show the addition of 4 and 5. The 4 and the 5 are called **addends.** The 9 is called the **sum.**

$4 + 5 = 9$

$$\begin{array}{r} 4 \\ +\,5 \\ \hline 9 \end{array}$$

In the number sentence below, what are the addends? What is the sum? $8 + 3 = 11$

4. Could you use addition to find the total cost of two items when you know the cost of each?

If a bicycle basket costs \$3 and a tire costs \$7, how much do both items cost?

Can you find the answer by solving this sentence? $3 + 7 = n$

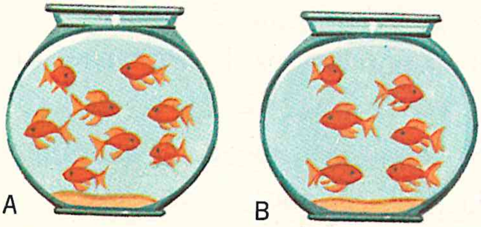
5. Could you use addition to find how far you have traveled in 2 days if you know how far you have gone each day?

If you ride 300 miles on Friday and 400 miles on Saturday, how many miles have you traveled?

Can you find the answer by solving this sentence? $300 + 400 = n$

6. Give an example of a problem in which you would use addition to find the answer. Then write a number sentence you could use to find the answer. Solve the sentence.

Addition and the commutative property



1. Jerry has 8 goldfish in fishbowl A. Mike has 6 in fishbowl B. They want to put them all in one bowl. If Mike puts his 6 in bowl A, what will be the number of the set of fish in fishbowl A? $8 + 6 = \underline{?}$

If Jerry puts his 8 fish in fishbowl B with Mike's 6 fish, what will be the number of the set of fish in fishbowl B? $6 + 8 = \underline{?}$

Is this number sentence true?

$$8 + 6 = 6 + 8$$

— Solve each of these sentences.

2. $3 + 9 = 9 + n$

3. $4 + 6 = \triangle + 4$

4. If $\square$ and $\triangle$ stand for any whole numbers, is this sentence always true? $\square + \triangle = \triangle + \square$

Your discovery

Tell your discovery

Does changing the order of the addends change the sum? This idea s called the **commutative property f addition.**

Use your discovery

$92 + 379 = 871$, so $379 + 492 = \underline{?}$

— Solve these open sentences.

5. $1{,}378 + 2{,}462 = n + 1{,}378$

6. $\square + 8{,}726 = 8{,}726 + 7{,}439$

7. Can you write an addition sentence in which the sum is the same as one of the addends? Try it.

— Solve these pairs of sentences.

8. $7 + 0 = n$ $\quad$ $0 + 7 = n$

9. $4 + 0 = \square$ $\quad$ $0 + 4 = \square$

10. $12 + 0 = \triangle$ $\quad$ $0 + 12 = \triangle$

11. What is the sum of 8 and 0? of 14 and 0? of 362 and 0?

Your discovery

Tell your discovery

Tell what you know about the sum of two addends if one of the addends is 0 (zero).

Use your discovery

- $1{,}492 + 0 = \underline{?}$
- $3{,}876 + \underline{?} = 3{,}876$

12. What is the number of members in the union of the empty set with a set that has 12 members? with a set that has 3,000 members?

— Solve each of these sentences.

	a	b
13.	$462 + 0 = n$	$375 + \square = 375$
14.	$n + 0 = 536$	$0 + 284 = \triangle$
15.	$0 + 1 = \square$	$0 + 0 = n$

Addition and the associative property

1. When you are finding the sum of more than two numbers, you add only two numbers at a time.

$$(4 + 2) + 3 = n$$

In the sentence above, the parentheses tell which two numbers to add first. $4 + 2 = \underline{?}$ What is the sum of this number and 3? Then what number makes this sentence true? $(4 + 2) + 3 = n$

2. Now look at this number sentence. $4 + (2 + 3) = \square$ What two numbers are you to add first? $2 + 3 = \underline{?}$ What is the sum of 4 and 5? Then what number makes this sentence true?

$$4 + (2 + 3) = \square$$

3. Is this number sentence true?

$$(4 + 2) + 3 = 4 + (2 + 3)$$

— Solve these pairs of sentences.

4. $(5+2)+8=n$ $\quad$ $5+(2+8)=n$

5. $(6+7)+3=\square$ $\quad$ $6+(7+3)=\square$

6. $(4+5)+9=\triangle$ $\quad$ $4+(5+9)=\triangle$

Your discovery

Tell your discovery

Tell what you know about pairing neighboring addends to find a sum. This idea is called the **associative property of addition.**

Use your discovery

$$(372 + 568) + 495 = 1{,}435,$$

so $372 + (568 + 495) = \underline{?}$

7. If □, △, and ◇ stand for any whole numbers, is this sentence true?

$$(\square + \triangle) + \Diamond = \square + (\triangle + \Diamond)$$

8. Number expressions that name the same number are **equivalent.**

- Are $3 + 5$ and $5 + 3$ equivalent?
- Are $(8 + 2) + 5$ and $8 + (2 + 5)$ equivalent?

9. How are the following expressions alike? How do they differ? Are they equivalent?

$2+(3+5)$	$2+(5+3)$	$(2+3)+5$
$3+(2+5)$	$3+(5+2)$	$(3+2)+5$
$(3+5)+2$	$(2+5)+3$	$5+(3+2)$
$5+(2+3)$	$(5+3)+2$	$(5+2)+3$

10. To find the sum of two or more numbers, can we group and add the numbers in any order we wish?

11. To find the sum of the numbers shown in the example at the right, start at the top and add down. Think: 5 plus 2 equals 7; 7 plus 6 equals $\underline{?}$.

$$\boxed{\begin{array}{r} 5 \\ 2 \\ \underline{+\,6} \end{array}}$$

If you start at the bottom and add up, should you get the same sum? Why?

12. Copy each example below. Add down to find the sum. Check your answer by adding up.

a	b	c	d	e
8	3	2	7	3
1	2	5	2	6
+ 4	+ 8	+ 4	+ 6	+ 4

Adding with hundreds, tens, and ones

1. Since 6 + 2 equals 8, is 26 + 2 equal to 28? Use what you know about 6 + 2 to help you find these sums.

$46 + 2 = \underline{\ ?\ }$ $76 + 2 = \underline{\ ?\ }$

$136 + 2 = \underline{\ ?\ }$ $206 + 2 = \underline{\ ?\ }$

2. What is 4 + 3? How does the sum of 4 and 3 help you complete this sentence? $34 + 3 = \underline{\ ?\ }$ Can you use the sum of 4 and 3 to help you find these sums?

$54 + 3 = \underline{\ ?\ }$ $74 + 3 = \underline{\ ?\ }$

$124 + 3 = \underline{\ ?\ }$ $244 + 3 = \underline{\ ?\ }$

— Find the sums.

	a	b	c	d
3.	$\begin{array}{r} 5 \\ +2 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ +2 \\ \hline \end{array}$	$\begin{array}{r} 85 \\ +2 \\ \hline \end{array}$	$\begin{array}{r} 165 \\ +2 \\ \hline \end{array}$
4.	$\begin{array}{r} 1 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 31 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 51 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 191 \\ +8 \\ \hline \end{array}$

5. Can you think of 20 + 70 as 2 tens + 7 tens? What is 2 + 7? Then what is 2 tens + 7 tens?

$$20 + 70 = \underline{\ ?\ }$$

6. Can you think of 24 as 2 tens 4 ones? as 20 + 4?

Show two ways to think about 73.

7. Copy and complete the sentences shown below.

$$\begin{aligned} 24 + 73 &= (20 + 4) + (70 + 3) \\ &= (20 + 70) + (4 + \underline{\ ?\ }) \\ &= \quad 90 \quad + \quad \underline{\ ?\ } \\ &= \underline{\ ?\ } \end{aligned}$$

8. When we add numbers such as 62 and 25, do we add the ones separately and the tens separately? Copy and complete the example below.

$$\begin{array}{r} 62 = 60 + 2 \\ \underline{+\,25} = \underline{20 + 5} \\ \boxed{?}\boxed{?} + 7, \text{ or } \underline{\ ?\ } \end{array}$$

9. Three ways to show the addition of 37 and 52 are given below. Explain each.

$$\begin{array}{lll} \text{3 tens 7 ones} & 30 + 7 & \quad 37 \\ \underline{\text{5 tens 2 ones}} & \underline{50 + 2} & \underline{+\,52} \\ \text{8 tens 9 ones,} & 80 + 9, & \quad 89 \\ \quad \text{or 89} & \quad \text{or 89} & \end{array}$$

— Copy and find the answers.

	a	b	c	d
10.	$\begin{array}{r} 48 \\ +\,41 \\ \hline \end{array}$	$\begin{array}{r} 63 \\ +\,24 \\ \hline \end{array}$	$\begin{array}{r} 37 \\ +\,22 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ +\,54 \\ \hline \end{array}$
11.	$\begin{array}{r} 32 \\ +\,33 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ +\,12 \\ \hline \end{array}$	$\begin{array}{r} 31 \\ +\,53 \\ \hline \end{array}$	$\begin{array}{r} 42 \\ +\,42 \\ \hline \end{array}$

12. Copy and complete.

$$\begin{array}{r} 324 = 300 + 20 + 4 \\ \underline{+\,472} = \underline{400 + 70 + 2} \\ \boxed{?}\boxed{?}\boxed{?} + \boxed{?}\boxed{?} + \boxed{?}, \text{ or } \underline{\ ?\ } \end{array}$$

— Copy and find the answers.

13.	$\begin{array}{r} 368 \\ +\,421 \\ \hline \end{array}$	$\begin{array}{r} 239 \\ +\,150 \\ \hline \end{array}$	$\begin{array}{r} 562 \\ +\,317 \\ \hline \end{array}$	$\begin{array}{r} 834 \\ +\,154 \\ \hline \end{array}$
14.	$\begin{array}{r} 666 \\ +\,302 \\ \hline \end{array}$	$\begin{array}{r} 742 \\ +\,105 \\ \hline \end{array}$	$\begin{array}{r} 961 \\ +\ \ 38 \\ \hline \end{array}$	$\begin{array}{r} 275 \\ +\,414 \\ \hline \end{array}$

If you need more practice, do Practice Sets 1 and 2, page 324.

Solving problems

1. Jim has 37 marbles in one bag and 52 marbles in another bag. How many marbles does Jim have in the two bags?

- Can this number sentence be used to solve the problem?

$$37 + 52 = n$$

- Can this number sentence also be used to solve the problem?

$$52 + 37 = n$$

- Use either sentence to find the correct answer to the problem.

2. Mary had saved 24 Lincoln head pennies. Her father gave her 12 more Lincoln head pennies. Then how many did Mary have?

- Can you use this number sentence to solve the problem?

$$24 + 12 = n$$

- Write another number sentence that can be used to solve the problem.
- Use either sentence to find the correct answer to the problem.

— Write two number sentences that can be used to solve each problem in Ex. 3–9. Use either one of the sentences to find the answer.

3. In his collection, Sam has 21 baseball cards and 34 football cards. How many cards are there in Sam's collection?

4. Fifteen children are seated in a school bus. There are 33 empty seats. How many seats are there in this school bus?

5. To raise funds for the school library, the fifth-grade class sold magazine subscriptions for two weeks. The first week they sold 235 subscriptions. The second week they sold 152 subscriptions. How many subscriptions did they sell during the two-week period?

6. In a football game, the Blue team gained 13 yards on one play, 4 yards on the next play, and 22 yards on the third play. How many yards did the Blue team gain on the three plays?

7. The two fifth-grade classes at Whitestone School are going on a trip to the museum. There are 28 children in one class and 31 in the other. How many children are going on the trip?

8. Amy baked 42 lemon cookies, 100 oatmeal cookies, and 54 chocolate cookies. How many cookies did she bake all together?

9. The school library checked out 375 books in September, 421 in October, and 802 in November. How many books were checked out during the three-month period?

Renaming numbers in addition

1. We often have to think of numbers in different ways when adding. Three ways to show the addition of 69 and 25 are given below.

A
6 tens 9 ones
2 tens 5 ones
8 tens 14 ones, or
9 tens 4 ones, or _?_

B
60 + 9
20 + 5
80 + 14, or
90 + 4, or _?_

C
$$\begin{array}{r} {\scriptstyle 1} \\ 69 \\ +\,25 \\ \hline 94 \end{array}$$

- In A, 69 is renamed as 6 tens 9 ones; 25 is thought of as 2 tens _?_ ones. What does 9 ones + 5 ones equal? What does 6 tens + 2 ens equal? Is 9 tens 4 ones another name for 8 tens 14 ones? Write a two-digit numeral for 9 tens 4 ones.
- Explain why 94 is the correct answer for B.
- In C, some of the computation is done mentally. The ones are added first. 9 + 5 = _?_ The 4 is written in the ones column. The 1 ten is added to the 6 tens and 2 tens. (1 + 6) + 2 = _?_ Then is 9 the correct digit to write in the tens column?

2. Two ways to show the addition of 368 and 275 are given below. Explain each. Tell what numbers to use to complete the examples.

A
3 hundreds 6 tens 8 ones
2 hundreds 7 tens 5 ones
5 hundreds 13 tens 13 ones, or
5 hundreds _?_ tens 3 ones, or
_?_ hundreds 4 tens 3 ones, or 643

B
300 + 60 + 8
200 + 70 + 5
500 + 130 + 13, or
500 + _?_ + 3, or
_?_ + 40 + 3, or 643

3. In the partially completed example at the right, explain why there should be a 4 in the tens place in the answer. Copy and complete the example.

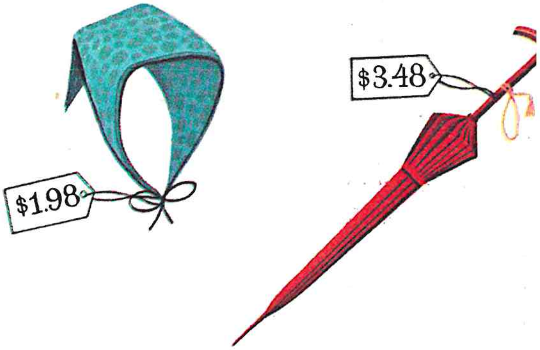
$$\begin{array}{r} {\scriptstyle 1\,1} \\ 368 \\ +\,275 \\ \hline \end{array}$$

4. Copy and find the sums.

a	b	c	d	e	f
68	256	685	7,946	3,702	6,085
+ 37	+ 698	+ 278	+ 5,285	+ 4,969	+ 3,948

If you need more practice, do Practice Set 3, page 324.

Sums involving money



1. Mary wishes to buy a rain scarf that costs $1.98 and an umbrella that costs $3.48. How much money does Mary need to buy both items?

- You can write this number sentence for the problem.

$$198 + 348 = n$$

- When you add, you will add the numbers 198 and 348. But write the example as shown at the right, with the dollar sign and cents points to show that the numbers stand for amounts of money. Notice that the dollar sign is written only with the top numeral. The cents point is in both numerals.

$$\begin{array}{r} \$1.98 \\ +\ 3.48 \\ \hline \end{array}$$

- Find the sum of 198 and 348. Write a dollar sign and a cents point to show that your answer stands for money.

If Mary has $5.00, does she have enough money to buy both the scarf and the umbrella?

— Copy, find the answers, and check. Be sure to write a dollar sign and cents point in the answers that stand for money.

	a	b	c	d	e	f
2.	26 + 87	88 + 54	976 + 268	$4.87 + 6.58	5,079 + 478	$69.48 + 23.71
3.	89 75 + 69	63 475 + 189	$6.79 .50 + 8.64	$7.42 .04 + 40.00	$8.64 .50 + .09	421 789 + 206
4.	52 35 + 72	644 835 + 170	974 658 + 372	408 66 + 960	$62.78 4.96 + 16.58	$85.39 4.76 + 16.97
5.	500 875 673 + 589	$9.80 5.07 8.95 + 7.54	$2.46 8.73 7.69 + 5.08	79 105 658 + 37	805 900 876 + 549	$10.00 5.98 1.09 + .67

Estimating sums

1. Before he adds, Robbie estimates the sum to get an idea of what his answer should be. For each addition example, he rounds the addends and finds the sum of these approximations. Then he finds the sum for the example itself. He compares the sum for the example with the estimate to see if his answer is sensible.

	EXAMPLE	ESTIMATE
A	78	80
	42	40
	+ 56	+ 60
	176	180
B	$159.60	$200
	215.70	200
	+ 320.18	+ 300
	$???.??	$700

- In A, Robbie rounded each addend to the nearest 10. Then he added these numbers to find an estimate of the sum. What is his estimate? What sum did he find for the example? Does his estimate show that the sum he found for the example is sensible?
- In B, Robbie rounded each addend to the nearest hundred dollars. What is his estimate of the sum for the example? Copy the example and find the sum. Use Robbie's estimate to see if your answer is sensible.

2. Before finding the sum for the example at the right, Bonnie rounded each addend to the nearest ?. What is her estimate of the sum? Does this mean that the answer to the example should be about 10,000?

EXAMPLE	ESTIMATE
6,305	6 thousand
878	1 thousand
46	0 thousand
+ 2,954	+ 3 thousand
?,???	10 thousand

- Bonnie found the sum of the numbers in the example. She says it is 8,183. Is this anwer sensible? How do you know?
- Copy the example and find the sum. Compare the sum you find with Bonnie's estimate. Is your answer sensible?

3. Estimate the sum for each example below. Then copy the xample and find the sum. Use your estimates to see if your answers re sensible.

a	b	c	d	e	f	g
30	49	386	199	3,412	$540.75	$1,325.48
28	82	515	250	2,967	375.78	3,162.15
+ 59	+ 78	+ 609	+ 79	+ 5,025	+ 628.15	+ 6,891.28

How well can you add?

— These examples will help you find out how well you can add. Copy each example and find the sum. Add down; to check, add up.

	a	b	c	d	e	f
1.	72 + 77	82 + 76	59 + 58	$1.37 + 2.59	54 67 + 89	85 8 + 67
2.	384 86 5 + 479	577 30 842 + 586	27 6,350 287 + 4,983	3,256 60 697 + 9,854	$19.00 83.52 9.76 + 83.94	$12.25 6.98 10.50 + 9.37
3.	$1.53 4.98 6.07 5.39 + .20	$5.98 .46 2.97 .08 + 6.35	$279.00 2.00 564.75 .82 + 46.73	$248.65 2.98 413.53 5.48 + 102.76	$ 9.58 3.57 46.24 .93 + 7.76	7,840 623 205 9,117 + 458

— These exercises will help you to add mentally.

4. To add 20 and 17, think: 20 plus 10 is 30, plus 7 more is ?. What is the sum of 30 and 18? of 40 and 22? of 60 and 37? of 54 and 20? of 17 and 40?

5. To add 45 and 14, think: 45 plus 10 is 55, plus 4 more is ?. What is the sum of 43 and 12? of 27 and 11? of 36 and 13? of 42 and 26? of 53 and 35?

6. To add 25 and 37, think: 20 plus 30 is 50; 5 plus 7 is 12; 50 plus 12 is ?.

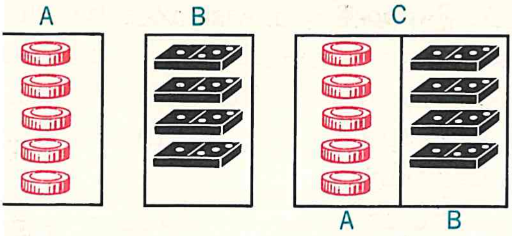
What does 15 + 17 equal? What does 28 + 34 equal? 14 + 58?

7. Tell these sums.

28 + 65	42 + 41	87 + 14	56 + 39	95 + 66	78 + 37

If you need more practice, do Practice Set 4, page 325.

The meaning of subtraction



1. Suppose set A is united with set B. Does the picture of set C show the union of set A with set B? What s the number of members in set A? What is the number of members in set B? What is the number of members in set C?

When we form the union of two different sets and find the number of members in the union of the two sets, we have added the number of members in one set to the number of members in the other set.

2. Study these number sentences.

a. $5 + \square = 9$ **b.** $\triangle + 4 = 9$

In sentence **a**, 9 is the sum and 5 is one of the addends. What number is the other addend?

In sentence **b**, what is the sum? Is an addend? What is the other ddend?

The operation of finding an ad-end when the sum and one addend re known is called *subtraction*.

- Solve these sentences.

	a	b
3.	$2 + n = 5$	$8 + n = 12$
4.	$n + 6 = 13$	$n + 3 = 8$

5. Study these two number sentences.

a. $n + 7 = 12$ **b.** $12 - 7 = n$

What addend makes sentence **a** true? Sentence **b** is a subtraction sentence that is related to sentence **a**. What number makes sentence **b** true? Then is 5 another name for $12 - 7$?

— Find the addends that make these sentences true.

	a	b
6.	$9 + \triangle = 12$	$12 - 9 = \triangle$
7.	$\square + 6 = 14$	$14 - 6 = \square$
8.	$3 + n = 7$	$7 - 3 = n$

9. In Ex. 6–8, is the subtraction sentence in column **b** related to the addition sentence in **a**? Explain.

— For each of the following, write a related subtraction sentence. Then solve the sentence.

10.	$8 + n = 13$	$4 + \triangle = 11$
11.	$\square + 7 = 16$	$n + 6 = 12$
12.	$22 + n = 22$	$46 - 0 = \square$

13. In the example shown below the number sentence in the box, what are the addends? What is the sum?

$7 - 4 = 3$

$$\begin{array}{r} 7 \\ -\ 4 \\ \hline 3 \end{array}$$

For each sentence in Ex. 10–12, write a complete example like the one shown in the box.

Inverse operations

$4 + 5 = 9 \qquad 9 - 5 = 4$

$5 + 4 = 9 \qquad 9 - 4 = 5$

1. Study the four number sentences shown above. How are they alike? How do they differ?

2. Solve each of the number sentences written below.

$7 + n = 13 \qquad 13 - n = 7$

$n + 7 = 13 \qquad 13 - 7 = n$

Did you find that the same number makes all four sentences true? Then are these sentences related?

3. Study this set of numbers: $\{7, 8, 15\}$. Write two addition sentences to show how these three numbers are related. Can you write two subtraction sentences to show how these numbers are related? Try it.

4. For a set of numbers such as $\{2, 5, 7\}$, four related number sentences can be written. Two related sentences for $\{2, 5, 7\}$ are shown below. Write the other two.

$2 + 5 = 7 \qquad 7 - 5 = 2$

5. Write four related number sentences for each set of numbers given below.

$\{6, 5, 11\} \qquad \{8, 9, 17\} \qquad \{4, 7, 11\}$

6. If you know the set of numbers involved in one sentence, can you use it to help you solve the related sentences? Try it.

$9 + 7 = 16 \qquad 16 - 7 = \triangle$

$7 + 9 = n \qquad 16 - 9 = \square$

7. Suppose you wish to solve this sentence. $17 - \triangle = 8$ Can you use any one of the following sentences to help you find the solution set? Why?

$\triangle + 8 = 17 \qquad 8 + \triangle = 17$

$17 - 8 = \triangle$

— For each sentence below, write three related number sentences. Then write the numeral for the number that makes the sentences true.

	a	b
8.	$14 - n = 8$	$\square - 5 = 6$
9.	$\triangle - 8 = 9$	$7 - n = 7$
10.	$16 - \square = 7$	$\triangle - 6 = 7$

— Solve the following sentences.

11. $(7 + 2) - 2 = n$

12. $(5 + 8) - 8 = n$

13. $(9 + 6) - 6 = n$

14. $(3 + 7) - 7 = n$

15. Does subtracting a number undo what adding that number does?

When one operation undoes what another operation does, each operation is called the **inverse** of the other. Is subtraction the inverse of addition? Explain.

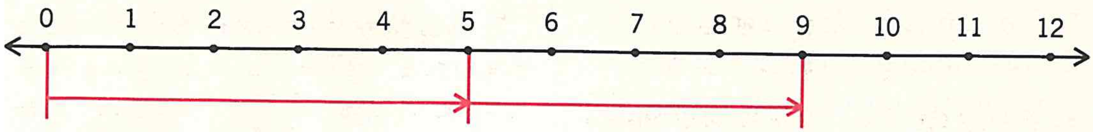
— Solve these open sentences.

16. $(27 + 15) - 15 = n$

17. $(48 + 32) - n = 48$

18. $(n + 63) - 63 = 12$

Addition and subtraction on the number line



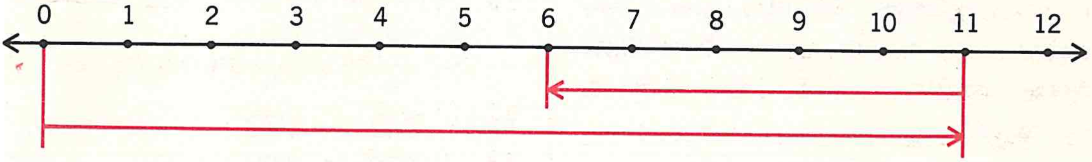
— These exercises will show you how to use the number line to show addition and subtraction.

1. To show the addition of 5 and 4 on the number line, start at 0 and draw an arrow to the right over 5 spaces. From this point, draw another arrow to the right over 4 spaces.

What numeral is at the tip of this second arrow? What is the sum of 5 and 4?

2. Draw a model of the number line and mark off points for the first twenty whole numbers. Use this model to show the addition of 7 and 8. What number is shown at the tip of your second arrow? What does 7 + 8 equal?

3. If you should forget an addition fact, such as the sum of 9 and 6, how could you use the number line to rediscover the fact?



4. If you wish to show the subtraction of 5 from 11 on the number line, start at 0 and draw an arrow to the right over 11 spaces. Is subtraction the inverse, or opposite, of addition? Then should the second arrow begin at 11 and go over 5 spaces to the right or to the left of 11? What number is shown at the tip of this second arrow?

11 − 5 = ?

5. Draw a model of the number line and show the subtraction of 7 from 15 on your model.

6. If you should forget a subtraction fact, for example, what 17 − 8 equals, how could you use the number line to rediscover the fact?

— Solve each number sentence below. Use the number line if you need help.

	a	b
7.	$14 - 5 = n$	$16 - 8 = n$
8.	$9 + 5 = n$	$13 - 7 = n$
9.	$18 - 8 = n$	$14 + 4 = n$
10.	$19 - 5 = n$	$17 - 3 = n$

Subtracting with hundreds, tens, and ones

1. You already know that you can add two numbers such as 75 and 23 by adding the ones and then the tens. Do you think you can subtract two numbers in the same way? Copy and complete the following example.

$$\begin{array}{rcl} 75 & = & 7 \text{ tens } 5 \text{ ones} \\ -\ 23 & = & 2 \text{ tens } 3 \text{ ones} \\ \hline & & ? \text{ tens } ? \text{ ones} \end{array}$$

Did you subtract 3 ones from 5 ones? 2 tens from 7 tens? Write a two-digit numeral for 5 tens 2 ones.

2. Copy the example at the right. What does 5 − 3 equal? Write a 2 in the ones column. What does 8 tens − 4 tens equal? Write a 4 in the tens column. 85 − 43 = ?

$$\boxed{\begin{array}{r} 85 \\ -\ 43 \\ \hline \end{array}}$$

3. Can you think of 145 as 14 tens 5 ones? Copy and complete each example below.

$$\begin{array}{rcl} 145 & = & 14 \text{ tens } 5 \text{ ones} \\ -\ 82 & = & 8 \text{ tens } 2 \text{ ones} \\ \hline & & ? \text{ tens } ? \text{ ones, or } ? \end{array} \qquad \begin{array}{r} 145 \\ -\ 82 \\ \hline ?? \end{array}$$

4. Copy and find the answers.

	a	b	c	d	e
	54	43	96	165	128
	− 31	− 12	− 35	− 40	− 58

5. Copy and complete this example.

$$\begin{array}{rcl} 349 & = & 300 + 40 + 9 \\ -\ 133 & = & 100 + 30 + 3 \\ \hline & & ??? + ?? + ?, \text{ or } ? \end{array}$$

6. Are these two sentences related?

$349 - 133 = n \qquad n + 133 = 349$

Then can you check subtraction by adding? Explain how.

— Copy and find the answers. Check by adding.

	a	b	c	d
7.	349 − 133	770 − 460	858 − 621	377 − 250
8.	722 − 301	412 − 210	685 − 342	883 − 152
9.	649 − 235	989 − 426	147 − 104	578 − 363

— See if you can discover a pattern in each exercise below.

	a	b	c	d	e
10.	$9 - 6 = n$	$19 - 6 = n$	$29 - 6 = n$	$39 - 6 = n$	$59 - 6 = n$
11.	$11 - 7 = n$	$21 - 7 = n$	$31 - 7 = n$	$51 - 7 = n$	$71 - 7 = n$
12.	$15 - 8 = n$	$25 - 8 = n$	$35 - 8 = n$	$65 - 8 = n$	$85 - 8 = n$

If you need more practice, do Practice Sets 13 and 14, page 328.

Renaming numbers in subtraction

1. When we wish to subtract 48 from 95, we discover that we are unable to subtract 8 ones from 5 ones and get a whole number for an answer. Study the example below.

$$\begin{array}{rcccl} 95 & = & 9\text{ tens }5\text{ ones} & = & 8\text{ tens }15\text{ ones}\\ \underline{-\ 48} & = & \underline{4\text{ tens }8\text{ ones}} & = & \underline{4\text{ tens }\ \ 8\text{ ones}}\\ & & & & \boxed{?}\text{ tens }\boxed{?}\text{ ones, or }\underline{\ ?\ } \end{array}$$

How is 95 renamed to make it possible to subtract 8 ones? Copy the example and complete it. What is the final answer?

2. Study the incomplete example shown below.

$$\begin{array}{rcccl} 754 & = & 700 + 50 + 4 & = & 600 + \boxed{?}\boxed{?}\boxed{?} + 4\\ \underline{-\ 382} & = & \underline{300 + 80 + 2} & = & \underline{300 + \ \ 80 + 2}\\ & & & & \boxed{?}\boxed{?}\boxed{?} + \ \boxed{?}\boxed{?} + \boxed{?}\text{, or }\underline{\ ?\ } \end{array}$$

Why is it necessary to rename 754 before you can subtract 382? How is 754 renamed? Copy and complete the example.

3. To subtract 676 from 842, you rename 842 as 7 hundreds 13 tens 12 ones. Why? How is this renaming shown in the example at the right? Copy the example.

$$\begin{array}{r} 7\ \ 13\ 12\\ \cancel{8}\ \cancel{4}\ \cancel{2}\\ \underline{-\ 6\ \ 7\ \ 6} \end{array}$$

- What does 12 − 6 equal? What numeral should you write in the ones column?
- What does 13 tens − 7 tens equal? What numeral should you write in the tens column?
- What does 7 hundreds − 6 hundreds equal? Where do you write the 1?

Is 166 the correct answer? Check by adding.

— Copy, find the answers, and check. See how many you can do without using the helping numerals when renaming.

	a	b	c	d	e	f
4.	34 − 17	52 − 38	563 − 135	478 − 192	913 − 745	825 − 797
5.	57 − 39	95 − 56	432 − 128	563 − 367	825 − 329	324 − 266

Subtraction involving 0

1. 800 = 8 hundreds, or 80 tens, or 800 ones
600 = 6 hundreds, or _?_ tens, or _?_ ones
400 = _?_ hundreds, or _?_ tens, or _?_ ones

2. 4,000 = 4 thousands, or 40 hundreds, or 400 tens, or _?_ ones
7,000 = 7 thousands, or _?_ hundreds, or _?_ tens, or _?_ ones
9,000 = _?_ thousands, or _?_ hundreds, or _?_ tens, or _?_ ones

3. In the example at the right, is 80 tens renamed as 79 tens 10 ones? Copy the example and subtract. Check by adding.

$$\begin{array}{r} \overset{7}{\cancel{8}}\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\,2\,4\,4 \\ \hline \end{array}$$

— Copy, compute, and check.

	a	b	c
4.	800 − 345	900 − 246	700 − 631
5.	804 − 527	700 − 74	600 − 187

6. To subtract 2,018 from 5,000, how should you rename 500 tens? Copy the example and subtract. Check your answer by adding.

$$\begin{array}{r} \overset{4}{\cancel{5}},\overset{9}{\cancel{0}}\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\,2,0\,1\,8 \\ \hline \end{array}$$

— Copy, compute, and check.

	a	b	c
7.	5,000 − 2,108	6,000 − 3,027	7,000 − 4,201
8.	8,000 − 4,123	9,000 − 5,999	4,000 − 250

9. Can we think of $30.00 as 3,000 cents? To subtract 1,865 from 3,000, how should you rename 3,000? Copy the example and find the answer. Write a dollar sign and cents point to show that the answer stands for money.

$$\begin{array}{r} \$30.00 \\ -\ 18.65 \\ \hline \end{array}$$

— Copy, find the answers, and check.

	a	b	c
10.	6,000 − 2,482	7,000 − 2,899	$50.00 − 16.84
11.	3,000 − 963	$90.00 − 7.83	$25.00 − 6.49
12.	5,020 − 3,542	4,010 − 987	$42.10 − 35.07
13.	1,025 − 650	$60.92 − 35.05	$21.00 − 8.69
14.	5,803 − 716	4,000 − 387	$70.50 − 6.89

If you need more practice, do Practice Set 15, page 328.

Estimating addends

1. Look at this number sentence. $29 + n = 48$ To find the other addend, can you use this related sentence? $48 - 29 = n$ What does $48 - 29$ equal?

2. When you are subtracting and want to see if your answer is sensible, you can estimate the addend. In A at the right, are the sum and the given addend rounded to the nearest ten? What is $50 - 30$? Then is 19 a sensible answer?

3. In example B, are the sum and the given addend rounded to the nearest hundred? What is $500 - 300$? Then should the other addend be about 200? Copy example B and subtract 285 from 482. Check your answer with the estimate to see if it is a sensible answer.

	EXAMPLE	ESTIMATE
A	48 − 29 19	50 − 30 ??
B	482 − 285 ???	500 − 300 ???

— Three numbers are named after each sentence below. Only one of the numbers makes the sentence true. Without using pencil and paper, estimate each answer. Then use your estimate to select the number that makes the sentence true. Write a numeral for the number you select.

	a	b
4.	$81 - 29 = n$ {52, 62, 72}	$101 - 38 = n$ {53, 63, 73}
5.	$159 - 91 = n$ {68, 78, 88}	$130 - 39 = n$ {71, 81, 91}
6.	$161 - 49 = n$ {92, 102, 112}	$304 - 175 = n$ {129, 139, 149}
7.	$249 - 38 = n$ {211, 221, 231}	$698 - 415 = n$ {183, 283, 383}
8.	$985 - 54 = n$ {441, 681, 931}	$582 - 196 = n$ {386, 486, 586}
9.	$714 - 64 = n$ {540, 650, 739}	$385 - 189 = n$ {196, 296, 396}
10.	$817 - 98 = n$ {629, 709, 719}	$444 - 233 = n$ {111, 211, 311}

11. Subtract to find the addend that makes each sentence true in Ex. 4–10. Compare your answers with the numbers you selected by estimating.

Using subtraction to solve problems

1. Joe has 25¢. He wants to buy a 98-cent tie. How much more money does he need?

- Joe thinks: "25¢ plus some amount equals 98¢." Does this sentence describe Joe's thinking?

$$25 + n = 98$$

- Can you use this related sentence to solve Joe's problem?

$$98 - 25 = n$$

What does 98 − 25 equal? Joe needs ? ¢ more.

2. Matthew is 52 inches tall. Mary is 48 inches tall. How many inches taller is Matthew?

- Can you use any one of the four sentences below to help you solve this problem? Which one would you use?

$48 + n = 52$ $52 - n = 48$

$n + 48 = 52$ $52 - 48 = n$

- Use the sentence you selected and find the answer.

3. Tom had $2.75. After buying a gift for his mother, he had $1.50 left. How much did the gift cost?

- Would this sentence help solve the problem?

$$275 - n = 150$$

- Would you get the same answer if you used this sentence? Explain.

$$275 - 150 = n$$

- What is the correct answer?

— For each of the following problems, write a number sentence that can be used to solve the problem. Then find the answer.

4. Ann has a quarter and some pennies. In all, she has 49¢. How many pennies does she have?

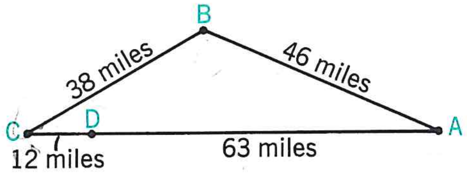
5. It is 37 blocks from where Mr. Abrams 24 lives to where he works. On nice mornings, Mr. Abrams walks to work. When he has walked 24 blocks, how many more blocks must he walk to reach his office?

6. Mary has saved $13.58. Cheryl has saved $38.42.

- Together, how much money have the two girls saved?
- What is the difference between the amount Mary has saved and the amount Cheryl has saved?

7. Mr. Jackson lives in City A. He drove to City B one morning and then drove from City B to City C, as shown on the map below. If he wants to go home by the shorter route, should he drive back through City B or should he drive through City D?

How many miles difference is there between the two routes?



Practice with subtraction

— Copy each example and find the answer. Check by adding.

	a	b	c	d	e	f	g
1.	88 − 48	67 − 35	149 − 62	137 − 54	109 − 45	118 − 72	300 − 89
2.	151 − 79	612 − 123	903 − 644	1,260 − 578	590 − 101	1,622 − 348	4,800 − 875
3.	1,108 − 396	800 − 237	965 − 896	7,003 − 616	5,000 − 4,708	6,876 − 3,890	8,476 − 3,209

— Copy, find the answers, and check.

	a	b	c	d	e	f
4.	\$149.25 − 83.75	\$459.85 − 94.68	\$86.12 − 75.58	\$80.00 − 59.23	\$765.75 − 87.77	\$137.50 − 49.75
5.	\$100.00 − 99.99	\$200.00 − 37.65	\$500.00 − 499.98	\$675.50 − 475.50	\$38.27 − 27.38	\$600.00 − 350.42

— Solve each of the following sentences.

	a	b	c	d
6.	$75 - 16 = n$	$54 - n = 28$	$83 - 19 = n$	$32 + n = 70$
7.	$n + 6 = 23$	$72 - n = 26$	$n + 24 = 82$	$47 - 18 = n$
8.	$67 - n = 21$	$98 - 59 = n$	$56 - n = 12$	$94 + n = 102$
9.	$832 - 63 = n$	$231 - n = 159$	$682 - 467 = n$	$111 - n = 68$
10.	$n + 528 = 800$	$644 - n = 320$	$500 + n = 964$	$606 - 332 = n$
11.	$246 + n = 688$	$793 - 408 = n$	$825 - n = 257$	$2{,}385 + n = 4{,}002$

Think twice!

— Tell whether the number that makes the sentence true will be an even number or an odd number.

12.	$324 + n = 640$	$119 + n = 720$	$n + 289 = 413$	$n + 226 = 563$
13.	$84 - 46 = n$	$71 - 17 = n$	$98 - 59 = n$	$43 - 28 = n$
14.	$474 - n = 237$	$512 - n = 354$	$645 - n = 119$	$801 - n = 574$

How well can you subtract?

— These examples will help you find out how well you can subtract. Copy each example and find the answer. Check by using addition.

	a	b	c	d	e
1.	75 − 43	84 − 55	117 − 35	$1.53 − .94	$1.40 − .63
2.	579 − 428	980 − 766	676 − 348	701 − 653	400 − 297
3.	1,540 − 875	6,000 − 3,476	4,006 − 458	5,104 − 672	2,700 − 2,613
4.	8,704 − 7,631	9,876 − 4,325	$46.08 − 39.89	$62.00 − 54.28	$48.69 − 46.75
5.	13,621 − 8,795	3,535 − 2,537	$70.00 − 65.08	$97.02 − 79.98	$50.00 − 29.50

Mathamusements

- John said, "I am thinking of a number. If you add 7 to it, the sum is 16. What is the number?"
- Jane said, "I am thinking of a number. If you subtract 7 from it, the answer is 23. What is the number?"
- The sum of two numbers is 96. One of the numbers is 38. What is the other number?
- The difference between two numbers is 15. The greater of the two numbers is 47. What is the smaller of the two numbers?
- The difference between two numbers is 18. One of the numbers is 44. Can you tell what the other number is? What two digits are in the numeral for the other number?

If you need more practice, do Practice Sets 16 and 17, page 329.

Adding and subtracting measures

1. Is 75 seconds a greater period of time than 1 minute? How many seconds greater is it?

75 sec. make 1 min. ? sec.

2. Is 38 ounces greater than or less than 1 pound? 2 pounds? 3 pounds? How many ounces greater than 2 pounds is 38 ounces?

38 oz. make 2 lb. ? oz.

3. Just as there are different names for a number, there are different ways to express a measurement.

- We can think of 2 hours as 1 hour ? minutes, or simply as ? minutes.
- We can think of 2 pounds as 1 pound ? ounces, or simply as ? ounces.

— Copy and complete.

4. 172 sec. make: 2 min. ? sec.
1 min. ? sec.

5. 52 oz. make: 3 lb. ? oz.
2 lb. ? oz.
1 lb. ? oz.

6. 2 yd. 4 ft. make: ? yd. 1 ft.
1 yd. ? ft.
? ft.

7. 2 qt. 1 pt. make: 1 qt. ? pt.
? pt.

8. 3 min. 20 sec. make: 2 min. ? sec.; or 1 min. ? sec.; or ? sec.

9. When we add measures, it is often necessary to rename the sum. Study the example below.

$$\begin{array}{rl} & 3\text{ hr. }45\text{ min.}\\ + & 2\text{ hr. }30\text{ min.}\\ \hline & 5\text{ hr. }75\text{ min., or }6\text{ hr. }15\text{ min.}\end{array}$$

Explain why 5 hours 75 minutes can be renamed as 6 hours 15 minutes.

— Copy and find the answers. Rename the sums.

	a	b
10.	4 hr. 55 min. + 5 hr. 42 min.	6 gal. 3 qt. + 2 gal. 2 qt.
11.	17 min. 45 sec. + 22 min. 58 sec.	12 yd. 2 ft. + 10 yd. 2 ft.

12. In subtraction, it is often necessary to rename the given sum before subtracting. Explain why, using the example below.

$$\begin{array}{rl} & 5\text{ hr. }10\text{ min.}\\ - & 3\text{ hr. }15\text{ min.}\\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 4\text{ hr. }70\text{ min.}\\ 3\text{ hr. }15\text{ min.}\\ \hline 1\text{ hr. }55\text{ min.}\end{array}$$

— Copy and find the answers.

13.	7 hr. 14 min. − 3 hr. 22 min.	12 lb. 6 oz. − 8 lb. 10 oz.
14.	3 ft. 4 in. − 1 ft. 8 in.	10 hr. 32 min. − 9 hr. 55 min.

If you need more practice, do Practice Set 5, page 325, and Practice Set 18, page 329.

PROBLEM-SOLVING HELP

Using addition and subtraction to solve problems

— In order to solve a problem you must decide what operation to use. Read each of the following problems and tell which of the two number sentences given can be used to solve the problem. Then find the answer.

1. Bob has 24 green marbles and 32 yellow marbles. How many marbles does he have in all?

$24 + 32 = n \qquad 32 - 24 = n$

2. Tom had 41 marbles. He gave away 6. How many marbles does he have left?

$41 + 6 = n \qquad 41 - 6 = n$

3. Laura has 41 different kinds of leaves in her nature collection. Sally has 34 different kinds. How many more does Laura have than Sally?

$41 + 34 = n \qquad 41 - 34 = n$

4. Nancy paid $1.49 for one record album and $2.98 for another. How much did Nancy pay for both of the albums?

$149 + 298 = n \qquad 298 - 149 = n$

5. Barbara has saved $5.45. Jean has saved $4.80. What is the difference between the amount saved by Barbara and the amount saved by Jean?

$545 + 480 = n \qquad 545 - 480 = n$

— Read each problem. Write a number sentence that can be used to solve the problem. Then find the answer.

6. Raymond has $13.00. If he spends $4.98 for a pair of jeans, how much will he have left?

7. Susan wants to buy a dress pattern for 75¢ and a package of needles for 49¢. How much money does she need?

8. Tom wants to buy a rabbit trap for 60¢. He has 29¢. How much more money does he need?

9. There are 37 pupils in the fifth grade. Only 28 are present today. How many are absent?

10. Allan weighed 69 pounds a year ago. Today he weighs 77 pounds. How many pounds has he gained during the year?

11. A boys' club is selling greeting cards. Jim sold 32 boxes; Peter sold 29 boxes; Jack, 53 boxes; Fred, 37 boxes; and Jerry, 25 boxes.

How many boxes have these five boys sold in all?

12. Barney weighs 82 pounds. Steve weighs 77 pounds. What is the difference between their weights?

Writing and solving problems

1. Bonnie has saved $3.75. Her brother Robbie has saved $5.20.

- One question that can be asked is: Together, how much money have Bonnie and Robbie saved?
- Another question that can be asked is: How much more money has Robbie saved than Bonnie?
- Find the answer to each of these questions.

— For each of the following exercises, write two questions. Then find the answers.

2. Charles spent 98¢ for a model-airplane kit. Jack spent 69¢ for a puzzle.

9

3. Joan plans to read two books. One book has 256 pages. The other book has 320 pages.

250

4. One week Mark sold 37 magazine subscriptions. The second week he sold 62 magazine subscriptions.

5. A bicycle sells for $59.95. A portable radio sells for $39.98.

6. Andy takes trumpet lessons. One week he practiced 8 hours 15 minutes. The second week he practiced 7 hours 55 minutes.

7. It took Jeff 1 hour 25 minutes to clean the cellar and 2 hours 15 minutes to wash and polish the car.

340

— In the following problems not enough information is given for you to answer the questions. Read each problem and tell what else you need to know to solve it.

8. Judy bought a ball-point pen for 49¢. How much money does she have left?

9. One hundred fifty-two people saw the school play on the opening night. How many more people saw the play on the second night?

10. Amy practiced 1 hour 10 minutes on Monday. How many more minutes did she practice on Tuesday?

— Each of the following stories gives too much information. Use only the information you need and solve the problem.

11. George has $3.50. Bill has $4.25. If George spends $1.98, how much will he have left?

12. Susan is reading a book that has 330 pages. The first day she read 62 pages. The second day she read 98 pages, and the third day she read 75 pages. How many pages did Susan read during those three days?

13. Murray has 18 inches of rope. If he buys another yard of rope at 60¢ a foot, how much rope will he have in all?

Inquiry into number

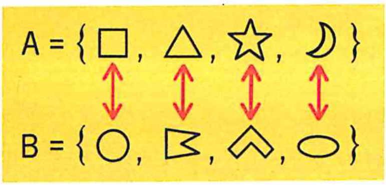
One-to-one correspondence

Shown at the right are two sets of numbers. Set C is the set of counting numbers. Set E is the set of even numbers. What do the three dots shown in each set mean?

$$C = \{1, 2, 3, 4, \cdots\}$$
$$E = \{0, 2, 4, 6, \cdots\}$$

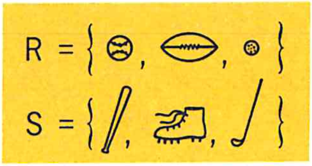
Which set do you think has the greater number of members, set C or set E? To check your answer, try the following exercises.

▶ Look at the picture of sets A and B. The arrows show each member of one set matched with only one member of the other set. When the members of two sets can be matched in this way, we say the sets are in one-to-one correspondence.

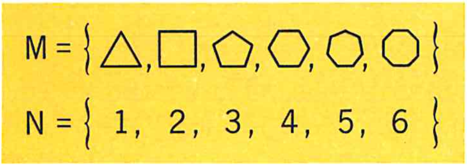


If two sets are in one-to-one correspondence, do the two sets have the same number of members? Explain.

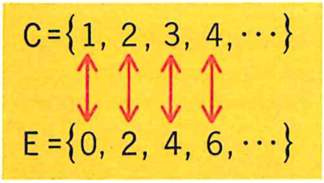
Which pairs of sets shown below are in one-to-one correspondence?



$$P = \{a, b, c\}$$
$$Q = \{w, x, y, z\}$$

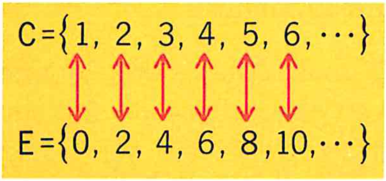


▶ Let's look again at the set of counting numbers and the set of even numbers. Do the arrows show a matching of the members? Do you think this matching can continue?



What even number is matched with the counting number 5? What counting number is matched with the even number 10?

What even number would be matched with the counting number 7? 8? 9?



COUNTING NUMBER		EVEN NUMBER
1	$(2 \times 1) - 2$	0
2	$(2 \times 2) - 2$	2
3	$(2 \times 3) - 2$	4
4	$(2 \times ?) - 2$	6
5	$(2 \times ?) - ?$	8
6	$(2 \times ?) - ?$	10

▶ Look for a pattern in the table shown at the right. For each counting number named, does the middle column in the table show you how to find a matching even number? What even number would be matched with the counting number 7? 9? 12? 27? 58?

Can you find a matching even number for each and every counting number? Are the set of counting numbers and the set of even numbers in one-to-one correspondence? Then do both sets contain the same number of members?

For further inquiry

$$\begin{array}{rccccl} E = \{ & 0, & 2, & 4, & 6, & \cdots\} \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ O = \{ & 1, & 3, & 5, & 7, & \cdots\} \end{array}$$

▶ Set E is the set of even numbers and set O is the set of odd numbers. Do you think they are in one-to-one correspondence?

What odd number in set O is matched with the even number 0? with the even number 2? 4? 6? In each case, is the matching odd number one more than the even number? What odd number would you match with 8? 10? 26? 130?

Are the set of even numbers and the set of odd numbers in one-to-one correspondence? Explain.

$$\begin{array}{rccccl} O = \{ & 1, & 3, & 5, & 7, & \cdots\} \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ K = \{ & 2, & 6, & 10, & 14, & \cdots\} \end{array}$$

▶ Study these two sets of numbers. Are they in one-to-one correspondence?

Do you see a pattern you can use to name a member of set K that could be matched with any member in set O that is named?

What member in K would you match with 9 in O? 15? 421?

▶ Try to find a pattern for matching and placing the pairs of sets below in one-to-one correspondence. Do both sets in each pair have the same number of members? Explain.

$S = \{2, 4, 6, 8, \cdots\}$

$T = \{4, 8, 12, 16, \cdots\}$

$X = \{1, 2, 3, 4, \cdots\}$

$Y = \{1, 4, 9, 16, \cdots\}$

PROBLEM TEST 1

1. Yesterday, 378 people took the eight o'clock train. Today, only 196 people took it. How many fewer people took the eight o'clock train today?

2. Tony wants to send for a book. The book costs $2.95. Postage will be 25¢, and there is a 9-cent tax. How much money should Tony send to the book company?

3. Ted spent 78¢ for two magnets to use in his science project. The larger magnet cost 49¢. How much did the smaller magnet cost?

4. One Saturday, 27,652 people attended a football game. The following Saturday, 24,875 people attended another game in the same stadium. Which game had the greater attendance? How much greater?

5. Going from Seattle, Washington, to Boston, Massachusetts, Mr. Sprat drove 3,098 miles. How far would he drive on a round trip between these two cities?

6. Sally's grandmother was born in 1909. How old will she be this year?

7. It took Stan 1 hour 35 minutes to mow his lawn and 45 minutes to trim the edges. How long did it take him to do both jobs?

8. Susan spent $5.25 for a skirt, $4.99 for a blouse, and $7.85 for a pair of shoes. How much money did she spend in all?

9. Eric caught a lake trout that measured 2 feet 1 inch long. His brother caught a lake trout 1 foot 7 inches long. How much longer was Eric's fish?

10. Dugan City has a population of 1,000,000. If each person in Dugan City had $1,000, how much money would the total population have?

There are seven tests like this one in the book. Start a Problem Test Record. Each time you take one of the Problem Tests, be sure to write your score on the record.

Mathamusements

- Have a friend think of any whole number and add 5 to it. Then tell him to multiply the sum by 4 and subtract 10. He should then divide by 2 and subtract 5. Now have him multiply this number by 3 and tell you his result. You can guess the number he first thought of by dividing the result he gives you by some whole number. What whole number?
- Try to discover why this game always works. Then make up your own number game.

IT'S CHECK–UP TIME

— Solve each of the following number sentences.

1. $45 + 72 = \triangle + 45$
2. $(64 + 33) + 12 = 64 + (\square + 12)$
3. $(99 + 17) - \triangle = 99$
4. $86 - n = 45$
5. $n + 62 = 104$
6. $n - 57 = 26$
7. $92 + 28 = n$
8. $35 - 9 = n$

— Copy each of these examples, find the answer, and check.

	a	b
9.	$\begin{array}{r} 52 \\ 67 \\ +\ 45 \\ \hline \end{array}$	$\begin{array}{r} 608 \\ 519 \\ +\ 762 \\ \hline \end{array}$
10.	$\begin{array}{r} 3{,}628 \\ +\ 4{,}972 \\ \hline \end{array}$	$\begin{array}{r} 72{,}849 \\ +\ 31{,}608 \\ \hline \end{array}$
11.	$\begin{array}{r} 495 \\ -\ 372 \\ \hline \end{array}$	$\begin{array}{r} 632 \\ -\ 276 \\ \hline \end{array}$
12.	$\begin{array}{r} 3{,}492 \\ -\ 1{,}578 \\ \hline \end{array}$	$\begin{array}{r} 8{,}000 \\ -\ 2{,}674 \\ \hline \end{array}$
13.	$\begin{array}{r} \$902.48 \\ +\ 607.95 \\ \hline \end{array}$	$\begin{array}{r} \$214.96 \\ +\ 587.64 \\ \hline \end{array}$
14.	$\begin{array}{r} \$872.64 \\ -\ 496.28 \\ \hline \end{array}$	$\begin{array}{r} \$1{,}000.00 \\ -\ 832.53 \\ \hline \end{array}$

— Write four related number sentences for each set of numbers shown below.

15. $\{3, 6, 9\}$
16. $\{8, 12, 20\}$

— Copy and find the answers.

	a	b
17.	$\begin{array}{r} \text{3 min. 18 sec.} \\ +\ \text{5 min. 22 sec.} \\ \hline \end{array}$	$\begin{array}{r} \text{6 ft. 8 in.} \\ +\ \text{3 ft. 5 in.} \\ \hline \end{array}$
18.	$\begin{array}{r} \text{17 hr. 45 min.} \\ -\ \text{9 hr. 38 min.} \\ \hline \end{array}$	$\begin{array}{r} \text{4 gal. 1 qt.} \\ -\ \text{1 gal. 3 qt.} \\ \hline \end{array}$

19. Jeff kept a record of how far he rode on his bicycle each week for a month. The first week he rode 17 miles, the second week 24 miles, the third week 12 miles, and the fourth week 19 miles. What is the total distance he rode during these four weeks?

20. Maureen bought a skirt for $6.38. How much change should she have received if she gave the salesman a ten-dollar bill?

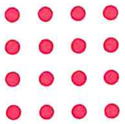
If you have time, try these —

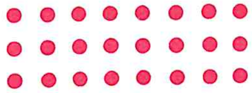
21. The sum of two numbers is 376. One of the numbers is 280. What is the other number?

22. The difference between two numbers is 48. The greater of the two numbers is 96. What is the smaller of the two numbers?

CHAPTER 3

Understanding multiplication of whole numbers



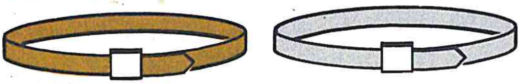


— Use the pictures above to help you solve these pairs of sentences.

1. $4 + 4 + 4 + 4 = \square \qquad 4 \times 4 = \square$

2. $8 + 8 + 8 = \triangle \qquad 3 \times 8 = \triangle$

3. How are multiplication and addition of whole numbers related?



4. A belt comes in brown or gray with a red, green, or blue letter. To find how many choices of color there are, look at the chart below.

letter	belt	letter	belt
red	brown	red	gray
green	brown	green	gray
blue	brown	blue	gray

What is the product of the number of members in the set of different-colored belts and the number of members in the set of different-colored letters?

5. Multiplication is a special way of thinking about two numbers and relating them to a third number.

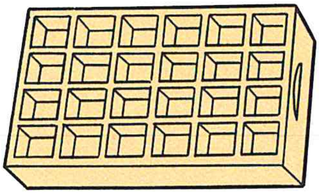
The numbers 4 and 6 are related by multiplication to what number?

$$4 \times 6 = 24$$

$$\begin{array}{r} 6 \\ \times\ 4 \\ \hline 24 \end{array}$$

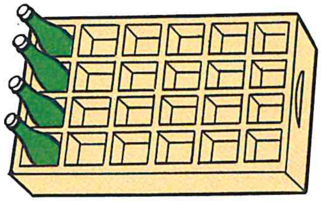
6. Here are two ways to show the multiplication of 6 by 4. As you know, 4 and 6 are called factors and 24 is called the product of 4 and 6. In each sentence below, name the factors and the product.

$9 \times 6 = 54 \qquad 3 \times 7 = 21$



7. How many rows are there in the case above? How many bottles are there in each row? $4 \times 0 = \underline{?}$

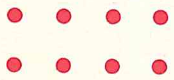
8. How many rows are there in this case? How many bottles are there in each row? $4 \times 1 = \underline{?}$



— Solve these sentences.

	a	b
9.	$7 \times 0 = n$	$9 \times 1 = \square$
10.	$42 \times 1 = \triangle$	$57 \times 0 = n$

11. Look at the set of dots pictured below. There are ? rows with ? dots in each row. Can you multiply to find the number of dots in this set? $2 \times 4 =$?



In the set of dots above, there are ? columns with ? dots in each column. Multiply again to find the number of dots in the set.

$$4 \times 2 = \text{?}$$

Is this sentence true?

$$2 \times 4 = 4 \times 2$$

— Solve these pairs of sentences.

12. $6 \times 3 = n$ $\qquad 3 \times 6 = n$
13. $8 \times 4 = \square$ $\qquad 4 \times 8 = \square$
14. $3 \times 9 = \triangle$ $\qquad 9 \times 3 = \triangle$

Your discovery

Tell your discovery

Does changing the order of the actors change the product? This dea is called the **commutative property of multiplication.**

Use your discovery

$391 \times 426 = 166{,}566$,
so $426 \times 391 =$?

15. What can you tell about the product of 1 and any whole number?

16. If either of two factors is 0, their product is always ?.

17. Solve this sentence.

$$(3 \times 2) \times 4 = n$$

- Solve this sentence.

$$3 \times (2 \times 4) = n$$

- Does $(3 \times 2) \times 4$ name the same number as $3 \times (2 \times 4)$?

18. If $\square$, $\triangle$, and $\Diamond$ stand for any whole numbers, is this sentence true?

$$(\square \times \triangle) \times \Diamond = \square \times (\triangle \times \Diamond)$$

Your discovery

Tell your discovery

Tell what you know about pairing neighboring factors to find a product. This idea is called the **associative property of multiplication.**

Use your discovery

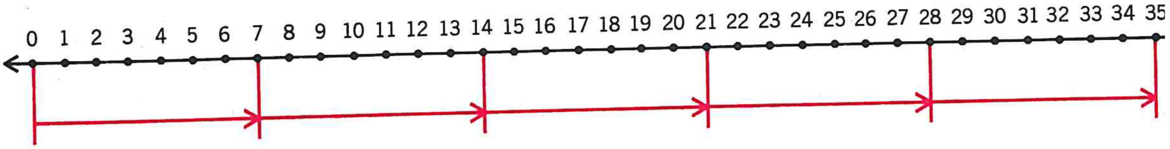
$(64 \times 39) \times 21 = 52{,}416$,
so $64 \times (39 \times 21) =$?

19. How are the following expressions alike? How do they differ? Are they equivalent?

$3\times(2\times4)$	$(3\times2)\times4$	$2\times(3\times4)$
$3\times(4\times2)$	$(3\times4)\times2$	$(2\times3)\times4$
$2\times(4\times3)$	$(2\times4)\times3$	$4\times(2\times3)$
$4\times(3\times2)$	$(4\times2)\times3$	$(4\times3)\times2$

To find the product of two or more numbers, can we multiply the numbers in any order that we wish?

Understanding more about multiplication



1. Find the answers for each of these sentences.

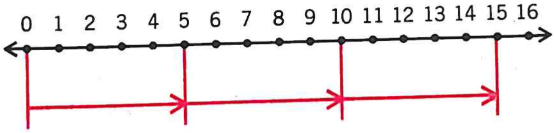
- $7 + 7 + 7 + 7 + 7 = \underline{?}$
- $5 \times 7 = \underline{?}$

Did you get the same answer for both sentences? Can you think of 5×7 as using 7 as an addend 5 times?

2. The multiplication of 7 by 5 is shown on the number line above. How many units long is each arrow? How many arrows are there? What number is shown at the tip of the last arrow?

3. To show the multiplication of 6 by 4 on the number line, first think of 6 used as an addend $\underline{?}$ times. Start at 0 and draw an arrow 6 units long. Draw arrows 6 units long one after the other until you have drawn $\underline{?}$ arrows. What number is shown at the tip of the last arrow?

4. Write a sentence that tells the multiplication fact shown on the number line pictured below.



5. Suppose you wish to know what 5×12 is. Write a number sentence that shows 12 used as an addend 5 times. What is the sum?

$$12+12+12+12+12=5\times12=\underline{?}$$

6. Find what 3×18 is by adding. Then what is 18×3? Why?

7. You can think of 10×6 as 6 used as an addend $\underline{?}$ times. What is 10×6? To find 11×6, can you add 6 to 10×6? Why?

$$11 \times 6 = (10 \times 6) + 6 = \underline{?}$$

What is 12×6? 13×6?

8. What is 9×9? 10×9? 11×9? Then what is 9×10? 9×11? 9×12?

9. Count by 5's from 0 to 50. The numbers you say are multiples of 5.

A **multiple** of 5 is the product of 5 and any whole number.

Is 0 a multiple of 5? is 15? is 40? List the first twelve multiples of 5.

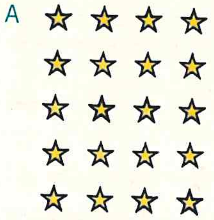
10. Count by 4's from 0 to 40. Are these numbers multiples of 4? List the first twelve multiples of 4.

11. List the first twelve multiples of 10. Does each of these multiples have 10 as one of its factors?

12. The set shown below is the set of multiples of what number?

$$\{0, 7, 14, 21, 28, \cdots\}$$

The distributive property


A

B

1. Look at the stars shown above. There are 5 rows of stars with $4 + 3$, or $\underline{?}$, stars in each row. How many stars are shown in all?

$$5 \times (4 + 3) = \underline{?}$$

2. In set A, there are 5 rows of stars with 4 stars in each row. In set B, there are 5 rows of stars with 3 stars in each row. What is 5×4? 5×3? What is the sum of these two products?

$$(5 \times 4) + (5 \times 3) = \underline{?}$$

Is this number sentence true?

$$5 \times (4 + 3) = (5 \times 4) + (5 \times 3)$$

— Solve these pairs of sentences.

3. $2 \times (6 + 2) = \triangle$
$(2 \times 6) + (2 \times 2) = \triangle$

4. $8 \times (5 + 4) = \square$
$(8 \times 5) + (8 \times 4) = \square$

Your discovery

ell your discovery

What have you found about mulplying the sum of two numbers?

se your discovery

$\times (30 + 2) = (5 \times \underline{?}) + (5 \times \underline{?})$

The **distributive property** tells you that $2 \times (4 + 6)$ names the same number as $(2 \times 4) + (2 \times 6)$; that $6 \times (3 + 4)$ names the same number as $(6 \times 3) + (6 \times 4)$; and so on.

— Solve these sentences.

5. $8 \times (2 + 5) = (8 \times 2) + (n \times 5)$

6. $4 \times (6 + 9) = (4 \times n) + (4 \times 9)$

7. $3 \times (n + 5) = (3 \times 7) + (3 \times 5)$

8. To find the product of 4 and 13, you may think of 13 as $(10 + 3)$.

$$4 \times 13 = 4 \times (10 + 3)$$

Then use the distributive property.

$$\begin{aligned} 4 \times (10 + 3) &= (4 \times 10) + (4 \times 3) \\ &= 40 + \underline{?} \\ &= \underline{?} \end{aligned}$$

Then 4×13 equals $\underline{?}$.

9. Is this sentence true?

$$4 \times 13 = 13 \times 4$$

Then complete this sentence.

$$\begin{aligned} 13 \times 4 &= (10 + 3) \times 4 \\ &= (10 \times 4) + (3 \times 4) \\ &= \underline{?} + \underline{?} \\ &= \underline{?} \end{aligned}$$

— Use the distributive property to solve each of these sentences.

	a	b
10.	$3 \times 14 = n$	$2 \times 19 = n$
11.	$8 \times 12 = n$	$6 \times 14 = n$
12.	$5 \times 15 = n$	$7 \times 13 = n$
13.	$4 \times 17 = n$	$9 \times 16 = n$
14.	$11 \times 9 = n$	$18 \times 5 = n$

Using the distributive property

1. What is 3×2? Then what is 3×2 tens? $3 \times 20 = \underline{\ ?\ }$

2. What is 4×2? Then what is 4×2 hundreds? $4 \times 200 = \underline{\ ?\ }$

— Solve each of these sentences.

	a	b
3.	$7 \times 20 = n$	$3 \times 40 = \square$
4.	$5 \times 300 = \triangle$	$6 \times 800 = n$

5. You can think of 78 as $70 + \underline{\ ?\ }$. To find the product of 3 and 78, copy and complete the following.

$$\begin{aligned} 3 \times 78 &= 3 \times (70 + 8) \\ &= (3 \times 70) + (3 \times \underline{\ ?\ }) \\ &= \underline{\ ?\ } + 24, \text{ or } \underline{\ ?\ } \end{aligned}$$

6. Study the example shown below.

$$\begin{array}{rcl} 78 & = & 7 \text{ tens} \quad 8 \text{ ones} \\ \underline{\times 3} & = & \underline{\qquad\qquad 3} \\ & & 21 \text{ tens } 24 \text{ ones, or } \underline{\ ?\ } \end{array}$$

What three-digit numeral names the product of 3 and 78?

7. Another way to show the multiplication of 78 by 3 is given at the right. Explain each step of the multiplication as shown in the example.

$$\boxed{\begin{array}{r} 78 \\ \underline{\times 3} \\ 24 \\ \underline{210} \\ 234 \end{array}}$$

8. Copy and find the products. Use any method you wish.

a	b	c	d	e	f
$\begin{array}{r} 34 \\ \underline{\times 2} \end{array}$	$\begin{array}{r} 51 \\ \underline{\times 6} \end{array}$	$\begin{array}{r} 76 \\ \underline{\times 4} \end{array}$	$\begin{array}{r} 43 \\ \underline{\times 8} \end{array}$	$\begin{array}{r} 95 \\ \underline{\times 7} \end{array}$	$\begin{array}{r} 19 \\ \underline{\times 5} \end{array}$

9. To find the product of 4 and 283, copy and complete the following.

$$\begin{aligned} 4 \times 283 &= 4 \times (200 + 80 + 3) \\ &= (4 \times 200) + (4 \times 80) \\ &\quad + (4 \times \underline{\ ?\ }) \\ &= 800 + \underline{\ ?\ } + 12, \text{ or } \underline{\ ?\ } \end{aligned}$$

10. Explain this way of showing the multiplication of 283 and 4.

$$\begin{array}{l} 2 \text{ hundreds} \quad 8 \text{ tens} \quad 3 \text{ ones} \\ \underline{\qquad\qquad\qquad\qquad\qquad \times \ 4} \\ 8 \text{ hundreds } 32 \text{ tens } 12 \text{ ones, or } \underline{\ ?\ } \end{array}$$

11. A third way to show the multiplication of 283 and 4 is given at the right. How are the 12 ones shown here? the 32 tens? the 8 hundreds? What is the sum of these numbers?

$$\boxed{\begin{array}{r} 283 \\ \underline{\times 4} \\ 12 \\ 320 \\ \underline{800} \\ ?,??? \end{array}}$$

12. Here is a shorter way to show the multiplication of 4 and 283. The digit in the tens place shows $\underline{\ ?\ }$ tens. Why? What two digits are missing? Explain how to find them.

$$\boxed{\begin{array}{r} 283 \\ \underline{\times 4} \\ ?,?32 \end{array}}$$

13. Copy and find the products. Use any method you wish.

a	b	c	d	e
$\begin{array}{r} 312 \\ \underline{\times 3} \end{array}$	$\begin{array}{r} 256 \\ \underline{\times 8} \end{array}$	$\begin{array}{r} 3{,}412 \\ \underline{\times 2} \end{array}$	$\begin{array}{r} 1{,}824 \\ \underline{\times 9} \end{array}$	$\begin{array}{r} 3{,}824 \\ \underline{\times 4} \end{array}$

If you need more practice, do Practice Set 27, page 332.

Multiplying by multiples of ten

1. Solve each of these number sentences. If necessary, use addition to find the solution set for each.

$1 \times 6 = n \qquad 10 \times 6 = n \qquad 100 \times 6 = n \qquad 1{,}000 \times 6 = n$

— Copy and complete Ex. 2–4.

2. $$\begin{aligned} 10 \times 436 &= (10 \times 400) + (10 \times 30) + (10 \times 6) \\ &= 4{,}000 + 300 + 60 \\ &= \underline{\ ?\ } \end{aligned}$$

3. $$\begin{aligned} 100 \times 528 &= (100 \times 500) + (100 \times 20) + (100 \times 8) \\ &= 50{,}000 + 2{,}000 + 800 \\ &= \underline{\ ?\ } \end{aligned}$$

4. $$\begin{aligned} 1{,}000 \times 742 &= (1{,}000 \times 700) + (1{,}000 \times 40) + (1{,}000 \times 2) \\ &= 700{,}000 + 40{,}000 + 2{,}000 \\ &= \underline{\ ?\ } \end{aligned}$$

5. Study this chart for a pattern.

GIVEN NUMBER	PRODUCT OF 10 AND GIVEN NUMBER	PRODUCT OF 100 AND GIVEN NUMBER	PRODUCT OF 1,000 AND GIVEN NUMBER
8	80	800	8,000
32	320	3,200	32,000
96	960	9,600	96,000
145	1,450	14,500	145,000
378	3,780	37,800	378,000
746	7,460	74,600	746,000

- Compare the place value of each digit in the numeral for the product of 10 and the given number with the place value of the same digit in the numeral for the given number. What do you discover?
- Compare the place value of each digit in the numeral for the product of 100 and the given number with the place value of the same digit in the numeral for the given number. What do you discover?
- Compare the place value of each digit in the numeral for the product of 1,000 and the given number with the place value of the same digit in the numeral for the given number. What do you discover?

Using the associative property

— Write only the numerals for the products.

	a	b
1.	$10 \times 27 = \underline{?}$	$10 \times 458 = \underline{?}$
2.	$100 \times 38 = \underline{?}$	$100 \times 972 = \underline{?}$
3.	$1{,}000 \times 56 = \underline{?}$	$1{,}000 \times 275 = \underline{?}$

4. Is 20 equal to 2 × 10? The following shows how to find the product of 20 and 34. Explain each step.

$$\begin{aligned} 20 \times 34 &= (2 \times 10) \times 34 \\ &= 2 \times (10 \times 34) \\ &= 2 \times \quad 340 \\ &= 680 \end{aligned}$$

5. Shown below is another way to find the product of 20 and 34. Explain it.

$$\begin{aligned} 20 \times 34 &= (10 \times 2) \times 34 \\ &= 10 \times (2 \times 34) \\ &= 10 \times \quad 68 \\ &= 680 \end{aligned}$$

6. Multiply each of these numbers by 20.

23 31 62 132 443 621

7. Tell a short way to multiply a number by 30; by 40.

Multiply the numbers named in Ex. 6 by 30; by 40.

8. Is 200 equal to 2 × 100? Copy and complete the following.

$$\begin{aligned} 200 \times 48 &= (2 \times 100) \times 48 \\ &= 2 \times (100 \times 48) \\ &= 2 \times \quad \underline{?} \\ &= \underline{?} \end{aligned}$$

9. Shown below is another way to find the product of 200 and 48. Explain it.

$$\begin{aligned} 200 \times 48 &= (100 \times 2) \times 48 \\ &= 100 \times (2 \times 48) \\ &= 100 \times \quad 96 \\ &= \underline{?} \end{aligned}$$

10. Multiply each of the numbers named in Ex. 6 by 200; by 300; by 600.

11. Copy and complete the following.

$$\begin{aligned} 4{,}000 \times 32 &= (4 \times 1{,}000) \times 32 \\ &= 4 \times (1{,}000 \times 32) \\ &= 4 \times \quad \underline{?} \\ &= \underline{?} \end{aligned}$$

12. Copy and complete the following.

$$\begin{aligned} 4{,}000 \times 32 &= (1{,}000 \times 4) \times 32 \\ &= 1{,}000 \times (4 \times 32) \\ &= 1{,}000 \times \quad \underline{?} \\ &= \underline{?} \end{aligned}$$

13. Multiply each of the numbers named in Ex. 6 by 2,000; by 4,000; by 5,000.

14. If you understand the pattern you discovered when multiplying a given number by 10, 100, or 1,000, you should be able simply to write a numeral for the product of 10,000 and each of the following. Try it.

16 58 235 164 528 123

15. Multiply each of the numbers named in Ex. 14 by 20,000; by 40,000.

Multiplying by tens and ones

1. Debby multiplied 46 by 32. She thought: "32 is 30 + 2, so 32 × 46 equals (30 + 2) × 46." She wrote her example as shown at the right. What is 2 × 46? What is the sum of that number and 1,380? 32 × 46 = ?

$$\begin{aligned} 32 \times 46 &= (30 + 2) \times 46 \\ &= (30 \times 46) + (2 \times 46) \\ &= \quad 1{,}380 \quad + \quad \underline{?} \\ &= \underline{?} \end{aligned}$$

2. John showed the multiplication of 32 and 46 by writing the example at the right. How are his example and Debby's example alike? Did they both multiply 46 by 2 and by 30?

What is 2 × 46? This answer is called a **partial product.** What is 30 × 46? This answer is also called a partial product. Why do you think they are called *partial* products?

$$\begin{array}{rl} & 46 \\ & \underline{\times\ 32} \\ 2 \times 46 \rightarrow & \left.\begin{array}{r} 92 \\ \underline{1{,}380} \end{array}\right] \text{partial products} \\ 30 \times 46 \rightarrow & \\ 32 \times 46 \rightarrow & 1{,}472 \text{ product} \end{array}$$

What is the sum of the partial products? 32 × 46 = ?

3. Tom multiplied 43 by 28.

- First he multiplied 43 by 8 ones. What is the partial product?
- Next he multiplied 43 by 2 tens. What is this partial product?

In the example at the right, where did Tom write the 8 hundreds? the 6 tens? Why? Why is it unnecessary to write the 0 in the ones place?

- What is the sum of the partial products? Copy and complete the example.

$$\begin{array}{r} 43 \\ \underline{\times\ 28} \\ 344 \\ \underline{86} \\ ?{,}??? \end{array}$$

Copy and find the products.

	a	b	c	d	e	f	g	h
4.	95 × 32	64 × 35	69 × 52	89 × 14	78 × 43	76 × 39	67 × 90	47 × 60
5.	235 × 42	307 × 54	578 × 36	$6.13 × 13	$5.08 × 25	$8.47 × 50	$6.80 × 67	$7.50 × 80

f you need more practice, do Practice Set 28, page 333.

Estimating products

1. Jim wanted to find the product of 6 and 28. Before he multiplied, he estimated the product.

EXAMPLE	ESTIMATE
28	30
× 6	× 6
168	180

He thought, "I can round 28 to 30. 6 × 30 = 180 So 6 × 28 should be about ?." Will 6 × 28 be greater than 180, or less than 180?

Then Jim multiplied. How far from his estimate is his product for the example?

2. To estimate the product of 7 and 63, round 63 to 60. What is 7 × 60? Then the product should be about ?. What is the product of 7 and 63?

3. Round 298 to the nearest hundred. Now estimate the product of 4 and 298. What is 4 × 298?

— Estimate each product. Then find the product and compare it with your estimate.

	a	b
4.	6 × 48 = ?	5 × 104 = ?
5.	5 × 51 = ?	3 × 489 = ?
6.	8 × 37 = ?	6 × 307 = ?

7. If the price of gasoline is 29¢ a gallon, what is a good estimate of the cost of 5 gallons? of 8 gallons?

— In Ex. 8–11, one of the numbers in each set shown at the right makes the sentence true. Estimate to find which number it is.

8. $7 \times 89 = n$ {623, 563}
9. $9 \times 22 = n$ {278, 198}
10. $8 \times 68 = n$ {484, 544}
11. $8 \times \$3.97 = n$ {\$25.16, \$31.76}

12. Round each factor in the example at the right to the nearest ten. What is 30 × 60? Then the product of 62 and 28 should be about ?. Copy the example and find the product.

$$\begin{array}{r} 62 \\ \times\ 28 \\ \hline \end{array}$$

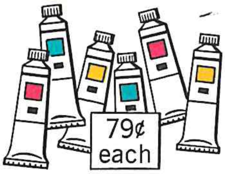
13. Estimate the products. Then find the products and compare them with your estimates.

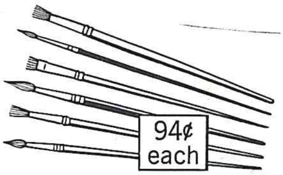
a	b	c	d	e
93	86	97	54	76
× 24	× 52	× 46	× 38	× 65

14. Jack plans to save 35¢ a week. In one year (52 weeks), about how much will he save?

What is 52 × 35?

15. Estimate the cost of 12 of these tubes of paint; of 26 of these paintbrushes.





Multiplying by hundreds, tens, and ones

1. Jean and Bill found the product of 364 and 216. Did they both get the same answer?

- Explain how Jean found each of her three partial products.
- Compare Bill's partial products with Jean's. Why can Bill write his second and third partial products as he does?

JEAN		BILL
216		216
× 364		× 364
864	← 4 × 216 →	864
12,960	← 60 × 216 →	12 96
64,800	← 300 × 216 →	64 8
78,624		78,624

2. Multiply using either way. Check by going over your work.

a	b	c	d	e	f	g	h	i
604	415	842	523	364	406	482	628	173
× 253	× 341	× 684	× 423	× 592	× 386	× 547	× 358	× 159

3. Shown here are two ways to multiply 357 by 280. Notice that the numeral for 280 has a 0 in the ones place.

- The first partial product shows _?_ × 357.
- In B why can the second partial product be written as it is? The digit 4 is written in which place?

A		B
357		357
× 280		× 280
28,560	← 80 × 357 →	28 560
71,400	← 200 × 357 →	71 4
99,960		99,960

4. Shown here are two ways to multiply 476 by 206.

- In C the first partial product shows 6 × 476; the second partial product shows _?_ × 476.
- In D the first partial product shows _?_ × 476. Explain how the second partial product shows 200 × 476.

C		D
476		476
× 206		× 206
2,856	← 6 × 476 →	2 856
95,200	← 200 × 476 →	95 2
98,056		98,056

5. Multiply using either way. Check by going over your work.

284	367	375	849	764	572	435	206	670
× 260	× 290	× 406	× 920	× 305	× 630	× 508	× 470	× 209

f you need more practice, do Practice Set 29, page 333.

Practice in multiplication

1. Multiply 246 by 172. Should you get the same answer if you multiply 172 by 246? Why? Explain how you can use the commutative property to check multiplication.

— Find the products. Check by using the commutative property.

	a	b	c	d	e	f	g	h	i	j
2.	38 × 2	60 × 8	17 × 3	58 × 6	27 × 7	45 × 5	16 × 4	43 × 9	57 × 4	64 × 6
3.	60 × 4	68 × 7	75 × 7	96 × 8	86 × 5	95 × 4	69 × 3	68 × 6	49 × 9	57 × 9

— Solve these open sentences.

	a	b	c	d	e
4.	$8 \times 53 = n$	$9 \times 18 = n$	$73 \times 6 = n$	$85 \times 4 = n$	$n = 6 \times 75$
5.	$5 \times 83 = n$	$n = 8 \times 56$	$7 \times 82 = n$	$58 \times 6 = n$	$9 \times 65 = n$
6.	$69 \times 7 = n$	$8 \times 98 = n$	$n = 79 \times 5$	$9 \times 80 = n$	$4 \times 89 = n$
7.	$n = \$.76 \times 9$	$6 \times \$.88 = n$	$8 \times \$.84 = n$	$n = 5 \times \$.98$	$6 \times \$.96 = n$

— Copy, find the products, and check.

	a	b	c	d	e	f	g
8.	27 × 36	69 × 45	58 × 54	95 × 60	67 × 35	47 × 46	38 × 40
9.	65 × 89	40 × 39	76 × 57	$.90 × 78	$.73 × 94	$.88 × 78	$.57 × 48
10.	732 × 323	890 × 76	907 × 865	978 × 360	626 × 740	799 × 465	574 × 890
11.	476 × 465	574 × 398	507 × 605	987 × 708	999 × 406	860 × 706	609 × 908
12.	$3.98 × 68	$9.56 × 95	$8.37 × 67	$6.78 × 86	$4.76 × 320	$84.58 × 9	$69.37 × 7

Multiplication and inequalities

1. What is 9 × 18? Is it greater than or less than 90? 180?

Which of the following number sentences are true?

$9 \times 18 > 90$ $\quad$ $9 \times 18 > 180$

$9 \times 18 < 180$

2. Is this number sentence true or is it false? $7 \times 49 < 350$ Think: 49 is less than 50, or 5 tens; 7 × 5 tens is equal to 35 tens, or 350; so 7 × 49 is less than 350.

— Tell whether each of the following number sentences is true or false.

	a	b
3.	$2 \times 27 > 40$	$2 \times 27 > 60$
4.	$5 \times 54 < 250$	$5 \times 54 < 300$
5.	$8 \times 32 > 240$	$8 \times 32 > 320$
6.	$7 \times 81 > 560$	$7 \times 81 < 630$

7. What do we call this set of numbers, $\{0, 2, 4, 6, 8, \cdots\}$? What numbers from this set will make this number sentence true? $3 \times n < 20$

8. What numbers from the set of odd numbers will make this sentence rue? $3 \times n < 20$

9. What are the next three members of this set? $\{0, 3, 6, 9, 12, \cdots\}$ What numbers from this set make his sentence true? $3 \times n < 20$

0. Name the first ten multiples of 4. What multiples of 4 make this sen-ence true? $8 \times n < 100$

11. What number in {1 ten, 2 tens, 3 tens, 4 tens} does *not* make these sentences true?

$8 \times n <$ 25 tens $\quad$ $8 \times n < 250$

— For each sentence below, tell which number in the set makes the sentence false.

12. $5 \times n <$ 25 tens {3 tens, 4 tens, 5 tens}

13. $n \times 4 >$ 7 tens $\{10, 20, 30, 40\}$

14. $12 \times n < 90$ $\{5, 6, 7, 8\}$

15. $n \times 28 > 90$ $\{3, 6, 9, 12\}$

— Tell which numbers in each set make the sentence true.

16. $7 \times n >$ 12 tens {3 tens, 4 tens, 5 tens}

17. $n \times 9 < 280$ $\{10, 20, 30, 40\}$

18. $15 \times n > 50$ $\{2, 4, 6, 8\}$

19. $n \times 42 < 450$ $\{3, 6, 9, 12\}$

20. For Ex. 16–19, tell the greatest number in each set that makes the sentence true.

21. What multiples of 10 make this sentence true? $7 \times n < 452$ What is the *greatest* multiple of 10 that makes the sentence true?

— Tell the greatest multiple of 10 that makes the sentence true.

	a	b
22.	$6 \times n <$ 32 tens	$6 \times n < 320$
23.	$8 \times n <$ 60 tens	$8 \times n < 600$
24.	$n \times 9 <$ 50 tens	$n \times 9 < 500$

Multiplication and problem solving

1. In a school auditorium there are 28 rows of seats with 36 seats in each row. How many seats are there in this auditorium?

- Can you use this number sentence to find the answer to the problem? $28 \times 36 = n$
- Can this sentence also be used to find the answer? $36 \times 28 = n$
- Solve either sentence and tell the answer to the problem.

2. On a loading platform, cases of canned peaches are arranged in stacks. Each stack is 5 cases high. There are 4 rows of stacks with 7 stacks in each row. How many cases are on the platform?

- Can you use this number sentence to find the answer to the problem? $(5 \times 4) \times 7 = n$
- Write at least two other sentences that could be used to find the answer.
- Solve any one of these sentences and tell the answer to the problem.

— Write a number sentence for each of the following problems. Then solve the sentence and write the answer to the problem.

3. During one hour, 35 cartons of books were loaded onto a truck. Each carton contained 48 books. How many books were loaded during the hour?

4. Each floor on the west side of a new apartment building has 27 windows. Each window has 3 panes of glass. There are 18 floors in the building. How many panes of glass were needed for the west side of the building?

5. Twenty-four cases are stacked for loading on a platform. Each case contains 96 cans of applesauce. How many cans of applesauce are there to be loaded?

6. A club votes to buy a jacket for each of the 16 members. Each jacket costs $4.95. How much money will the club need in order to buy the jackets?

7. On a supermarket shelf, soup is arranged in stacks with 5 cans in each stack. There are 7 rows of stacks with 22 stacks in each row. How many cans of soup are on the shelf?

8. A ticket for the school play costs 75¢. If 220 tickets are sold, how much money will be collected?

9. Jean spends 18¢ a day to ride the bus. How much does she spend in a year if she rides on the bus 5 days a week for 42 weeks a year?

10. A school gymnasium has 14 rows of bleachers. Each row can seat 47 people. How many people can sit on the bleachers?

How well can you multiply?

— These examples will help you find out how well you can multiply. Check each answer by going over your work.

	a	b	c	d	e	f
1.	67 × 5	87 × 6	97 × 58	74 × 78	$.98 × 94	$.36 × 62
2.	579 × 90	480 × 97	806 × 69	634 × 53	$3.58 × 68	$6.79 × 34
3.	679 × 843	526 × 126	509 × 673	725 × 497	$8.69 × 254	$7.30 × 172
4.	865 × 700	526 × 205	6,245 × 94	8,399 × 48	$84.27 × 86	$70.08 × 74

— These exercises will help you to multiply mentally.

5. To multiply 46 by 2 mentally, think:

$$(2 \times 40) + (2 \times 6) = 80 + 12 = \underline{?}$$

6. Try these without pencil and paper.

a	b	c	d	e	f	g	h
11 × 9	23 × 3	48 × 2	36 × 5	15 × 4	68 × 6	97 × 7	72 × 6

7. Mike's father drives 14 miles to and from work every day. In days, how many miles would he drive to and from work?

8. A club uniform costs $5.85. Would $60.00 be enough to buy 12 f these club uniforms?

9. The Browns' station wagon will travel about 14 miles on 1 gallon gasoline. Would 7 gallons of gasoline be enough to drive 80 miles? 0 miles? 100 miles?

you need more practice, do Practice Set 30, page 333.

The meaning of division

1. Maureen is going to bake 32 cupcakes. She can put 8 cupcakes in one tin. How many tins will she need?

- Does the number of tins times the number of cupcakes in each tin equal the total number of cupcakes?
- Does this sentence tell about the problem? $n \times 8 = 32$ In this sentence, do you know one factor and the product? What is the other factor?
- Does this sentence also tell about the problem? $32 \div 8 = n$ What number makes this sentence true?

2. Mary has 6 boxes of candy canes with the same number of canes in each box. In all, she has 48 canes. How many candy canes are there in each box?

- Does the number of boxes times the number of candy canes in each box equal the total number of canes?
- Does this sentence tell about the problem? $6 \times n = 48$ In this sentence, do you know one factor and the product? What is the other factor?
- Does this sentence also tell about the problem? $48 \div 6 = n$ What number makes this sentence true?

3. How do you find a factor when you know one factor and the product? What number makes each of the following sentences true?

$n \times 9 = 54$ $\quad$ $54 \div 9 = n$

— Solve these pairs of sentences.

4. $8 \times n = 24$ $\quad$ $24 \div 8 = n$

5. $\triangle \times 5 = 45$ $\quad$ $45 \div 5 = \triangle$

6. $7 \times \square = 28$ $\quad$ $28 \div 7 = \square$

7. $n \times 6 = 36$ $\quad$ $36 \div 6 = n$

Are multiplication and division related operations?

— Solve these sentences.

8. $(8 \times 4) \div 4 = n$

9. $(5 \times 7) \div 7 = \square$

10. Does dividing by a number undo what multiplying the number does? Then division is the inverse of $\underline{?}$.

11. In the number sentence and division example at the right, 32 is the **dividend**, 8 is the **divisor**, and 4 is the **quotient**.

$32 \div 8 = 4$

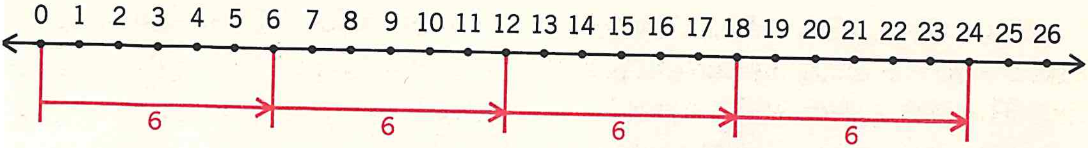
$$\begin{array}{r} 4 \\ 8\overline{)32} \end{array}$$

What is the dividend in this sentence? $8 = 56 \div 7$ What is the divisor? the quotient?

— Tell the dividend and the divisor in each of the following. Then find the quotient.

	a	b
12.	$6\overline{)42}$	$18 \div 3 = \square$
13.	$8\overline{)72}$	$20 \div 4 = n$
14.	$7\overline{)49}$	$\triangle = 12 \div 3$
15.	$3\overline{)27}$	$14 \div 2 = \square$
16.	$8\overline{)64}$	$n = 25 \div 5$

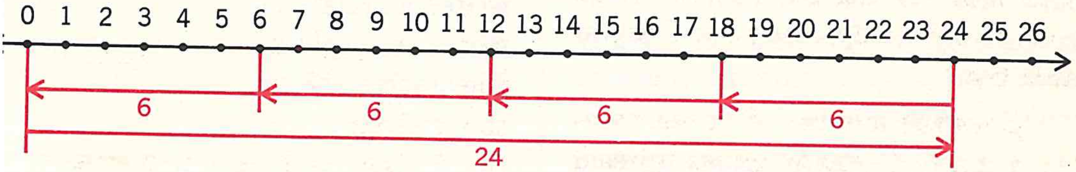
Division and the number line



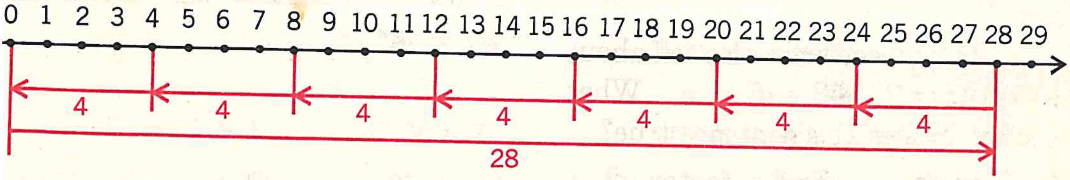
1. You can sometimes find the product of whole numbers by using ddition. You can think of 4 × 6 as 6 used as an addend ? times. Explain how the arrows on the number line above can be used to find he product of 6 and 4.

Why can't you use addition to help find the product of 1 and 6? he product of 0 and 6?

2. In the same way that some multiplication examples are related to epeated addition, some division examples can be solved by repeated ubtraction. To find the quotient for 24 ÷ 6, you can subtract 6 rom 24 and continue to subtract until the result is 0. How many imes can you subtract 6? Then what is the quotient? Explain ow this is shown on the number line below.

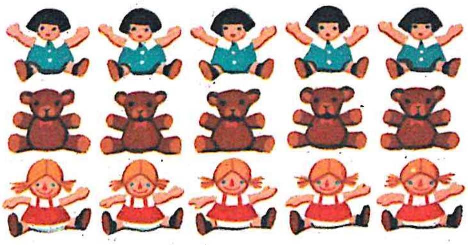


3. What division fact is shown on the number line below? What umber is being subtracted? How many times is it subtracted to rrive at 0? What number is the dividend? the divisor? the quoent?



. To find what 42 ÷ 3 is, subtract 3 from 42 and continue to btract until you arrive at 0. How many times do you subtract 3? hen what is 42 ÷ 3? Use a model of the number line to show the vision of 42 by 3.

Related sentences



1. How many rows of dolls are there in the set shown above? How many dolls are there in each row? To find how many dolls are in the set, can you use this sentence?

$$3 \times 5 = n$$

Can you also use this one? Explain.

$$5 \times 3 = n$$

2. Suppose you know there are 15 dolls arranged in rows with 5 dolls in each row. To find how many rows there are, can you use this sentence? $15 \div 5 = n$

3. There are 15 dolls arranged in 3 rows with the same number of dolls in each row. To find how many dolls are in each row, can you use this sentence? $15 \div 3 = n$

4. Do the four related sentences below use the same set of numbers? What are the numbers in the set?

$3 \times 5 = 15$ $5 \times 3 = 15$
$15 \div 3 = 5$ $15 \div 5 = 3$

5. Write four related number sentences using each of these sets of numbers.

$\{5, 7, 35\}$ $\{6, 8, 48\}$ $\{6, 9, 54\}$

6. John said, "I am thinking of some number. The product of this number and 6 is 48. What is the number?"

- In this multiplication sentence, does n stand for the number John is thinking about? $n \times 6 = 48$
- Is the following division sentence related to the multiplication sentence above? $48 \div 6 = n$ Does n stand for the quotient of $48 \div 6$?
- What does $48 \div 6$ equal? Does this number make both the division sentence and the multiplication sentence above true?

— For each number sentence below, write a related division sentence where n stands for the quotient. Then solve the sentence.

	a	b
7.	$n \times 8 = 64$	$n \times 4 = 32$
8.	$5 \times n = 50$	$3 \times n = 18$
9.	$n \times 7 = 49$	$6 \times n = 42$

10. Jim said, "I'm thinking of two factors of 40. One of the factors is 8. What is the other factor?"

11. Susan said, "Four times some number is 28. What is the number?"

12. Since multiplication and division are related — that is, they are inverse operations — explain how you can use multiplication to check your answer for a division sentence.

Using division to solve problems

1. It takes 9 players to make up a baseball team. How many teams can be made up from 36 players?

- Ann thought, "The number of teams is the number of 9's in 36." She wrote this sentence.

$$36 \div 9 = n$$

- Tony thought, "The number of teams times 9 equals 36." He wrote this sentence.

$$n \times 9 = 36$$

- Should Ann and Tony both get the same answer? Explain. What is the correct answer for both of these sentences?

2. Forty-eight children are to march in groups of 8. How many of these groups of 8 will there be?

- Can you use this multiplication sentence to find the answer?

$$n \times 8 = 48$$

- Is this sentence related to the multiplication sentence above?

$$48 \div 8 = n$$

- What number does n stand for in both of these sentences?

3. How many 5-cent pencils can you buy for 45¢?

Can you use this sentence to find he answer? $45 \div 5 = n$ Does n tand for a number of cents or a umber of pencils? What number oes n stand for?

4. Stanley paid 30¢ for 6 candy bars. He paid the same amount for each bar. How much did each bar cost?

Can this sentence be used to find the answer? $30 \div 6 = n$ Does n stand for a number of cents or a number of candy bars? What number does n stand for?

5. Ben is planting trees in rows of 6. If he plants 54 trees, he will have $\underline{?}$ rows.

64 ÷ 8 =

6. How many pounds of potatoes at 8¢ a pound can be bought for 64¢?

How many pounds of these potatoes can be bought for 72¢? for 56¢?

7. Jim placed 72 chairs in rows with 8 chairs in each row. How many rows of chairs were there?

8. Joe placed 72 chairs in 9 rows with the same number of chairs in each row. How many chairs were in each row?

9. Suppose you know the number of members in a set, and this set is separated into subsets with the same number of members in each subset.

- If you know the number of members in each subset, what do you do to find the number of subsets?
- If you know the number of subsets, what do you do to find the number of members in each subset?

Division with remainders that are not zero

— For each of the following number sentences, find the *greatest* whole number that makes the sentence true.

	a	b
1.	$6 \times n < 45$	$n \times 3 < 20$
2.	$n \times 4 < 35$	$8 \times n < 25$
3.	$5 \times n < 22$	$n \times 7 < 45$
4.	$n \times 2 < 15$	$9 \times n < 50$
5.	$7 \times n < 22$	$n \times 3 < 28$
6.	$n \times 5 < 36$	$9 \times n < 12$

$$\boxed{7\overline{)25}}$$

7. The example at the right asks you to find the greatest whole number of 7's in 25. Are there at least 2 sevens in 25? Are there at least 3 sevens in 25? Are there as many as 4 sevens in 25? Then $\underline{\ ?\ }$ is the greatest whole number of 7's in 25.

What number remains after you subtract 3 sevens from 25? This number is called the **remainder.**

$$25 = (3 \times 7) + \underline{\ ?\ }$$

$$\boxed{5\overline{)48}}$$

8. Which of the following number sentences are true?

$8 \times 5 < 48$ $9 \times 5 < 48$

$10 \times 5 < 48$

Which of the sentences above tells you the greatest whole number of 5's in 48? What is the remainder after you subtract 9 fives from 48?

$$48 = (9 \times 5) + \underline{\ ?\ }$$

9. To find the quotient for $6\overline{)26}$, first find the greatest whole number of 6's in 26. Is it 3? 4? 5? Now subtract this number of 6's from 26. What is the remainder?

Copy this sentence and replace the $\square$ with a numeral for the greatest whole number of 6's in 26 and the $\triangle$ with a numeral for the remainder.

$$26 = (\square \times 6) + \triangle$$

— Copy the following exercises. Replace each $\square$ with a numeral for the greatest whole number of divisors in the dividend and each $\triangle$ with a numeral for the remainder.

10. $6\overline{)26}$ $26 = (\square \times 6) + \triangle$

11. $7\overline{)25}$ $25 = (\square \times 7) + \triangle$

12. $4\overline{)15}$ $15 = (\square \times 4) + \triangle$

13. $5\overline{)37}$ $37 = (\square \times 5) + \triangle$

14. $8\overline{)43}$ $43 = (\square \times 8) + \triangle$

15. $3\overline{)20}$ $20 = (\square \times 3) + \triangle$

16. $2\overline{)13}$ $13 = (\square \times 2) + \triangle$

17. $9\overline{)50}$ $50 = (\square \times 9) + \triangle$

18. $7\overline{)56}$ $56 = (\square \times 7) + \triangle$

19. $4\overline{)31}$ $31 = (\square \times 4) + \triangle$

20. $6\overline{)42}$ $42 = (\square \times 6) + \triangle$

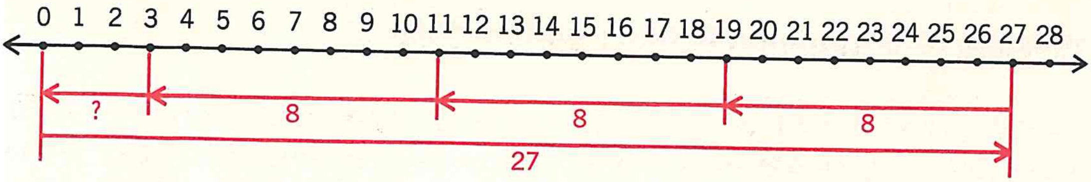
21. $28 = (\square \times 4) + \triangle$

22. $29 = (\square \times 4) + \triangle$

23. $30 = (\square \times 4) + \triangle$

24. $31 = (\square \times 4) + \triangle$

25. $32 = (\square \times 4) + \triangle$



26. The drawing above shows you what happens when you divide 27 by 8. What is the greatest whole number of 8's in 27? What is the remainder when you subtract this number of 8's from 27?

$$27 = (\square \times 8) + \triangle$$

27. Draw a model of the number line. On your model, show what happens when you divide 18 by 5.

28. What is the greatest whole number of 6's in 46? What is the remainder when you subtract this number of 6's from 46? The example at the ight shows how we xpress the answer to 6 ÷ 6.

7 r4
$6\overline{)46}$

— Copy the following examples and how the answers in a way similar to hat used in Ex. 28.

	a	b	c	d
29.	$4\overline{)23}$	$6\overline{)52}$	$3\overline{)20}$	$7\overline{)45}$
30.	$5\overline{)42}$	$8\overline{)38}$	$7\overline{)54}$	$9\overline{)19}$

31. In Ex. 28, the answer is 7 r4. an you use this sentence to check he answer?

$$46 = (7 \times 6) + 4$$

Write a similar sentence to check ach answer in Ex. 29 and 30.

32. In the example at the right, what is the divisor? What is the remainder? Is the remainder greater than the divisor? Then is 5 the greatest whole number of 8's in 50? What is the greatest whole number of 8's in 50? What is the remainder?

INCORRECT
5 r10
$8\overline{)50}$

33. In an answer to a division example, the remainder should never be equal to or greater than the ?. Explain.

— Copy each example below and find the answer. Write a true number sentence to check each answer.

	a	b	c	d
34.	$9\overline{)30}$	$7\overline{)38}$	$8\overline{)66}$	$4\overline{)34}$
35.	$6\overline{)40}$	$5\overline{)22}$	$3\overline{)28}$	$2\overline{)15}$
36.	$7\overline{)25}$	$9\overline{)34}$	$4\overline{)37}$	$3\overline{)14}$

Think twice!

37. What has been done to each number shown in row A to get the number shown in row B? Copy and complete the table.

A	3	8	5	1	7	4	6
B	11	26	17	5	?	?	?

PROBLEM-SOLVING HELP

Division with remainders and problem solving

1. Five boys bought 5)36 pencils. They plan to share the pencils equally. Each boy should receive _?_ pencils.

- What is the greatest whole number of 5's in 36? Then how many pencils should each boy receive?
- Is there a remainder? What is it? Is the remainder the number of pencils left over?
- Explain how this sentence can be used to answer the problem.

$$36 = (\square \times 5) + \triangle$$

2. Twenty-six cartons are to be stacked on a loading platform. No stack should contain more than 4 cartons. How many stacks, 4 cartons high, can be made from the 26 cartons?

- Are you asked to find the greatest whole number of 4's in 26? Will there be a remainder?
- Explain how this sentence can be used to answer the problem.

$$26 = (\square \times 4) + \triangle$$

- The remainder means that one stack will contain only _?_ cartons.

3. A candy bar costs 6¢. How many candy bars can Elizabeth buy with 25¢?

- Write a number sentence and use it to answer the problem.
- Does the remainder mean 1 cent or 1 candy bar?

4. On a jet plane, there are 5 seats in each row. Forty-eight passengers can fill _?_ of these rows.

5. Mr. Carter wants each box of tomatoes to contain exactly 4 tomatoes. How many boxes can he fill using 35 tomatoes? How many tomatoes will he have left over?

6)52

6. Fifty-two trading cards are to be shared by 6 boys. Each boy is to receive the same number of cards. What is the least number of cards that will be left over?

7. Each baseball team requires 9 players. How many teams can be formed with 50 players?

8. Mary wants to buy as many pencils as she can with 45¢. Each pencil costs 8¢. How many can she buy?

9. Leonard and his three brothers caught 28 fish. If each boy caught the same number of fish, how many did Leonard catch?

10. Barbara and Betsy are giving a party. There will be 7 girls at the party. Barbara is baking cookies — 3 pans with 8 cookies on each pan. Betsy is baking 5 pans of cookies with 6 cookies on each pan.

24

30

7)54

If the girls at the party share the cookies equally, how many will each get? How many cookies will be left over?

Keeping up in mathematics

Oral

1. What number is 8 greater than 26?

2. The product of 8 and 9 is ?.

3. One factor of 56 is 7; the other actor is ?.

4. The difference between 33 and 8 is ?.

5. What number is 25 greater than 2?

6. Complete these sentences.

- 87 sec. make ? min. 27 sec.
- 148 sec. make 2 min. ? sec.

- Solve these open sentences.

	a	b
7.	$n - 17 = 40$	$50 - n = 12$
8.	$15 + n = 60$	$n - 7 = 42$

- Read each of these numerals.

9.	268,496	406,843
10.	8,000,000	2,005,090
11.	2,703,800	3,000,000,000
12.	1,008,360	2,765,298,432

13. Count by:

- 9's from 35 to 80.
- 500's from 37,000 to 40,000.

. Tell the first ten multiples of 2; 3; of 5.

. What number is:

- 10,000 greater than 497,000?
- 100,000 less than 1 million?

Written

— Copy, compute, and check.

	a	b	c
1.	241 58 + 37	670 512 + 902	4,687 9,504 + 863
2.	883 − 59	940 − 515	700 − 286
3.	68 × 79	482 × 97	509 × 64

— For each sentence, list the set of whole numbers that make it true.

	a	b
4.	$195 - 38 = n$	$400 - 163 = \square$
5.	$106 - 47 = \square$	$546 - 307 = n$
6.	$7 + \square > 10$	$725 + 809 = \square$
7.	$n \times 4 > 37$	$\square - 386 = 592$
8.	$3 + \square < 8$	$314 - 298 > n$
9.	$n + 5 < 6$	$456 + 394 = n$
10.	$6 \times n < 18$	$672 + n = 845$

Think twice!

— What number is halfway between:

- 10,000 and 11,000?
- 10,000 and 20,000?
- 10,000 and 15,000?
- 200,000 and 100,000?
- 500,000 and 1,000,000?

111/

Comparing numbers

1. Sally is 8 years old. Her mother is 32 years old. What is the difference between Sally's age and her mother's age?

To find the answer, do you add, subtract, multiply, or divide?

2. Fred is 9 years old. His father is 36 years old. How many times as old as Fred is his father?

To find the answer, do you add, subtract, multiply, or divide?

3. In which exercise above did you compare two numbers by subtraction?

In which of the exercises above did you compare two numbers by division?

4. Would you use subtraction or division to find:

- how much more a coat costs than a hat?
- how many times as much a coat costs as a hat?

5. Bill is 9 years old. His grandfather is 63 years old.

- What question can Bill answer by solving sentence **a** below?
- What question can Bill answer by solving sentence **b** below?

a. $63 \div 9 = n$ **b.** $63 - 9 = n$

6. A quarter is worth how much more than a nickel? how many times as much as a nickel?

7. A tablet costs 30¢. A pencil costs 5¢. What is the 25 difference between the costs of these two items?

How many times as much as the cost of the pencil is the cost of the tablet?

8. An hour is how many times as long as 10 minutes?

An hour is how much longer than 10 minutes?

9. A yard is how many times as long as 4 inches?

A yard is how much longer than 4 inches?

10. Six yards is how much longer than 2 feet?

Six yards is how many times as long as 2 feet?

11. Mike has 56 trading cards. Harry has 8 trading cards.

- Joe said, "Mike has 48 more cards than Harry." Did he compare the numbers of the sets of cards by subtraction or by division?
- Mary said, "Mike has 7 times as many cards as Harry." Did Mary use subtraction or division to compare the two numbers?

12. Fred swam 50 feet. Sally swam 10 feet.

$$\begin{array}{r}5\\10\overline{)50}\end{array}$$

- Fred swam how many times as far as Sally?
- Fred swam how much farther than Sally?

Multiplying measures

1. What property of multiplication lo you use when you rewrite

3 × (5 tens + 2 ones)

as (3 × 5 tens) + (3 × 2 ones)?

When you multiply tens and ones y 3, you multiply the ones by 3 and he ? by 3.

2. One day Jack ran to school in 5 inutes 2 seconds. It took Jill 3 imes as long to walk the same disance. How long did it take Jill to alk to school? Explain this example.

5 min. 2 sec.
× 3
15 min. 6 sec.

How is multiplying a number of inutes and a number of seconds by similar to multiplying a number of ns and a number of ones by 3?

Multiply each of these by 3.

	a	b
.	3 min. 4 sec.	2 gal. 1 qt.
.	5 lb. 2 oz.	3 ft. 2 in.

. Bob spends 7 hours 15 minutes a y in school. He used this example find how much time he spends in hool during one week.

7 hr. 15 min.
× 5
35 hr. 75 min., or 36 hr. 15 min.

How did Bob rename 35 hours 75 nutes as 36 hours 15 minutes?

6. Helen practices the piano 1 hour 15 minutes each day. In 5 days she practices ? hours ? minutes.

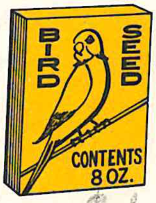
7. Sandra's class spends 45 minutes on mathematics, 45 minutes on science, and 45 minutes on reading each day. These three subjects take ? hours ? minutes a day.

Copy these examples and write the answers.

	a	b
8.	3 wk. 4 da. × 5	2 gal. 3 qt. × 6
9.	4 ft. 5 in. × 9	6 lb. 9 oz. × 4
10.	10 min. 28 sec. × 3	2 qt. 1 pt. × 5

11. There are ? hours in 3 days. There are ? minutes in 3 days.

12. How many pounds of bird seed will Brian be buying if he buys 4 packages like the one in the picture below?



8 1
8

If you need more practice, do Practice Set 31, page 334.

1
24
24

True or false

— Read statements 1–20 very carefully. Then tell if each is *true* or *false*. If you say *false*, give an example to show why.

1. The product of two factors that are both whole numbers is always greater than either factor.

2. If some number is added to one of two factors and subtracted from the other factor, the product remains unchanged.

3. Multiples of 6 are divisible exactly (the remainder is 0) by 2 and by 3.

4. The quotient is always less than the divisor.

5. The difference between two numbers is always greater than one of the numbers.

6. The commutative property applies to division.

7. The associative property applies to subtraction.

8. An approximation is always less than the number that is being approximated.

9. If two addends are equal, their sum is the same as two times one of the addends.

10. The commutative property applies to subtraction.

11. A number of days is the same length of time as 24 times the same number of hours.

12. A number of minutes is the same length of time as 60 times the same number of hours.

13. The difference between two numbers is always less than one of the numbers.

14. If each of two addends is increased by the same number, their sum will be increased by two times that number.

15. Numbers whose numerals have 0 in the ones place are exactly divisible (the remainder is 0) by 2, 5, and 10.

16. Numbers whose numerals have 3 in the ones place are exactly divisible by 3.

17. In a decimal numeral for a whole number, the first digit at the left has a greater place value than any of the other digits.

18. Since numbers exactly divisibl by 2 are even numbers, then numbers whose numerals have 0, 2, 4, 6, or 8 in the ones place are even numbers.

19. Since numbers not exactly divisible by 2 are odd numbers, ther numbers whose numerals have 1, 3 5, 7, or 9 in the ones place are od numbers.

20. If an addend and a sum are eac increased by the same amount, thei difference is decreased by tha amount.

SELF-HELP TEST 1

If you make a mistake on a Self-Help Test, you can find help yourself. If your mistake is in Ex. 1, you can find help on page 10. Where can you find help for Ex. 2?

1. The set that has no members is called the ? set. [10]

2. Write numerals for the next five members of each set shown below. [11]

- {1, 3, 5, 7, · · ·}
- {0, 2, 4, 6, · · ·}

— Solve these sentences.

3. $321 + 485 = \triangle + 321$ [21]

4. $(75 + n) + 40 = 75 + (35 + 40)$ [22]

5. Write a 3-digit numeral to name he number shown below. [3]

$7 \times 10 \times 10) + (3 \times 10) + (6 \times 1)$

6. In a decimal numeral, the value of each place is ? times the value of the place at its right. [2]

7. 352 = 3 hundreds 4 tens ? ones [13]

8. 600 = 5 hundreds ? tens 10 ones [13]

9. Which of the sets of numbers shown below is a subset of the set of even numbers? [11]

{4, 7, 10} {12, 14, 16} {1, 2, 3}

10. If the 7 in 27,632,844,590 is changed to a 9, the number named is ? greater than 27,632,844,590. [5]

ELF-HELP TEST 2

— Copy and find the answers.

1.
$$\begin{array}{r} 317 \\ +\,240 \\ \hline \end{array}$$
[23]

2.
$$\begin{array}{r} 3{,}759 \\ 8{,}467 \\ +\,4{,}285 \\ \hline \end{array}$$
[25]

.
$$\begin{array}{r} 793 \\ -\,411 \\ \hline \end{array}$$
[32]

4.
$$\begin{array}{r} 9{,}860 \\ -\,3{,}876 \\ \hline \end{array}$$
[34]

.
$$\begin{array}{r} \text{3 ft. 8 in.} \\ +\,\text{6 ft. 9 in.} \\ \hline \end{array}$$
[39]

.
$$\begin{array}{r} \text{5 hr. 20 min.} \\ -\,\text{2 hr. 45 min.} \\ \hline \end{array}$$
[39]

7. Find the solution set from the set of whole numbers for each of the following number sentences. [18]

$n + 7 < 36$ $n + 12 > 30$

— Round each of the numbers named below to the nearest ten; to the nearest hundred. [14–15]

	a	b	c
8.	3,451	426,011	72,938
9.	5,007	321,998	40,055

10. Write a Roman numeral for 631; for 87; for 258. [6–7]

IT'S CHECK–UP TIME

— Solve each of the following open sentences.

1. $13 \times \triangle = 18 \times 13$

2. $(15 \times 3) \times 5 = 15 \times (\square \times 5)$

3. $6 \times (3 + 4) = (6 \times \square) + (6 \times 4)$

4. $n \times 4 = 36$

5. $64 \div n = 8$

6. $28 + 28 + 28 = \triangle \times 28$

7. $58 = (9 \times 6) + n$

8. Write four related number sentences for the set of numbers shown below.

$$\{8, 5, 40\}$$

— Write only the numerals for the products.

	a	b
9.	$7 \times 300 = \underline{\ ?\ }$	$436 \times 1{,}000 = \underline{\ ?\ }$
10.	$200 \times 24 = \underline{\ ?\ }$	$6{,}000 \times 11 = \underline{\ ?\ }$

— Copy each of these examples, find the answer, and check.

11.	$\begin{array}{r} 493 \\ \times\ 5 \\ \hline \end{array}$	$\begin{array}{r} 271 \\ \times\ 8 \\ \hline \end{array}$
12.	$\begin{array}{r} 984 \\ \times\ 276 \\ \hline \end{array}$	$\begin{array}{r} 543 \\ \times\ 918 \\ \hline \end{array}$
13.	$\begin{array}{r} \$2.06 \\ \times\ 57 \\ \hline \end{array}$	$\begin{array}{r} \$3.28 \\ \times\ 94 \\ \hline \end{array}$
14.	$7\overline{)53}$	$4\overline{)35}$

— Copy each example below and find the answer.

	a	b
15.	$\begin{array}{r} \text{2 min. 6 sec.} \\ \times\ 4 \\ \hline \end{array}$	$\begin{array}{r} \text{4 lb. 4 oz.} \\ \times\ 6 \\ \hline \end{array}$
16.	$\begin{array}{r} \text{5 gal. 3 qt.} \\ \times\ 8 \\ \hline \end{array}$	$\begin{array}{r} \text{2 ft. 7 in.} \\ \times\ 5 \\ \hline \end{array}$

17. List the set whose members are the first five multiples of 7.

18. A dollar is worth how much more than a dime?

A dollar is worth how many times as much as a dime?

19. A shelf contains 8 layers of boxes. Each layer has 12 rows with 7 boxes in each row. How many boxes are on the shelf?

20. Each floor in an apartment building has 8 windows. There are 32 windows in the building. How many floors are there?

If you have time, try these —

21. The product of 24 and 43 is 1,032. What number would you add to 1,032 to find the product of 25 and 43?

22. The quotient for 742 divided by 14 is 53. What number makes this sentence true?

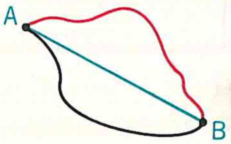
$$(742 - 14) \div 14 = n$$

CHAPTER 4

Points, paths, and lines

1. The picture above shows a set of tracks made in the snow by a squirrel. Suppose other squirrels run between the two trees, making more tracks along those made by the first squirrel. Will a path soon be worn in the snow between the two trees? Will this path be a straight path?

2. The two dots named *A* and *B* in this picture represent two points.

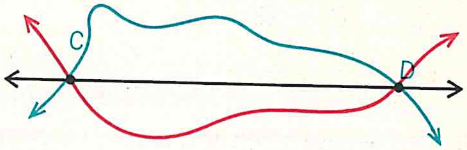


A **point** is an exact location.

Points *A* and *B* mark the locations f the trees in the picture at the top f the page. In this drawing, three aths are shown connecting the two oints. Which path in the drawing o you think represents the one made y the squirrels in the snow? Which f the three paths would you call a *raight* path? Use the straight edge f your ruler to check your answer.

3. In the drawing for Ex. 2, all three paths contain points *A* and *B*. Do the three paths also contain many other points? Can we think of a path then as a set of points?

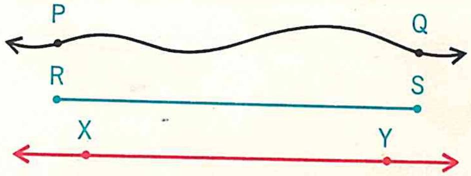
4. Look at the three paths represented in this picture.



Do all three paths contain points *C* and *D*? What do you think the arrowheads show? Which of these paths would you call a straight path?

The set of all points contained in a straight path that goes on and on in both directions without end is called a line.

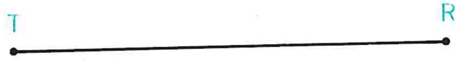
5. Look at the paths shown below.



Is the path containing points *P* and *Q* a line? Is the path containing points *R* and *S* a line? points *X* and *Y*? Explain.

12
12
24

Points, lines, and line segments



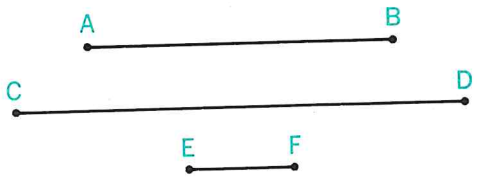
1. Place the straight edge of your ruler on the dots representing points T and R. Is the path that joins T and R a straight path? Does it contain points T and R? Can we call the straight path a line? Explain.

Does the path shown above end at points T and R? Then is this path only part of a line?

A straight path that has two endpoints is called a line segment.

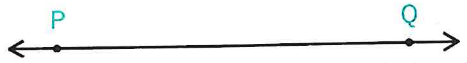
2. Name the two endpoints of the line segment shown in Ex. 1. We call the line segment shown above line segment TR or line segment RT. We write these names as $\overline{TR}$ or $\overline{RT}$.

Two ways to name the first line segment shown below are $\overline{AB}$ and $\underline{\ ?\ }$. Write two names for each of the other line segments shown.



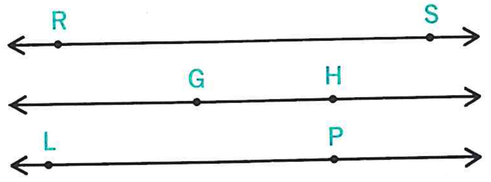
3. Fold a piece of paper in half. Does the crease represent a line segment? Name some other things in your classroom that remind you of line segments.

4. Through what two points does the line shown below pass?

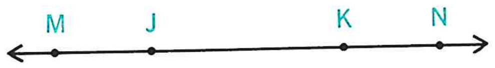


We call the line shown above, line PQ or line QP. We write these names as $\overleftrightarrow{PQ}$ or $\overleftrightarrow{QP}$. How does the symbol for a line differ from the symbol for a line segment?

Two names for the first line shown below are $\overleftrightarrow{RS}$ and $\underline{\ ?\ }$. Write two names for each of the other lines shown.



5. Is the set of points named $\overline{MN}$ in the drawing below contained in the set of points named $\overleftrightarrow{MN}$? Then is $\overline{MN}$ a subset of $\overleftrightarrow{MN}$?



- Name four other line segments that are subsets of $\overleftrightarrow{MN}$.
- Is point M a subset of $\overleftrightarrow{MN}$? Name three other points that are subsets of $\overleftrightarrow{MN}$.
- Name two points that are subsets of $\overline{MN}$; $\overline{MK}$; $\overline{JN}$; $\overline{JK}$.

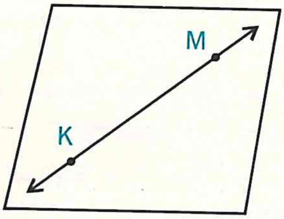
Points, lines, and planes

1. When you describe a table top, you say that it has a flat surface. Is the surface of a floor flat?

Imagine a perfectly flat surface extending on and on without end in all directions. When we imagine a perfectly flat surface extending without end, we are thinking about a **plane.**

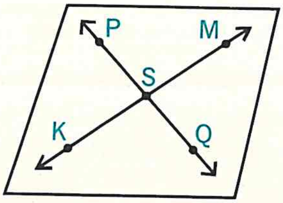
Many objects around us represent planes. A sheet of paper is not a plane, but it can represent a plane. Name some other objects that can represent planes.

2. In this pic-ure, are points K and M in the lane? Then is $\overrightarrow{KM}$ in the lane?



If two points are in a plane, then the ine containing the two points is in the ame plane.

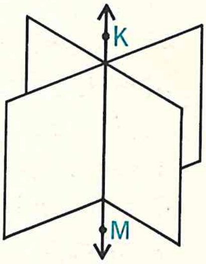
3. This picture hows two lines a plane. Do he lines share a oint? Name it.



We say $\overleftrightarrow{PQ}$ and $\overleftrightarrow{KM}$ intersect. Point is the point of intersection of $\overleftrightarrow{PQ}$ and $\overrightarrow{M}$.

4. Can two different lines share more than one point? If two different lines intersect, in how many points do they intersect? Explain.

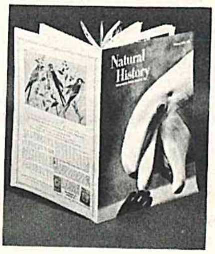
5. Here is a picture representing two planes that cross. Do the two planes share a line? Name the line.



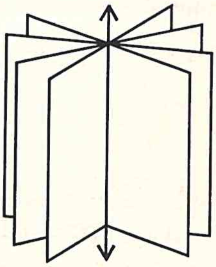
If two planes cross, they intersect in a line. The line is called the line of intersection of the two planes.

6. Can two different planes intersect in more than one line? Explain.

7. Look at this picture of a book. Do the covers and pages remind you of planes? Do the planes represented by the covers and pages intersect in only one line or in many lines?

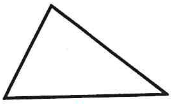


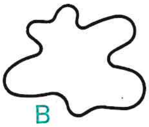
8. In this picture, how many planes are shown intersecting in one line?

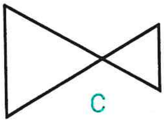


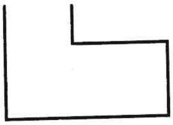
9. How many planes do you think can intersect in one line?

Polygons


A


B


C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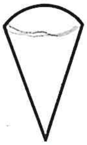
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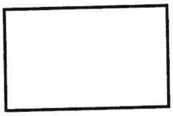
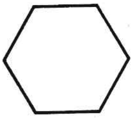
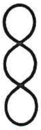
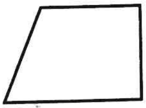

F


G


H


I

1. Do all the drawings above represent paths? Which drawings show a path that begins and ends at the same point?

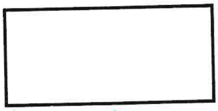
A path that begins and ends at the same point is called a **closed path.**

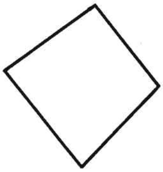
2. Which of the closed paths above are formed by line segments only? Which closed paths do not cross themselves?

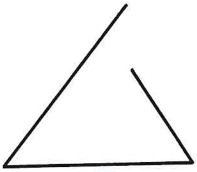
3. How many line segments form figure A? figure F? figure G? figure I?

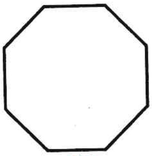
A closed path made of line segments that do not cross themselves, such as figure A, F, G, or I, is called a polygon.

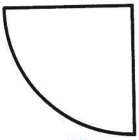
4. Study the figures shown below.


J


K


L


M


N

- Explain why figures L and N are not polygons.
- The line segments that form a polygon are called the **sides** of the polygon. How many sides does figure J have? figure K? figure M?

5. Using your pencil and ruler, draw four different closed paths that are polygons.

6. Can a polygon have as many as 8 sides? as many as 12 sides? Draw a polygon with 8 sides. Draw a polygon with 12 sides.

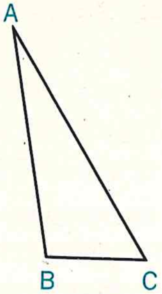
7. Can a polygon have as many as 20 sides? 30 sides? 100 sides?

Is there any end to the number of sides a polygon can have?

8. Explain why a polygon cannot have fewer than 3 sides.

Some special kinds of polygons

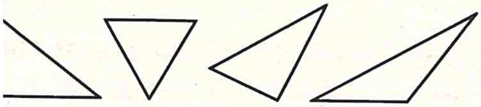
1. The polygon shown at the right has exactly ? sides. Name an endpoint of one of the sides. Is this point also an endpoint for another side of the polygon?



An endpoint shared by two sides of a polygon is called a **vertex**. The vord *vertices* means more than one vertex.

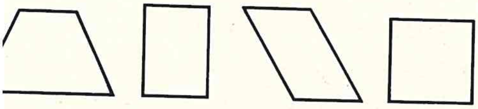
The polygon shown above has ? vertices.

2. Four different polygons are hown below. Each of them, however, has exactly ? sides.



A polygon that has exactly 3 sides called a **triangle**. How many vertices does every triangle have?

3. How many sides does each of the olygons shown below have?



A polygon that has exactly 4 sides called a **quadrilateral**. How many rtices does every quadrilateral ve?

4. Draw three different polygons that have just 5 sides. A polygon that has exactly 5 sides is called a **pentagon**. How many vertices does every pentagon have?

5. The Greek word for six is *hexa*. A polygon that has exactly 6 sides is called a **hexagon**. Draw three different hexagons. How many vertices does every hexagon have?

6. How many vertices do you think a 7-sided polygon has? Draw some pictures to illustrate your answer.

★ Your discovery

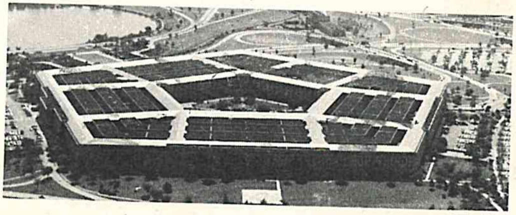
Tell your discovery

How is the number of vertices of any polygon related to the number of sides of the polygon?

Use your discovery

How many vertices has an 8-sided polygon? a 20-sided polygon? a 100-sided polygon?

7. This photograph shows a famous building near Washington, D.C. Do you know the name of this building? Why was it given this name?

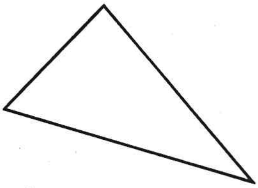


U.S. AIR FORCE

Plane figures and regions

When you think of a geometric figure such as a line, a line segment, or a polygon, you are thinking about a set of points in a plane. We call each of these sets of points a **plane figure.**

1. Is a triangle a set of points?



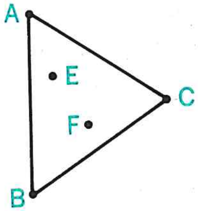
Think of this page of your book as a model of a plane. Are all the points of the triangle shown above in this plane? Is a triangle a plane figure?

2. Think of each of the plane figures shown on this page as a set of points. Do all of these points lie in the same plane? Is each figure shown on this page a plane figure?

3. Look at the four edges of the chalkboard in your classroom. Can you think of these edges as representing a quadrilateral? Does this quadrilateral lie in the plane represented by the wall?

4. In what plane is the quadrilateral represented by the edges of a post card that is lying on a desk top?

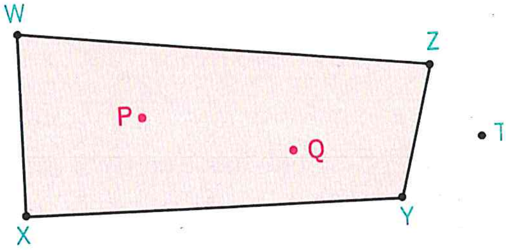
5. Think of some other models of plane figures in your classroom. Tell in what plane each figure lies.



6. How many line segments form the triangle shown here? Name the endpoints of each of these line segments. Is each of these endpoints on the triangle?

Name the three points marked on the triangle shown above. Name two points marked in the drawing that are not on the triangle. We say these two points are *enclosed* by the triangle. Are there other points enclosed by the triangle? How many? Explain.

7. These four line segments, shown in black, form what kind of polygon?

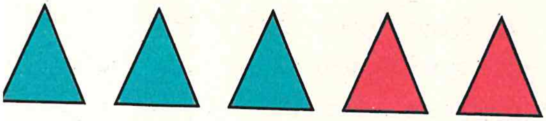


The part of the drawing in red represents the set of points enclosed by the quadrilateral. Are points P and Q enclosed by the quadrilateral? Is point T?

We call a closed path and the se of points enclosed by the path a **region.**

8. See how many models of region you can find in your classroom.

Number pairs



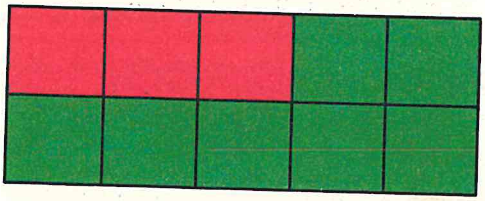
1. The drawing above shows a number of triangular regions of the same size and shape.

- How many triangular regions are shown in the drawing?
- How many regions are red?
- In the number pair (2, 5) which number tells the total number of triangular regions in the drawing? Which tells the number of these regions that are red?

2. Which of the following number pairs tells the number of triangular regions in the drawing above Ex. 1 that are blue and the total number of triangular regions?

(2, 5) (3, 5) (2, 3)

3. The large region shown below has been separated into a number of smaller regions with the same size and shape.

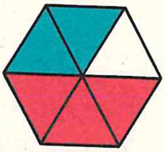


Which number pair below tells he number of regions of equal size hat are green and the total number f regions of equal size?

(3, 10) (7, 10) (3, 7)

4. What does the number pair (3, 10) tell you about the drawing for Ex. 3?

5. The hexagonal region at the right is separated into __?__ triangular regions of the same size and shape.

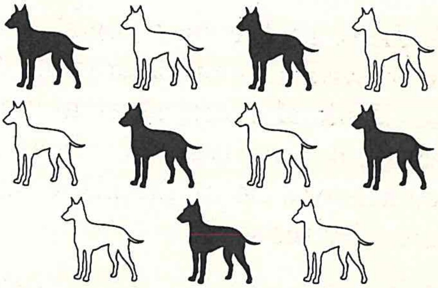


- Which of the following number pairs tells the number of triangular regions that are blue and the total number of triangular regions?

(2, 6) (1, 6) (3, 6)

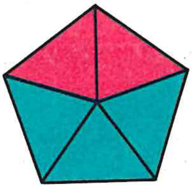
- What number pair tells the number of regions that are red and the total number of regions?
- What number pair tells the number of regions that are white and the total number of regions?

6. How many dogs are shown in the drawing below?



What number pair tells the number of dogs that are white and the total number of dogs? the number of dogs that are black and the total number of dogs?

Fractional numbers



1. The pentagonal region shown above has been separated into $\underline{?}$ triangular regions of the same size and shape. There are $\underline{?}$ blue regions. The number pair that tells the number of regions that are blue and the total number of regions is $\underline{?}$.

2. Is the number of regions colored blue in the drawing above a whole number?

Is the total number of regions in the drawing a whole number?

The number three fifths is not a whole number. It is a **fractional number.** It tells you what part of the pentagonal region is blue.

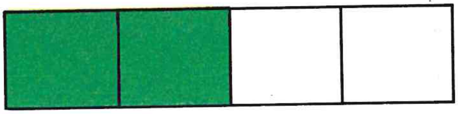
The numeral for three fifths is $\frac{3}{5}$. It is called a **fraction.** Which number shown in the fraction $\frac{3}{5}$ tells the number of blue regions? This number is called the **numerator.**

Which number shown in $\frac{3}{5}$ tells the total number of regions? This number is called the **denominator.**

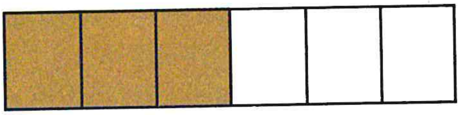
3. Is part of the pentagonal region shown in Ex. 1 blue? Is the entire pentagonal region blue? Then the number $\frac{3}{5}$ is greater than 0 and less than $\underline{?}$.

4. How many triangular regions are red in the figure for Ex. 1? Which of these fractions names the number that tells what part of the pentagonal region is red?

$\frac{3}{5}$ $\quad \frac{2}{3}$ $\quad \frac{2}{5}$ $\quad \frac{1}{2}$



5. The bar in the drawing above is separated into 4 parts of the same size and shape. Is $\frac{2}{4}$ of the bar green? Is $\frac{1}{2}$ of the bar green? Is $\frac{1}{2}$ of the bar the same as $\frac{2}{4}$ of the bar?



6. The bar in this drawing is separated into $\underline{?}$ parts of the same size and shape. How many sixths of the bar are brown? Is $\frac{1}{2}$ of the bar brown? Does $\frac{3}{6}$ name the same part of the bar as $\frac{1}{2}$?

7. Use your answers to Ex. 5 and 6 to tell if this sentence is true. $\frac{3}{6} = \frac{2}{4}$

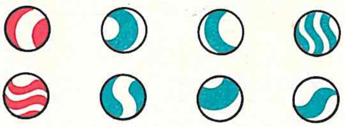
8. Draw a bar similar to those above to show that $\frac{4}{8}$ of the bar is the same part of the bar as $\frac{1}{2}$ of it.

9. How many halves are there in 1 whole bar? Does $\frac{2}{2}$ equal 1?

10. How many fourths are there in 1 whole bar? Does $\frac{4}{4}$ equal 1?

11. Copy and complete.

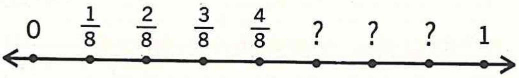
$\frac{?}{6} = 1 \qquad \frac{?}{8} = 1$



12. What is the number of marbles in the set shown above? Is $\frac{2}{8}$ of the set of marbles red? What part of the set of marbles is blue?

13. Think of the marbles as being arranged in pairs. How many pairs of marbles are there? Is 1 of these pairs red? Then is $\frac{1}{4}$ of the set of marbles red? How many fourths of the set of marbles are blue?

Does $\frac{2}{8}$ name the same part of the set of marbles as $\frac{1}{4}$? Does $\frac{6}{8}$ name the same part as $\frac{3}{4}$?



14. The line segment between 0 and 1 on this number line is separated into $\underline{?}$ segments of the same length. Each small segment is what part of the whole segment from 0 to 1?

Copy the drawing and replace the question marks with the correct fractions.

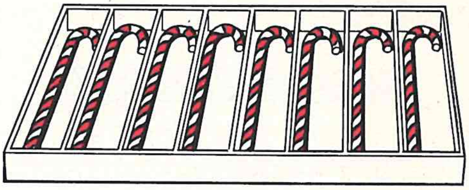
15. Is the segment from 0 to $\frac{4}{8}$ equal in length to the segment from $\frac{4}{8}$ to 1? Is this segment $\frac{1}{2}$ of the whole segment? Is this sentence true? $\frac{4}{8} = \frac{1}{2}$

16. Use the number line above Ex. 14 to tell what numbers will make the following sentences true.

$\frac{2}{8} = \frac{?}{4} \qquad \frac{4}{8} = \frac{?}{4} \qquad \frac{6}{8} = \frac{?}{4}$

$\frac{?}{8} = 1 \qquad \frac{?}{4} = 1 \qquad \frac{?}{2} = 1$

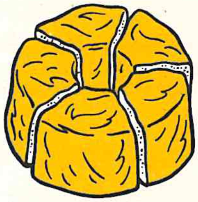
17. Between what two whole numbers is $\frac{1}{2}$? $\frac{1}{4}$? $\frac{1}{8}$? $\frac{3}{4}$? $\frac{5}{8}$? $\frac{2}{4}$? $\frac{7}{8}$?



18. Jane bought the box of candy canes shown above. She gave away $\frac{1}{2}$ of the set of canes. How many canes did she give away?

19. Jane gave 1 candy cane to Shirley. What part of the set of canes did she give to Shirley?

20. Jane gave $\frac{1}{4}$ of the set of candy canes to Mary. How many canes did Mary receive?

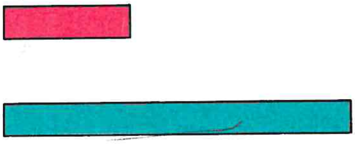


21. Charles cut this coffee cake into $\underline{?}$ pieces of about the same size.

- He gave 1 piece to Peter. What part of the coffee cake did he give to Peter?
- He gave 2 pieces to Henry. What part of the cake did Henry receive?
- What part of the cake did Charles keep for himself?

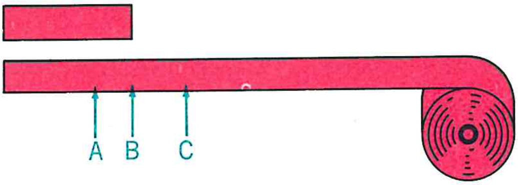
Units of measure

1. Martha has two pieces of ribbon. She wishes to know which piece is longer. If she places the two pieces side by side as shown below, can she tell which is the longer piece?



Does Martha *compare* the two pieces of ribbon to find the longer piece?

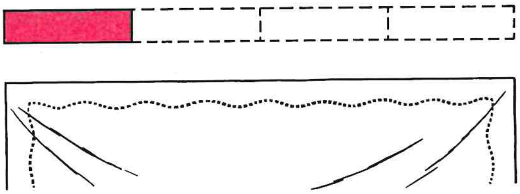
2. Martha wants another piece of red ribbon the same length as the one she has now. She places the first piece and the roll of ribbon side by side as shown below.



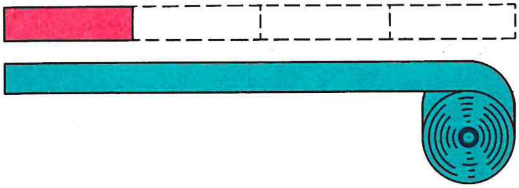
- At what point should Martha cut the ribbon, *A*, *B*, or *C*? Explain. Does Martha compare to find at what point she should cut the ribbon?
- Is Martha using the first piece of ribbon as a **unit of measure** to find where to cut the second piece?

3. Explain how Martha can use the first piece of red ribbon to get a second piece that is 3 times as long as the first.

4. Martha plans to sew a piece of blue ribbon along the edge of a white tablecloth. She uses a piece of red ribbon to measure the edge of the tablecloth, as shown below.



- Does she use the red ribbon as a unit of measure? The edge of the tablecloth is $\underline{\ ?\ }$ units long.



- Martha uses the same piece of red ribbon to find where to cut the roll of blue ribbon. The piece of blue ribbon should be $\underline{\ ?\ }$ units long.

5. In Ex. 4, can you think of the edge of the tablecloth and the pieces of ribbon as models of line segments? To measure the length of a line segment, did Martha use a line segment as a unit?

6. Use the length of your shoe as a unit to measure the width of the classroom. Then name some other models of line segments you can use to measure the length of objects.

Standard units of length

1. Jack and his older brother measured the height of their father's desk, each using his hand as a unit of measure. Jack said the height of the desk is 12 units. His older brother said the height is 8 units. Why do you think the boys gave different measures for the height of the desk?

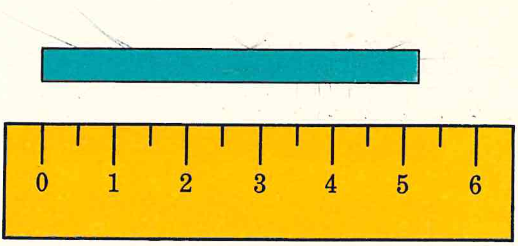
2. Suppose Jack and his older brother each wrote a letter to their uncle and told him the height of the desk, using their hands as a unit. Would the uncle know how high the desk is? Explain.

Would the uncle know how high the desk is if the boys' father wrote that it is 28 inches high? Explain why.

3. The *inch* is called a **standard unit** of length. How does a standard ınit of length differ from the units ısed by Jack and his brother? Why lo you think people use standard ınits of length?

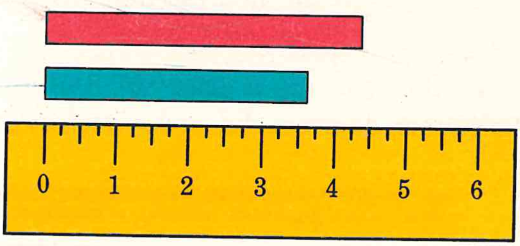
4. Which of the following are stanlard units of length? Why?

- a stone's throw
- a yard
- a foot
- an arm's length
- a five-minute walk
- a mile
- a hop, skip, and a jump



5. The ruler shown above represents a line segment separated into smaller segments, each $\frac{1}{2}$ inch long.

- Is the blue ribbon 5 inches long, a little more than 5 inches long, or a little less than 5 inches long?
- We say, "The ribbon is 5 inches long to the nearest inch." What does *to the nearest inch* mean?

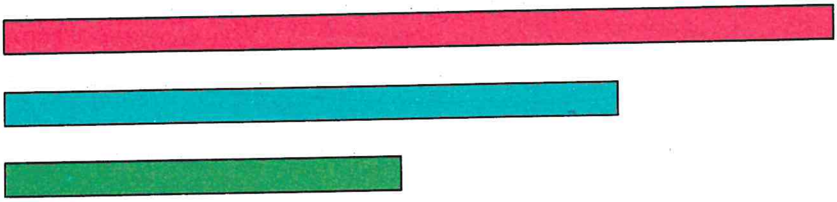


6. Each one-inch segment on the ruler above is separated into ? smaller segments of equal length. The length of the segment from 0 to the first mark is $\frac{1}{4}$ inch. The length of the segment from 0 to the second mark is ? inch.

- To the nearest inch, is the length of the red ribbon 4 inches or 5 inches? To the nearest half inch, is the length 4 inches or $4\frac{1}{2}$ inches?
- What is the length of the blue ribbon to the nearest inch? to the nearest half inch?

Using different units to measure

— Use your ruler to measure the three ribbons shown below.

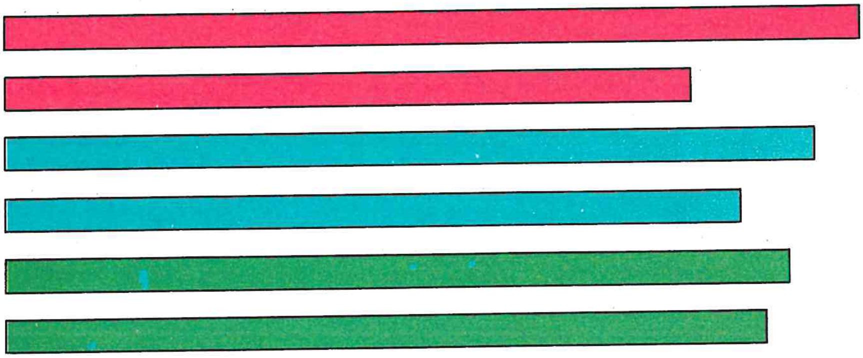


1. The length of the red ribbon is ? inches to the nearest inch; ? inches to the nearest half inch; ? inches to the nearest quarter inch.

2. The length of the blue ribbon is ? inches to the nearest inch; ? inches to the nearest half inch; ? inches to the nearest quarter inch.

3. The length of the green ribbon is ? inches to the nearest inch; ? inches to the nearest half inch; ? inches to the nearest quarter inch.

4. Did you give three different answers for Ex. 1? How many different answers did you give for Ex. 2? for Ex. 3?



5. Measure each of the two red ribbons shown above to the nearest inch. Compare your answers. What do you discover?

6. Measure each of the two blue ribbons to the nearest half inch. Compare your answers. What do you discover?

7. Measure each of the two green ribbons to the nearest quarter inch. Compare your answers. What do you discover?

8. Does there appear to be a difference in length between the two ribbons in each pair shown above Ex. 5 on page 84? In which pair is this difference the smallest? Is each ribbon in this pair closer in length to 4 inches than any of the other ribbons? Did you measure this pair to the nearest inch, to the nearest half inch, or to the nearest quarter inch?

The smaller the unit of measure you use, the more precise the measurement will be.

9. Which is more precise:

- measuring to the nearest inch or to the nearest half inch?
- measuring to the nearest half nch or to the nearest quarter inch?
- measuring to the nearest quarter nch or to the nearest eighth inch?

10. Suppose you are cutting glass to replace a broken window. Would t be better to measure the glass to he nearest inch or to the nearest quarter inch? Why?

1. The *foot* is another standard unit of length. It is equivalent to 2 inches; that is, 12 inches make foot.

How many inches are equivalent o 2 feet? 3 feet? 6 feet?

2. What is your height, to the earest foot? to the nearest inch?

13. What is the width of your desk, to the nearest foot? to the nearest inch? to the nearest half inch?

14. Which is more precise: a measurement made to the nearest foot or a measurement made to the nearest inch? Why?

15. Another standard unit of length is the *yard.* One yard is equivalent to 3 feet, or ? inches.

How many feet are equivalent to 2 yards? 15 yards? 200 yards?

16. The length of a football field is 100 yards. This is equivalent to ? feet, or ? inches.

17. The distance between first base and second base on a baseball diamond is 90 feet. This is equivalent to ? yards, or ? inches.

18. If Carol buys 3 yards and 1 foot of material, she will be buying ? feet of material; ? inches.

— Copy and complete.

19. 4 ft. 5 in. make 3 ft. ? in.

20. 3 yd. 2 ft. make 2 yd. ? ft.

21. 5 yd. 7 ft. make ? yd. 1 ft.

22. To make a shelf for a closet, would it be best to measure the board to the nearest inch, to the nearest half inch, or to the nearest quarter inch? Why?

23. Boise, Idaho, is 2,741 feet above sea level. Do you think this is to the nearest foot or to the nearest inch? Explain.

Other units of measure

1. Units of measure such as the ounce, the cup, the pint, the quart, and the gallon are *standard* units of measure.

Other units, such as a scoop, a dipper, and a handful, are called *indefinite* units of measure. Why?

Name some other indefinite units of measure.

2. Some standard units of liquid measure that you should know are given in this chart.

8 fluid ounces (fl. oz.) make 1 cup (c.).
2 cups (c.) make 1 pint (pt.).
2 pints (pt.) make 1 quart (qt.).
4 quarts (qt.) make 1 gallon (gal.).

- How many fluid ounces are equivalent to 1 quart? 1 gallon?
- How many pints are equivalent to 3 quarts? 5 gallons?

— Copy and complete.

3. 1 gal. 3 qt. make ? qt.

4. 3 qt. 1 pt. make 2 qt. ? pt.

5. 5 qt. 3 pt. make ? gal. 2 qt. 1 pt.

6. 2 gal. 3 qt. 4 pt. make ? gal. 1 qt.

7. The label on a bottle of soda reads, "Contents: 12 fl. oz." Do you think the contents have been measured only to the nearest ounce, or with greater precision? Explain.

8. Some standard units of weight that you should know are given in this chart.

16 ounces (oz.) make 1 pound (lb.).
2,000 pounds (lb.) make 1 ton (T.).
2,240 pounds make 1 long ton.

- How many ounces are equivalent to 5 pounds? 62 pounds? 100 pounds?
- How many pounds are equivalent to 3 tons? 3 long tons?

9. What is the difference in pounds between 1 ton and 1 long ton?

— Copy and complete.

10. 5 lb. 12 oz. make 4 lb. ? oz.

11. 2 T. 250 lb. make 1 T. ? lb.

12. 7 lb. 30 oz. make ? lb. 14 oz.

13. 3 T. 5,400 lb. make ? T. 1,400 lb.

14. Harry says that he weighs 68 pounds. This is equivalent to ? ounces.

15. Fred says that he weighs 72 pounds. Do you think he means to the nearest pound or to the nearest ounce? Explain.

16. If you order 3 pounds of ground meat, will the butcher weigh the meat to the nearest pound, or to the nearest ounce? Explain.

Using measures

68
16
408
8

1. Joe needs 5 feet 7 inches of weather stripping for each of 3 doors. How much weather stripping does he need in all?

Explain Joe's multiplication.

5 ft. 7 in.
×3
15 ft. 21 in., or 16 ft. 9 in.

How did Joe rename 15 feet 21 inches as 16 feet 9 inches?

2. A large can of tomato juice holds 1 quart 1 pint of juice. How much juice do 5 cans hold?

3. A case holds 24 cans of meat. The case weighs 1 pound. Each can weighs 1 pound 2 ounces. The case of meat weighs _?_ pounds.

4. Three of Mary's library books weigh 1 pound 6 ounces, 12 ounces, and 2 pounds 4 ounces. Together the three books weigh _?_ pounds _?_ ounces.

5. A can of orange juice contains 32 fluid ounces of juice. If Judy fills an 8-ounce glass from the can, how much juice will be left in the can?

6. One box weighs 6 pounds 4 ounces. Another box weighs 2 pounds 9 ounces. What is the difference between the weights of the two boxes?

7. Harry is about 4 feet tall. He says the living room of his house is about twice as high. About how high is the living room of Harry's house?

8. Helen says her hand is about 3 inches wide. She says her book is two hands wide. About how many inches wide is Helen's book?

9. How wide is your hand? Use your hand to estimate the width of your desk.

Check your estimate by measuring the desk with your ruler.

10. How tall are you? Can you use this information to estimate the width of the classroom? Try it.

Check your estimate by measuring with a ruler.

11. Estimate the length of your classroom. Check by measuring.

12. Estimate the width of your classroom door. Estimate the height of the door.

Check your estimates by measuring with a ruler.

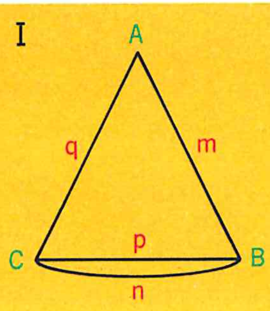
13. Fred says he can walk 1 mile in 15 minutes. About how long will it take Fred to walk 3 miles?

14. June can walk 1 mile in 20 minutes. How far can she walk in an hour?

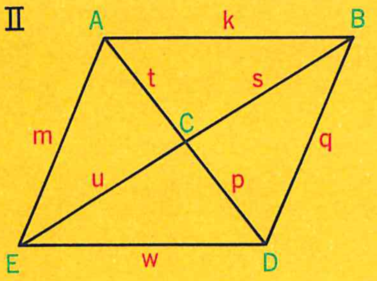
Inquiry into geometry

Networks

I



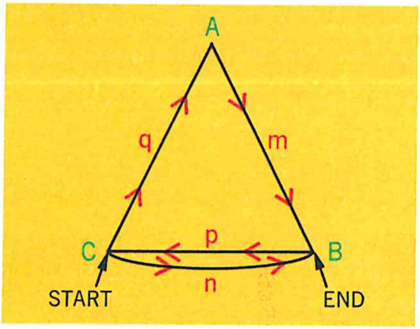
II



In mathematics, figures such as the ones shown above are called **networks**. A network is a figure made up of straight paths (segments), curved paths, or both straight and curved paths that intersect only at endpoints. On these two pages, all endpoints are named by capital letters and all paths are named by small letters.

If you can copy a network without lifting your pencil after you place it on your paper and without retracing any path, we say you have **traveled** the network. You can pass through any endpoint more than once to travel a network.

Do you think you can travel the network shown in figure I? On your paper, copy figure I and try to travel it before reading further.



▶ Look at the drawing at the right. Does it show you one way to travel the network in figure I? In this drawing, we started at endpoint *C* and ended at endpoint ?.

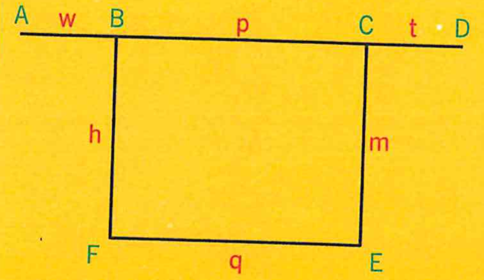
The paths named in the order they were traced are *q*, *m*, *p*, and *n*.

Find another way to travel this network. Tell how you do it by naming the paths traced in order.

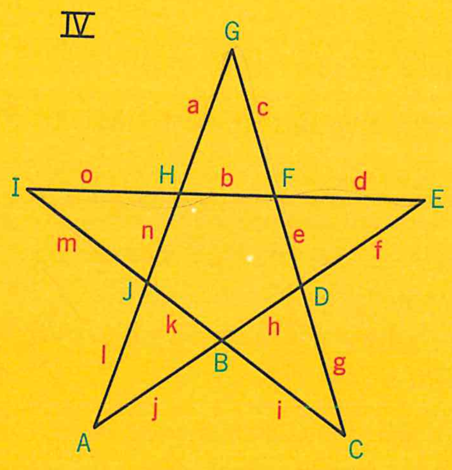
▶ Now copy figure II shown at the top of the opposite page. Try to travel this network. Can you do it? Remember, when you travel a network, you do not lift your pencil after placing it on the paper, and you mark over each path once but no more than once.

▶ Copy figures III, IV, V, and VI shown below. Try to find out which ones can be traveled. If a network can be traveled, show a way it can be done by writing the names of the paths in the order that you traced over them.

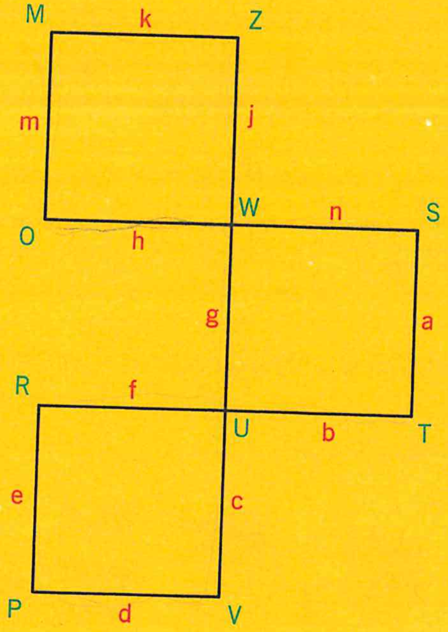
III



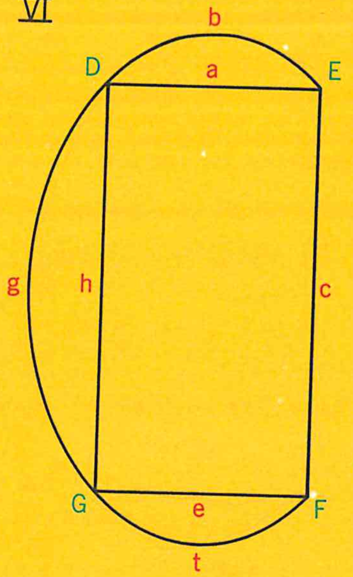
IV



V



VI



Keeping up in mathematics

Oral

1. The set shown below is the set of $\underline{?}$ numbers.

$$\{1, 3, 5, 7, \cdots\}$$

Tell the next five numbers in this set.

2. This set is the set of multiples of $\underline{?}$.

$$\{0, 4, 8, 12, \cdots\}$$

Tell the next five numbers in this set.

3. In the decimal numeral 555, the value of the 5 at the left is $\underline{?}$ times as much as the value of the 5 in the middle; $\underline{?}$ times as much as the value of the 5 at the right.

— For each sentence, find the set of whole numbers that make it true.

4. $n \times 5 < 52$

5. $17 + n < 23$

6. $408 \div 8 < n$

7. $n > 14 + 15$

— Tell whether = or ≠ should replace the ● to make the sentence true.

8. (33 + 18) + 42 ● 33 + (42 + 18)

9. 258 − 189 ● 189 − 258

10. 15 × 27 ● 27 × 15

11. (10 + 7) × 32 ● 17 × (30 + 2)

12. 68 ÷ 15 ● 15 ÷ 68

Written

	a	b	c
1.	$\begin{array}{r} 67 \\ +\ 22 \\ \hline \end{array}$	$\begin{array}{r} 483 \\ 317 \\ +\ 94 \\ \hline \end{array}$	$\begin{array}{r} 73{,}675 \\ 47{,}549 \\ 16{,}227 \\ +\ 6{,}978 \\ \hline \end{array}$
2.	$\begin{array}{r} 384 \\ -\ 73 \\ \hline \end{array}$	$\begin{array}{r} 414 \\ -\ 388 \\ \hline \end{array}$	$\begin{array}{r} 11{,}700 \\ -\ 6{,}528 \\ \hline \end{array}$
3.	$\begin{array}{r} 35 \\ \times\ 6 \\ \hline \end{array}$	$\begin{array}{r} 951 \\ \times\ 18 \\ \hline \end{array}$	$\begin{array}{r} 624 \\ \times\ 378 \\ \hline \end{array}$
4.	$3\overline{)24}$	$6\overline{)13}$	$8\overline{)75}$

— Copy and complete. 24

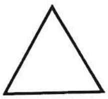
5. 4 ft. 4 in. make $\underline{?}$ ft. 28 in.

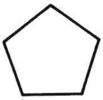
6. 3 lb. 23 oz. make 4 lb. $\underline{?}$ oz.

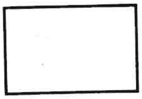
7. 8 hr. 7 min. make 6 hr. $\underline{?}$ min.

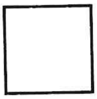
8. 2 T. 575 lb. make $\underline{?}$ lb.

9. Write the name of each kind of polygon shown below.

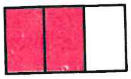

A

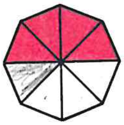

B

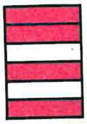

C

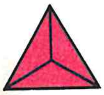

D

10. What fractional number tells the part of each region below that is colored red?

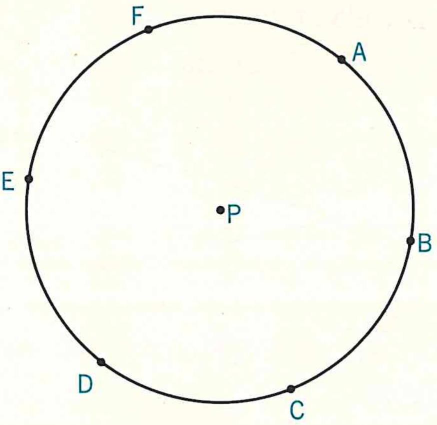

E


F


G


H

Circles



1. Look at the path shown in the drawing above. Is this path a closed path? Is any part of the path straight?

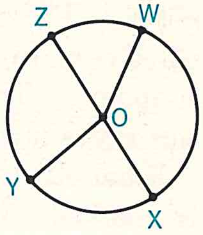
- Is point *A* on the path? Is point *B*? point *C*? Name three other points on the path that are marked n the drawing.
- Point *P* is not on the path. It s enclosed by the path. Use your ruler to measure the distance from *P* to *A*; from *P* to *B*; *P* to *C*; *P* to *D*. What do you discover?
- Without measuring, what do you hink is the distance from *P* to *E*? *P* to *F*? *P* to any point on the path?

If every point on a closed path in a lane is the same distance from one articular point enclosed by the path, he figure is called a circle. The point nclosed by the figure is called the cen-r of the circle.

2. Find point *A* on the circle shown in Ex. 1. Run your finger along the circle from point *A* past point *B* to point *C*. Is this part of the circle a set of points? Is *B* a member of this set? How many other points are members of this set? Explain.

A part of a circle, such as the part that joins points *A* and *C* and includes *B*, is an **arc**. This arc is called arc *ABC*.

- Find arc *DEF* in the drawing. What are the endpoints of this arc?
- Find and name the endpoints of arc *BCD*; arc *CFD*; arc *ACE*.



3. A line segment from the center of a circle to any point on the circle is called a **radius**. *Radii* is the plural of *radius*.

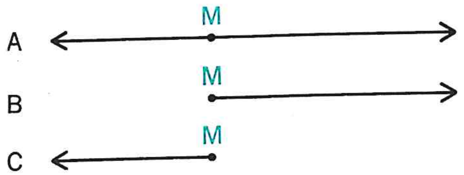
One radius is $\overline{OW}$. Name three other radii shown above.

4. Measure radius *OW*. Without measuring, tell the length of radius *OX*. How do you know? Is every radius of this circle the same length?

5. Does $\overline{ZX}$ pass through the center of the circle shown in Ex. 3? Are the endpoints of $\overline{ZX}$ points on the circle? A line segment such as $\overline{ZX}$ is called a **diameter** of the circle.

Is the length of a diameter of a circle twice the length of a radius of that circle? Explain.

Rays and angles

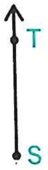


1. What do we call the figure shown in drawing A? Does drawing B look exactly like drawing A? Explain.

Does drawing B show a point *M* on a line and all other points to the right of *M* on the line? What does drawing C show?

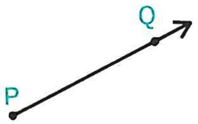
A figure like the one shown in B or in C is called a **ray.**

How many rays do you see in drawing A? Is a ray a subset of a line? Explain.



2. Every ray has just one endpoint. The endpoint of this ray is named $\underline{?}$.

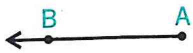
What other point on the ray is marked in the drawing? We name this ray by writing $\overrightarrow{ST}$. The name of the endpoint is always written first.

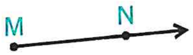


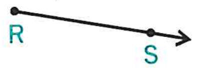
3. Which of the following is the name for this ray?

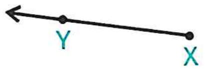
$\overrightarrow{PQ}$ $\overrightarrow{QP}$ $\overleftrightarrow{PQ}$

4. Write the name for each ray shown below.

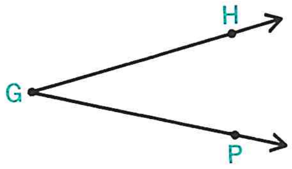






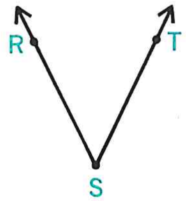


5. In this drawing, the name of one ray is $\overrightarrow{GH}$. Name the other ray.



Do $\overrightarrow{GH}$ and $\overrightarrow{GP}$ have the same endpoint?

Two rays that have the same endpoint form an angle. The common endpoint is called the vertex of the angle.

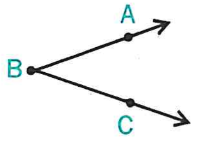


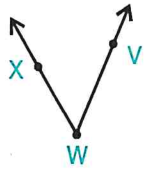
6. Name the two rays shown in this picture. Do they form an angle? What is the vertex of this angle?

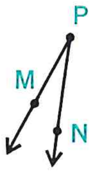
We name the angle pictured above angle *RST* or angle *TSR*. The name of the vertex is always in the middle.

One way to write the name for the angle is $\angle RST$. The symbol $\angle$ means "angle." Is $\angle TSR$ another way to name the same angle?

7. Write two names for each angle shown below.



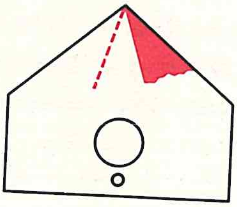




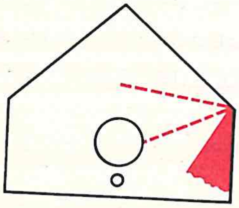
Angle measurement

1. To measure a line segment, we use another line segment as a standard unit. To measure an angle, what do you think we shall use as a standard unit?

2. To make a birdhouse like the one his father had made, Joe needed to measure the angle at the peak and the two angles at the sides. To do this, Joe used the edges of a piece of cardboard cut in the shape of an angle as a unit of measure.



- He found that
e could fit the
ardboard into the
ngle at the peak
times. He said
he size of this an-
le is ? units.

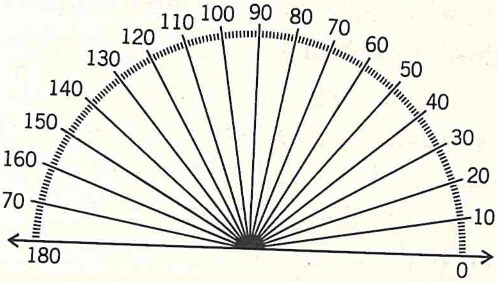


- The cardboard
t into each angle
t the sides 4
mes. Each of
ese angles meas-
ed ? units.

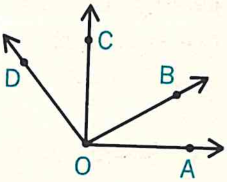
. Joe's brother cut another piece cardboard in the shape of an angle. e used his cardboard model as a it and said the size of the angle at e peak of the birdhouse is 2 units. hy does his measurement differ m Joe's?

4. Joe's friend Larry wanted to make a birdhouse. If Joe had told him that the angle at the peak should measure 3 units, would Larry have known what size the angle should be? Explain why it would have been better for Joe to use a standard unit of measure to tell Larry the size of the angle.

The standard unit of measure for an angle is an angle called a **degree.** The symbol for degree is °. One hundred eighty of these angles will fit on one side of a line.



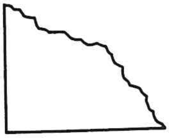
5. What is $\frac{1}{2}$ of 180? An angle whose measurement is 90° is called a **right angle.**



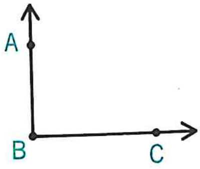
- Angle *AOC* is a right angle. Is the measurement of ∠*AOB* greater than or less than 90°?

- Which angle in the drawing above has a measurement greater than 90°?

Right angles and perpendicular and parallel lines

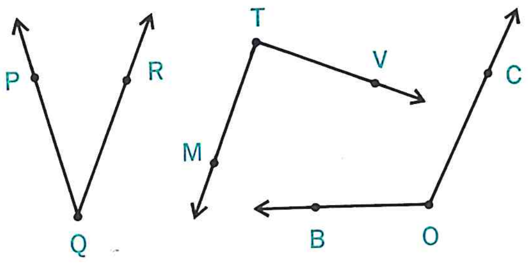


Cut off the corner of an envelope like this. The angle represented by the creases is a right angle. You can use this corner as a model of a right angle.



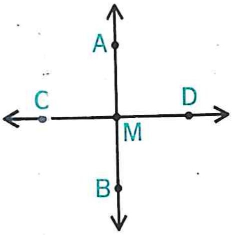
1. Place the vertex of your right-angle model at the vertex of $\angle ABC$ and place one crease along $\overrightarrow{BC}$. Does the other crease fall along $\overrightarrow{BA}$? Then is $\angle ABC$ a right angle?

2. Use your model to find which of these is a drawing of a right angle.



- Which angle shown above has a measurement greater than 90°?
- Which angle has a measurement less than 90°?

3. Use your model to draw a right angle. Now using only a pencil and a ruler, draw two angles with measurements less than 90°. Draw two angles with measurements greater than 90°.

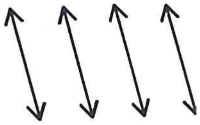


4. This drawing shows two lines that intersect at point M. Name them. Name two rays and three line segments that are subsets of $\overleftrightarrow{AB}$; of $\overleftrightarrow{CD}$.

Do you see that $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ form $\angle AMD$, $\angle DMB$, $\angle BMC$, and $\angle CMA$? Are they all right angles?

Two lines that intersect to form right angles are called perpendicular lines. Two rays or two line segments are perpendicular if they intersect and are subsets of perpendicular lines.

5. In the drawing for Ex. 1, are $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ perpendicular? Explain.



6. Do these lines appear to have any points in common?

Two or more lines in a plane that have no points in common are parallel lines. Line segments or rays are parallel if they are subsets of parallel lines.

7. What things in your classroom remind you of perpendicular lines? of parallel lines?

12
×9
108
9
117

PROBLEM TEST 2

1. Jerry earns $2.75 a week. How much money will he earn in 6 weeks?

2. Jim needs 9 feet 9 inches of molding for each of the 4 walls of his room. How many feet of molding does he need in all?

3. Three boys are to carry 24 spelling books to their classroom. If they share the work equally, how many books should each boy carry?

4. Baskets of apples are stacked on a truck for shipping. There are 8 rows of stacks with 12 stacks in each row. Each stack contains 9 baskets. How many baskets of apples are stacked on the truck?

5. If you deal out 35 cards into 4 piles with the same number of cards in each, there will be ? cards in each pile, with ? left over.

6. Each pupil in a class needs 12 pieces of construction paper for a project. There are 28 pupils in the class. How many pieces of construction paper are needed?

7. At 95¢ a ticket, how much money will be collected if 325 tickets are sold?

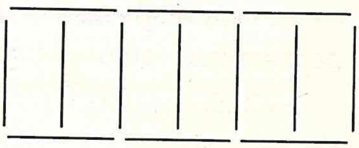
8. Paul earns $3 a week. How many weeks will it take him to earn $27?

9. How many 8-cent stamps can Mary buy with 50¢? How much money will she have left over?

10. If 7 boys share equally the cost of a 28-dollar archery set, how much should each boy pay?

Write your score on your Problem Test Record.

Mathamusements



- The 13 toothpicks shown at the right are all the same length, and they form 6 polygons all the same size and shape. Can you take 1 toothpick away and rearrange the rest to show 6 polygons? These polygons must be all the same size and shape, but different in size and shape from the polygons shown here. The toothpicks cannot cross each other.

- A frog is at the bottom of a 30-foot well. He climbs 3 feet in 1 hour and then rests for 2 hours. How long will it take the frog to reach the top of the well?

9 2)27

36
36
108

77

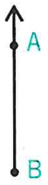
5

12
18

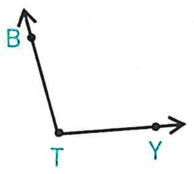
IT'S CHECK-UP TIME

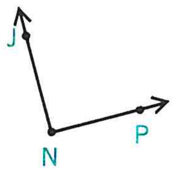
1. Which of the following names the figure at the right?

$\overleftrightarrow{AB}$ $\overrightarrow{BA}$ $\overrightarrow{AB}$ $\overline{BA}$



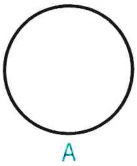
2. Name the vertex of each angle shown below.

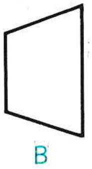


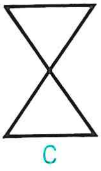


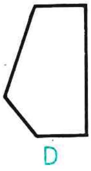
3. Which angle shown above is a right angle?

4. Which figures shown below are polygons?



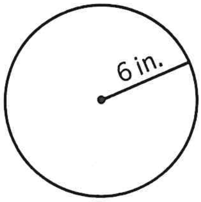




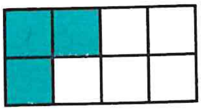


5. Which figure shown above is a quadrilateral? a pentagon?

6. What is the length of a diameter of this circle?



7. Write a fraction for the number that tells what part of this rectangular region is shown in blue.



8. The denominator of $\frac{2}{5}$ is ?.

9. The numerator of $\frac{3}{10}$ is ?.

— Copy and complete.

10. $\frac{2}{4} = \frac{?}{2}$ 11. $\frac{4}{?} = \frac{2}{4}$ 12. $\frac{?}{4} = \frac{8}{8}$

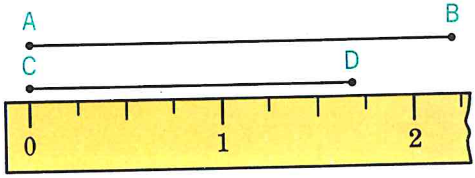
13. 6 lb. 12 oz. make 5 lb. ? oz.

14. 5,000 lb. make 2 T. ? lb.

15. 5 gal. 9 qt. make ? gal. 1 qt.

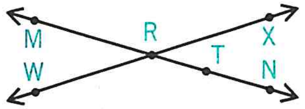
16. 3 yd. 4 in. make ? in.

17. To the nearest inch, what is the length of $\overline{AB}$? of $\overline{CD}$?



18. To the nearest half inch, what is the length of $\overline{AB}$? of $\overline{CD}$?

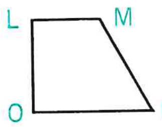
19. Name two segments that are subsets of $\overline{MN}$.

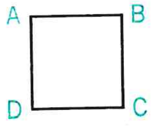


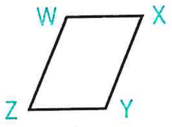
20. What is the point of intersection of $\overline{MN}$ and $\overline{WX}$?

If you have time, try these —

21. Name five sets of parallel line segments shown in these figures.







22. Name six right angles shown in the figures above.

CHAPTER 5

Solving inequalities

1. Suppose we wish to find the set of whole numbers that make the following number sentence true.

$$n \times 6 < 18$$

Is the sentence true if n is replaced by 0? by 1? 2? 3? 4? Then if we consider only whole numbers, the solution set for the number sentence above is $\{0, 1, 2\}$.

— Suppose n stands for any whole number. Find the solution set for each of the following sentences.

	a	b
2.	$n \times 8 < 35$	$n \times 4 < 28$
3.	$n \times 9 < 27$	$n \times 5 < 42$
4.	$n \times 2 < 13$	$n \times 7 < 49$

5. What is the set of whole numbers that make this sentence true? $n \times 8 < 43$ What is the greatest whole number that makes the sentence true?

— For each number sentence below, find the greatest whole number that makes the sentence true.

6.	$n \times 7 < 45$	$n \times 2 < 10$
7.	$n \times 5 < 33$	$n \times 9 < 64$
8.	$n \times 3 < 21$	$n \times 6 < 42$

9. Find the greatest whole number hat makes each sentence in Ex. –4 true.

10. The set shown below is the set of multiples of what number? Name three other members of this set.

$$\{0, 10, 20, 30, 40, \cdots\}$$

11. Suppose n in the sentences below stands for any multiple of 10.

$n \times 6 < 13$ tens $\qquad n \times 6 < 134$

Will 0 make the sentences true? Will 10? 20? 30? 40? Then if we consider only multiples of 10, the solution set for these number sentences is $\{0, 10, 20\}$.

— Suppose n stands for any multiple of 10. Find the solution set for each of the following sentences.

	a	b
12.	$n \times 2 < 4$ tens	$n \times 9 < 22$ tens
13.	$n \times 4 < 90$	$n \times 3 < 100$

14. What is the greatest multiple of 10 that makes this sentence true? $n \times 4 < 176$ Think: 176 is about 170, or 17 tens; 4×4 tens equals 16 tens, or 160; 4×5 tens equals 20 tens, or 200; so the greatest multiple is 40.

— For each of the following number sentences, find the greatest multiple of 10 that makes the sentence true.

15.	$n \times 7 < 64$ tens	$n \times 5 < 23$ tens
16.	$n \times 8 < 168$	$n \times 6 < 95$

Solving a new kind of number sentence

1. For all of the questions in this exercise, n stands for any whole number.

- What is the solution set for this number sentence? $n \times 4 < 12$
- What is the solution set for this number sentence? $n \times 4 = 12$
- Suppose the product of n and 4 is less than or equal to 12. This idea is expressed in the following number sentence.

$$n \times 4 \leq 12$$

The symbol $\leq$ is read "is less than or equal to." Does 0 make the sentence true? Does 1? 2? 3? 4? Explain why $\{0, 1, 2, 3\}$ is the solution set for this sentence. $n \times 4 \leq 12$

2. The following number sentence says that the product of n and 6 is less than or equal to 24.

$$n \times 6 \leq 24$$

If n stands for any whole number, which of the following sets is the solution set for this sentence? Explain.

$\{0, 1, 2, 3\}$ $\{0, 1, 2, 3, 4\}$

3. Here are two number sentences in which n stands for any whole number.

a. $n \times 5 < 30$ **b.** $n \times 5 \leq 30$

Show the solution set for each of these sentences. Explain why the solution set for sentence **a** is not the same as the solution set for sentence **b**.

— If n stands for any whole number, find the solution set for each of the following number sentences.

	a	b
4.	$n \times 5 \leq 25$	$n \times 3 \leq 21$
5.	$n \times 8 \leq 56$	$n \times 6 \leq 54$
6.	$n \times 2 \leq 15$	$n \times 2 < 15$
7.	$n \times 7 \leq 28$	$n \times 7 < 28$

8. For each number sentence in Ex. 4–7, find the greatest whole number that makes the sentence true.

9. In the following sentences, n stands for any multiple of 10.

$n \times 7 \leq 21$ tens $n \times 7 \leq 210$

Which of these sets is the solution set for these sentences?

$\{0, 10, 20\}$ $\{0, 10, 20, 30\}$

10. In these two number sentences, n stands for any multiple of 10.

$n \times 4 < 32$ tens $n \times 4 \leq 32$ tens

Show the solution set for each of these sentences. Explain why the solution sets are not the same.

— If n stands for any multiple of 10, find the solution set for each of the following number sentences.

11. $n \times 6 \leq 30$ tens $n \times 9 \leq 72$ tens

12. $n \times 3 \leq 18$ tens $n \times 5 \leq 25$ tens

13. $n \times 2 \leq 140$ $n \times 8 \leq 640$

14. Find the greatest multiple of 10 that makes each sentence in Ex. 11–13 true.

Estimating quotients

— Copy and solve each of these sentences.

1. $8 \div 2 = \triangle$ $\quad$ 8 tens $\div 2 = \underline{?}$ tens $\quad$ $80 \div 2 = \square$

2. $9 \div 3 = \triangle$ $\quad$ 9 tens $\div 3 = \underline{?}$ tens $\quad$ $90 \div 3 = \square$

3. $49 \div 7 = \triangle$ $\quad$ 49 tens $\div 7 = \underline{?}$ tens $\quad$ $490 \div 7 = \square$

4. $36 \div 4 = \triangle$ $\quad$ 36 tens $\div 4 = \underline{?}$ tens $\quad$ $360 \div 4 = \square$

	a	b
5.	$8 \div 4 = 2$, so $80 \div 4 = n$	$45 \div 9 = 5$, so $450 \div 9 = n$
6.	$10 \div 5 = 2$, so $100 \div 5 = n$	$81 \div 9 = 9$, so $810 \div 9 = n$

7. What is the greatest multiple of 10 that makes these sentences true?

$n \times 3 \leq 24$ tens $\qquad n \times 3 \leq 240$

Does 80×3 equal 240? Then is $240 \div 3$ equal to 80?

8. What is the greatest multiple of 10 that makes these sentences true?

$n \times 3 \leq 24$ tens $\qquad n \times 3 \leq 246$

Is 80×3 less than 246? Then is $246 \div 3$ equal to 80 plus some number, or 80-some?

9. What is the greatest multiple of 10 that makes these sentences true?

$n \times 3 \leq 25$ tens $\qquad n \times 3 \leq 255$

Then $255 \div 3$ is equal to $\underline{?}$-some.

— Copy these pairs of sentences and ind the greatest multiple of 10 that nakes the sentences true.

0. $n \times 5 \leq 408$ $\quad$ $408 \div 5$ is n-some

1. $n \times 3 \leq 126$ $\quad$ $126 \div 3$ is n-some

2. $n \times 7 \leq 437$ $\quad$ $437 \div 7$ is n-some

3. $n \times 6 \leq 162$ $\quad$ $162 \div 6$ is n-some

14. If 328 marbles are separated into 8 sets with the same number of marbles in each set, will there be 30-some, 40-some, or 50-some marbles in each set?

15. Josephine, Martin, and Alexander plan to share equally 246 seashells. Estimate to the nearest multiple of 10 the number of shells each will receive.

16. Five boys are going to buy a basketball for $4.60. They will share the cost equally. Estimate to the nearest 10 cents how much each boy will pay.

— Name a multiple of 10 that would be a good estimate for each quotient below. For Ex. 17a think: Since 20×9 equals 18 tens, then 20×9 is less than or equal to 189; so $189 \div 9$ is 20-some.

	a	b
17.	$189 \div 9 = \underline{?}$	$330 \div 4 = \underline{?}$
18.	$195 \div 3 = \underline{?}$	$576 \div 9 = \underline{?}$
19.	$218 \div 6 = \underline{?}$	$111 \div 2 = \underline{?}$

Dividing tens and ones

TENS	ONES
///	//
///	//
///	//

1. The tens-ones chart above shows 96 separated into 3 sets with the same number in each set. Ninety-six separated into 3 sets with the same number in each set means 96 ÷ ?. The number in each set is ?.

2. The chart above shows ? tens separated into 3 sets with the same number in each set. What is the number of each set of tens?

$$90 \div 3 = ?$$

The chart shows ? ones separated into 3 sets with the same number in each set. What is the number of each set of ones?

$$6 \div 3 = ?$$

The sum of the quotients of 90 ÷ 3 and 6 ÷ 3 is 30 + ?, or ?. Is this sentence true? Explain.

$$96 \div 3 = (90 \div 3) + (6 \div 3)$$

3. To divide 84 by 2, you can divide 80 by 2 and 4 by 2, and add.

$$\begin{aligned} 84 \div 2 &= (80 + 4) \div 2 \\ &= (80 \div 2) + (4 \div 2) \\ &= \underline{?} + \underline{?} \\ &= \underline{?} \end{aligned}$$

4. To find what 427 ÷ 7 is, copy and complete this example. Explain each step.

$$\begin{aligned} 427 \div 7 &= (420 + 7) \div 7 \\ &= (420 \div 7) + (7 \div 7) \\ &= \underline{?} + \underline{?} \\ &= \underline{?} \end{aligned}$$

5. $96 \div 4 = n$ Can you rename 96 as 9 tens 6 ones? Can you rename 96 as 8 tens 16 ones? Which way do you think you should rename 96 to divide 96 by 4? Explain.

$$\begin{aligned} 96 \div 4 &= (80 + 16) \div 4 \\ &= (80 \div 4) + (16 \div 4) \\ &= \underline{?} + \underline{?} \\ &= \underline{?} \end{aligned}$$

6. Copy and complete.

$$\begin{aligned} 372 \div 6 &= (360 + \underline{?}) \div 6 \\ &= (360 \div 6) + (\underline{?} \div 6) \\ &= \underline{?} + \underline{?} \\ &= \underline{?} \end{aligned}$$

Why was 372 renamed as 360 + 12 rather than as 370 + 2?

— Find the quotient for each of the following examples, using the way shown above.

	a	b
7.	66 ÷ 2 = ?	96 ÷ 6 = ?
8.	540 ÷ 9 = ?	415 ÷ 5 = ?
9.	168 ÷ 8 = ?	287 ÷ 7 = ?
10.	128 ÷ 4 = ?	312 ÷ 8 = ?
11.	192 ÷ 6 = ?	684 ÷ 9 = ?
12.	186 ÷ 2 = ?	246 ÷ 3 = ?

13. One way to find the quotient of 246 divided by 3 is like this.

$$\begin{aligned} 246 \div 3 &= (240 \div 3) + (6 \div 3) \\ &= \quad 80 \quad + \quad 2, \qquad \text{or } \underline{\ ?\ } \end{aligned}$$

A

$$\begin{array}{lr} \text{80 threes} \longrightarrow & 80 \\ & 3\overline{)246} \end{array}$$

B

$$\begin{array}{lr} \text{80 threes} \longrightarrow & 80 \\ & 3\overline{)246} \\ 80 \times 3 \longrightarrow & \underline{240} \\ & 6 \end{array}$$

C

$$\begin{array}{lr} \text{(80 + 2) threes} \longrightarrow & 82 \\ \text{2 more threes} \longrightarrow & 2 \\ \text{80 threes} \longrightarrow & 80 \\ & 3\overline{)246} \\ 80 \times 3 \longrightarrow & \underline{240} \\ & 6 \\ 2 \times 3 \longrightarrow & \underline{6} \\ & 0 \end{array}$$

14. Here is another way to find the quotient of 246 divided by 3.

- Find the greatest multiple of 10 that makes this sentence true. $n \times 3 \leq 246$ Think: 246 is about 24 tens; 8 tens × 3 equals 24 tens; so n must represent 8 tens, or 80.

Write 80 above 246 with 8 in the tens place, as shown in example A.

- Think: 80 × 3 equals $\underline{\ ?\ }$. Write the answer, 240, under 246, as shown in example B. Subtract. What is the remainder?
- Find the greatest whole number that makes this sentence true. $n \times 3 \leq 6$ Write the answer in the ones place above the 80, as shown in example C.

Think: 2 × 3 equals $\underline{\ ?\ }$. Write the answer, 6, under the remainder 6. What is the new remainder when you subtract 2 threes from 6?

- Add the two parts of the quotient and write the answer on top.
- Check by multiplying. 82 × 3 = $\underline{\ ?\ }$

5. Looking at the book only to check your work, use the method shown in Ex. 14 to divide 246 by 3.

– Use the method shown in Ex. 14 to find the quotient for each example. Check your answers by multiplying.

	a	b	c	d	e	f	g	h
6.	$4\overline{)44}$	$3\overline{)45}$	$7\overline{)84}$	$3\overline{)87}$	$6\overline{)72}$	$9\overline{)108}$	$5\overline{)105}$	$4\overline{)124}$
7.	$5\overline{)155}$	$6\overline{)366}$	$4\overline{)184}$	$9\overline{)369}$	$3\overline{)192}$	$9\overline{)558}$	$7\overline{)497}$	$8\overline{)656}$
8.	$7\overline{)245}$	$2\overline{)186}$	$3\overline{)294}$	$6\overline{)204}$	$4\overline{)156}$	$8\overline{)152}$	$5\overline{)460}$	$2\overline{)98}$
9.	$4\overline{)336}$	$5\overline{)265}$	$3\overline{)138}$	$8\overline{)424}$	$4\overline{)276}$	$6\overline{)186}$	$3\overline{)201}$	$2\overline{)112}$
0.	$9\overline{)747}$	$7\overline{)119}$	$2\overline{)174}$	$7\overline{)357}$	$6\overline{)450}$	$9\overline{)666}$	$5\overline{)420}$	$8\overline{)704}$

Division with remainders that are not zero

1. Suppose you wish to divide 167 by 3. Study the example below to see how the division can be done.

- First, find the greatest multiple of 10 that makes this sentence true. $n \times 3 \leq 167$ Think: 167 is about 16 tens; 16 tens ÷ 3 is about 5 tens, or 50. Where is this number shown in the example? What is the remainder when you subtract 50 threes from 167?
- What is the greatest whole number of 3's in 17? Where is this number shown in the example? What is the remainder when you subtract 5 threes from 17?
- Add the two parts of the quotient and write the remainder.
- Check your answer by seeing if this sentence is true. $167 = (55 \times 3) + 2$

$$\begin{array}{lr}
(50+5)\text{ threes} + 2 \rightarrow & \overline{??}\text{ r } ?\\
5\text{ more threes} \longrightarrow & 5\\
50\text{ threes} \longrightarrow & 50\\
 & 3\overline{)167}\\
50 \times 3 \longrightarrow & \underline{150}\\
 & 17\\
5 \times 3 \longrightarrow & \underline{15}\\
 & ?
\end{array}$$

2. Copy and complete the example shown below. It is not necessary to write the part shown in red.

$$\begin{array}{lr}
(20+1)\text{ sevens} + 2 \longrightarrow & \overline{??}\text{ r } ?\\
1\text{ more seven} \longrightarrow & ?\\
20\text{ sevens} \longrightarrow & 20\\
 & 7\overline{)149}\\
20 \times 7 \longrightarrow & \underline{140}\\
 & 9\\
1 \times 7 \longrightarrow & \underline{?}\\
 & ?
\end{array}$$

Check your answer by seeing if the following number sentence is true.

$$149 = (21 \times 7) + 2$$

3. Use this number sentence to write a division example similar to the one in Ex. 2.

$$335 = (83 \times 4) + 3$$

— Copy and divide. Check by writing a sentence like the one in Ex. 3.

	a	b	c	d
4.	$7\overline{)95}$	$4\overline{)78}$	$5\overline{)87}$	$2\overline{)43}$
5.	$2\overline{)165}$	$9\overline{)279}$	$6\overline{)463}$	$4\overline{)354}$
6.	$3\overline{)127}$	$8\overline{)649}$	$5\overline{)472}$	$7\overline{)225}$
7.	$4\overline{)327}$	$8\overline{)255}$	$6\overline{)403}$	$5\overline{)327}$
8.	$9\overline{)418}$	$3\overline{)278}$	$7\overline{)583}$	$4\overline{)102}$
9.	$8\overline{)656}$	$6\overline{)205}$	$5\overline{)127}$	$2\overline{)145}$
10.	$7\overline{)327}$	$9\overline{)624}$	$3\overline{)145}$	$6\overline{)186}$

Estimating quotients named by three-digit numerals

1. The set shown below is the set of multiples of what number?

$\{0, 100, 200, 300, 400, \cdots\}$

Name the next five members of this set.

2. What multiples of 100 will make hese sentences true?

$n \times 4 \le 8$ hundreds $\qquad n \times 4 \le 800$

What is the greatest multiple of 100 hat makes the sentences true?

3. What multiples of 100 will make hese sentences true?

$n \times 4 \le 8$ hundreds $\qquad n \times 4 \le 884$

What is the greatest multiple?

— For each of the following sentences, name the greatest multiple of 00 that makes the sentence true.

	a	b
4.	$n \times 2 \le 700$	$n \times 4 \le 1{,}700$
5.	$n \times 5 \le 1{,}500$	$n \times 7 \le 1{,}400$
6.	$n \times 3 \le 2{,}163$	$n \times 9 \le 7{,}218$
7.	$n \times 6 \le 3{,}642$	$n \times 8 \le 6{,}568$

8. To estimate what $1{,}684 \div 4$ is, hink: 1,684 is about 16 hundreds; 6 hundreds $\div$ 4 is 4 hundreds, or 00.

What is the greatest multiple of 00 that makes this sentence true? $\times 4 \le 1{,}684$ Then is $1{,}684 \div 4$ ual to 400-some?

. Is $1{,}568 \div 5$ equal to 100-some, 0-some, 300-some, or 400-some?

— Copy and replace n in these pairs of sentences with a numeral for the greatest multiple of 100 that makes the sentences true.

10. $n \times 2 \le 456$
$456 \div 2$ is n-some

11. $n \times 7 \le 2{,}184$
$2{,}184 \div 7$ is n-some

12. $n \times 5 \le 4{,}675$
$4{,}675 \div 5$ is n-some

13. You can think of 2,484 as 24 hundreds $\underline{?}$ tens 4 ones. To find what $2{,}484 \div 4$ is, copy and complete the sentences below.

$$\begin{aligned} 2{,}484 \div 4 &= (2{,}400 + 80 + 4) \div 4 \\ &= (2{,}400 \div 4) + (80 \div 4) \\ &\qquad + (4 \div 4) \\ &= 600 + \underline{?} + 1, \text{ or } \underline{?} \end{aligned}$$

14. To divide 4,065 by 5, would you rename 4,065 as $4{,}000 + 60 + 5$, or as $4{,}000 + 50 + 15$? Why? Copy and complete these sentences.

$$\begin{aligned} 4{,}065 \div 5 &= (4{,}000 + 50 + 15) \div 5 \\ &= (4{,}000 \div 5) + (50 \div 5) \\ &\qquad + (15 \div 5) \\ &= \underline{?} + 10 + 3, \text{ or } \underline{?} \end{aligned}$$

$$\begin{aligned} 3{,}068 \div 4 &= (2{,}800 + 240 + 28) \div 4 \\ &= (2{,}800 \div 4) + (\underline{?} \div 4) \\ &\qquad + (28 \div 4) \\ &= \underline{?} + \underline{?} + \underline{?} \\ &= \underline{?} \end{aligned}$$

15. Estimate the quotients. Then copy and divide.

$7{,}288 \div 8 = \underline{?} \qquad 3{,}225 \div 5 = \underline{?}$

Dividing hundreds, tens, and ones

1. Susan estimated the quotient for 2,772 ÷ 3 to be 90-some. Why is her estimate not sensible?

2. To estimate the number of hundreds in the quotient for 2,772 ÷ 3, think: 2,772 is about 27 hundreds; 27 hundreds ÷ 3 is 9 hundreds, or 900. Where is this number shown in the example below?

$$\begin{array}{lr}
(900+20+4)\text{ threes} \longrightarrow & \underline{924} \\
4\text{ more threes} \longrightarrow & 4 \\
20\text{ more threes} \longrightarrow & 20 \\
900\text{ threes} \longrightarrow & \underline{900} \\
 & 3\overline{)2{,}772} \\
900\times 3 \longrightarrow & \underline{2{,}700} \\
 & 72 \\
20\times 3 \longrightarrow & \underline{60} \\
 & 12 \\
4\times 3 \longrightarrow & \underline{12} \\
 & 0
\end{array}$$

- What is the remainder when you subtract 900 threes from 2,772?
- To estimate the number of tens in the quotient, think: 7 tens ÷ 3 is about 2 tens, or 20. Where is this number shown in the example?
- How many 3's are there in 12? Where is this number shown in the example?
- Add the parts of the quotient.
- To check, see if this sentence is true. 2,772 = (924 × 3) + 0

3. Estimate what 2,256 ÷ 7 is. Copy and complete this example.

$$\begin{array}{lr}
(300+20+2)\text{ sevens}+2 \longrightarrow & \underline{???}\text{ r}? \\
2\text{ more sevens} \longrightarrow & ? \\
20\text{ more sevens} \longrightarrow & 20 \\
300\text{ sevens} \longrightarrow & \underline{300} \\
 & 7\overline{)2{,}256} \\
300\times 7 \longrightarrow & \underline{2{,}100} \\
 & 156 \\
20\times 7 \longrightarrow & \underline{140} \\
 & 16 \\
2\times 7 \longrightarrow & \underline{??} \\
 & ?
\end{array}$$

To check, see if this sentence is true. 2,256 = (322 × 7) + 2

4. Study the example below. What numbers do *a*, *b*, and *c* stand for? Copy and complete the example.

$$\begin{array}{lr}
(a+b+c)\text{ fives}+2 \longrightarrow & \underline{692}\text{ r}? \\
c\text{ more fives} \longrightarrow & c \\
b\text{ more fives} \longrightarrow & b \\
a\text{ fives} \longrightarrow & \underline{a} \\
 & 5\overline{)3{,}462} \\
a\times 5 \longrightarrow & \underline{3{,}000} \\
 & 462 \\
b\times 5 \longrightarrow & \underline{450} \\
 & 12 \\
c\times 5 \longrightarrow & \underline{10} \\
 & 2
\end{array}$$

5. What is 2,957 ÷ 5? 3,176 ÷ 4 2,184 ÷ 7? 8,199 ÷ 9?

Practice with division

— Estimate the quotients. Then copy and divide. Check your answers by multiplying.

	a	b	c
1.	$6\overline{)372}$	$5\overline{)565}$	$8\overline{)3,368}$
2.	$7\overline{)644}$	$3\overline{)765}$	$2\overline{)1,842}$
3.	$4\overline{)128}$	$9\overline{)999}$	$5\overline{)4,625}$
4.	$8\overline{)232}$	$6\overline{)672}$	$9\overline{)7,398}$
5.	$2\overline{)156}$	$4\overline{)528}$	$3\overline{)1,626}$

— Name a multiple of 100 that vould be a good estimate for the juotient for each example below.

6.	$3\overline{)378}$	$2\overline{)462}$	$8\overline{)968}$
7.	$5\overline{)1,655}$	$4\overline{)3,824}$	$3\overline{)1,746}$
8.	$7\overline{)2,184}$	$6\overline{)4,872}$	$9\overline{)7,389}$

9. Without using a pencil or paper, ame a multiple of 10 that would be good estimate for the quotient for ach example in Ex. 6–8.

0. Copy the examples in Ex. 6–8 nd find the quotients. Compare our answers with the estimates you ave in Ex. 9. Check each quotient y multiplying.

. Divide 1,332 by 4. What is the mainder? Explain how you can e the following sentence to check ur answer.

$$1,332 = (333 \times 4) + 0$$

— Copy and find the quotients. To check, write a true sentence like the one in Ex. 11 using the divisor, quotient, and remainder.

2

	a	b	c
12.	$6\overline{)3,279}$	$4\overline{)378}$	$8\overline{)3,350}$
13.	$5\overline{)642}$	$9\overline{)5,049}$	$7\overline{)792}$
14.	$8\overline{)2,600}$	$3\overline{)2,464}$	$2\overline{)1,263}$
15.	$7\overline{)2,724}$	$5\overline{)2,468}$	$6\overline{)5,245}$

16. Fred can paste 6 pictures on each page of his photo album. He has 324 pictures. How many pages of his album can he fill?

17. Wilma can paste 8 pictures on each page of her photo album. She has 495 pictures. How many pages of her album can she fill? How many pictures will she have left over?

Think twice!

18. 1,000 ÷ 4 = 250, so 1,004 ÷ 4 = ?

19. 780 ÷ 3 = 260, so 786 ÷ 3 = ?

20. 450 ÷ 25 = 18, so 475 ÷ 25 = ?

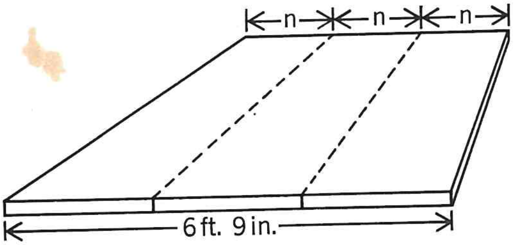
21. 400 ÷ 40 = 10, so 400 ÷ 80 = ?

22. 300 ÷ 30 = 10, so 300 ÷ 15 = ?

If you need more practice, do Practice Set 36, page 335.

PROBLEM-SOLVING HELP

Dividing measures



1. George has a board 6 feet 9 inches long. He wishes to cut it into 3 pieces of equal length. What should be the length of each piece?

- If *n* represents the length of each of the 3 pieces of board, does this sentence tell how to find *n*?

$$n = (6 \text{ ft. } 9 \text{ in.}) \div 3$$

- To divide 48 by 2, can you divide 4 tens by 2 and then 8 ones by 2? In the same way, to divide 6 feet 9 inches by 3, you can divide 6 feet by 3 and then divide 9 inches by 3.

- This example shows how to divide 6 feet 9 inches by 3. Each piece of board will be 2 feet _?_ inches long.

```
   2 ft. 3 in.
3)6 ft. 9 in.
```

- How can you show that 2 feet 3 inches is the correct answer?

— Copy and find the answers.

	a	b
2.	2)4 ft. 10 in.	4)8 hr. 40 min.
3.	2)12 lb. 8 oz.	3)12 hr. 15 min.
4.	4)8 yd. 12 in.	3)15 gal. 3 qt.
5.	5)5 lb. 10 oz.	4)4 min. 20 sec.

6. Can you divide 3 feet by 2 and get a whole number of feet for the answer? Three feet is equivalent to 2 feet _?_ inches. Then one way to show the division of 3 feet by 2 is to rename the 3 feet as 2 feet _?_ inches. Copy and complete this example. 2)2 ft. 12 in.

7. How would you show the division of 3 feet 4 inches by 2?

8. 2)3 lb. 8 oz. = 2)2 lb. _?_ oz.

9. 3)5 gal. 1 qt. = 3)3 gal. _?_ qt.

10. 4)1 hr. 20 min. = 4)_?_ min.

11. 5)12 ft. 6 in. = 5)10 ft. _?_ in.

12. Another way to show the division of 3 feet by 2 is to rename the dividend this way: 3 feet is equivalent to 36 inches.

Copy and divide. 2)36 in.

What quotient did you find for the example in Ex. 6? Is 1 foot 6 inches equivalent to 18 inches?

— Copy and find the quotients. Use the method you prefer.

	a	b
13.	2)5 ft. 8 in.	5)12 min. 10 sec.
14.	3)7 qt. 1 pt.	7)22 hr. 3 min.
15.	4)2 ft. 4 in.	2)9 gal. 2 qt.

If you need more practice, do Practice Set 37, page 335.

34

Using division to solve problems

1. Six boys are saving the money they earn to buy a boat. The boat they want costs $79.50. If they plan to share the cost equally, how much must each boy save?

- Can you use this number sentence to help find the answer to the problem?

$$7{,}950 \div 6 = n$$

- You can think of $79.50 as __?__ ¢. Study the example below and explain each step.

```
                      ????
                         5
                        20
                      3 00
                     10 00
                  6)$79.50
1,000 × 6 ——→        60 00
                     19 50
  300 × 6 ——→        18 00
                      1 50
   20 × 6 ——→         1 20
                        30
    5 × 6 ——→           30
                         0
```

- When you add the parts of the otient, what is the sum? Write a meral for the sum and put in the llar sign and cents point to show at it stands for money.

2. Eight girls are giving a party that will cost $15.36. If the girls share the cost equally, how much will each girl pay?

- Which of the following number sentences should you solve to find the answer?

$1{,}536 \div 8 = n$ $\qquad$ $8 \times 1{,}536 = n$

- What is the correct answer?

3. The science club wants to buy a chemistry set that costs $72. The club collects $4 a week in dues. How many weeks will it take them to collect enough money to buy the chemistry set?

- If n stands for the number of weeks, will the solution set for the following number sentence be the answer to the problem?

$$n \times 4 = 72$$

- Solve the sentence. How many weeks will it take the club to collect the money?

4. Each week Mary's family puts aside the same amount of money for a vacation trip. In 9 weeks they have saved $139.50. How much money have they been putting aside each week?

5. Seven boys plan to share equally the cost of a gift for their club adviser. If the gift costs $12.25, what is each boy's share?

How well can you divide?

1. On a round trip between St. Louis, Missouri, and Harrisburg, Pennsylvania, a train traveled 1,712 miles. How far is it one way between these two cities?

2. On 2 round trips between Boston, Massachusetts, and Washington, D.C., a plane flew 1,572 miles. If the plane flew the same distance each time, how far did it fly one way?

Which of the following number sentences should you solve to answer the question? Solve the sentence.

$1{,}572 \div 2 = n$ $1{,}572 \div 4 = n$

3. Mary Lou has saved the same amount of money each week for 6 weeks. If she has saved $7.50, how much did she save each week?

4. Seven boy scouts collected 1,622 pounds of paper during a paper drive. If each boy collected about the same amount of paper, how much paper did each boy collect?

5. Five girls plan to share equally the cost of a party. If the party costs $12.25, what is each girl's share?

— These examples will help you find out how well you can divide. Copy, find the answers, and check.

	a	b	c	d	e
6.	7)645	8)675	9)801	6)574	8)765
7.	9)702	6)497	4)617	8)550	7)339
8.	5)4,055	9)5,563	6)1,632	7)3,269	8)5,000
9.	9)2,895	8)6,492	9)5,645	6)4,089	7)6,799
10.	8)6,888	7)5,597	6)4,288	7)5,257	9)7,669
11.	8)$53.29	6)$40.32	8)$66.39	5)$89.90	7)$41.1
12.	3)$16.72	5)$34.57	4)$11.88	7)$27.47	9)$83.1

If you need more practice, do Practice Sets 38 and 39, page 336.

Solving two-step problems

1. Sam bought 6 kites at 45¢ each. How much change should he receive from $5.00?

- The first step is to find the cost of the 6 kites. 6 × 45 = ?
- The second step is to find the ifference between $5.00 and $2.70.

500 − 270 = ?

- If n stands for the amount of hange, does this number sentence orrectly combine the first and sec- nd steps shown above into one umber sentence? Explain.

$n = 500 - (6 \times 45)$

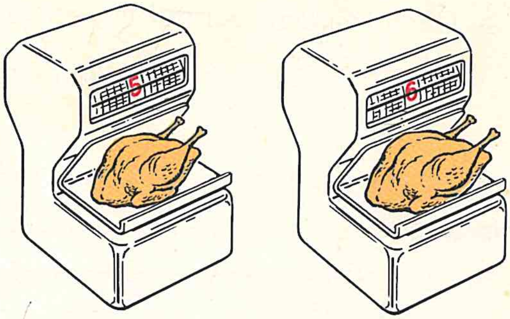
- Solve the sentence.

2. Nancy wants 4 yards of lace, osting 38¢ a yard, and a roll of tape, osting 15¢. What will be the total ost of the two purchases?

- If Nancy lets n stand for the otal cost, explain how she can use is sentence to find the answer to e problem. $n = (4 \times 38) + 15$
- Solve the sentence.

For each of the following prob- ms, write one number sentence at you can solve to answer the oblem. Let n stand for the num- r you want to find. Then solve e sentence.

.) Ann bought overshoes for $1.79 d a rain hat for $1.45. How much ange should she receive from a 10- llar bill?



4. At 39¢ a pound, how much will these two chickens cost?

5. David delivers 16 papers each morning and 18 each evening. How many does he deliver in 6 days?

6. Mr. Adams started on a 956-mile trip. The first day he drove 283 miles. The second day he drove 316 miles. The third day he drove 331 miles. How much farther does he have to drive?

7. John bought 50 magazines for $1.00. He sold them for 5¢ each. How much money did he make on all the magazines?

30

8. Movie tickets for adults cost 60¢. Children's tickets are half price. Five children's tickets will cost ?.

9. Susan needs 3 yards of vinyl plastic to make a rain cape and 1 yard for book covers. How much will the plastic cost at 40¢ a yard?

10. Roy wants six 25-cent batteries and a 19-cent wheel for his model airplane. How much will they cost?

Working mentally

— Good thinkers can do these examples mentally. Can you?

1. Is 8 × $3.02 nearer to $24.00 or to $24.10?

2. Is $28.65 ÷ 7 nearer to $4.00 or to $4.25?

3. A club had $111.11 in its treasury. How much was left after paying a bill amounting to $110.10?

4. How much less than $30.00 is the cost of 6 footballs at $4.95 each?

5. Does 1,980 ÷ 9 equal about 200 or about 2,000?

6. If 6 pounds 10 ounces is multiplied by 2, the result will be 13 pounds _?_ ounces.

7. If 2 hours 28 minutes is separated into 2 equal periods of time, how long will each period be?

8. Shown below is one way to find 9 × $4.25 without using a pencil. What is 9 × $4.25?

9 × $4.00 = $36.00
9 × $ .25 = $2.25
so 9 × $4.25 = _?_

— Use the way shown in the box above to find these products.

	a	b
9.	9 × $5.25 = _?_	8 × $3.20 = _?_
10.	6 × $9.80 = _?_	7 × $6.50 = _?_

— Try to solve these problems without a pencil and paper. Read each problem carefully to see what it asks you to find. Next, decide how to solve the problem. Then find the answer.

11. Eighteen pupils from a class are going to visit a museum; 8 pupils cannot go. How many pupils are there in the class?

12. There are 36 pupils in Jerry's class. All but 7 have visited the Egyptian room at the museum. How many have visited this room?

13. The temperature outside the museum was 82 degrees. Inside it was 68 degrees. How many degrees cooler was it inside?

14. Three boys plan to buy a 75 cent book. If they share the cos equally, each boy should pay _?_ ¢

15. Don has 36¢. He wants to bu two 25-cent pictures of the Nil River. How much more mone does he need?

16. At the souvenir counter, a mode of an Egyptian mummy costs $1.5 and post cards cost 5¢ each. Ho much would 1 model of a mumm and 10 post cards cost?

331 239

Keeping up in mathematics

Oral

— Tell which division examples have remainders of 0.

	a	b	c	d
1.	$4\overline{)32}$	$3\overline{)28}$	$5\overline{)21}$	$2\overline{)19}$
2.	$4\overline{)26}$	$6\overline{)30}$	$8\overline{)50}$	$7\overline{)51}$
3.	$8\overline{)73}$	$7\overline{)56}$	$9\overline{)65}$	$9\overline{)82}$
4.	$8\overline{)49}$	$6\overline{)54}$	$7\overline{)44}$	$8\overline{)64}$

5. Tell which division examples in Ex. 1–4 have remainders of 1; of 2.

6. Give the quotient and remainder for each division example in Ex. 1–4.

— In Ex. 7–10 tell how many digits there will be in each answer.

	a	b
7.	100 × 75 = ?	7 × 7,097 = ?
8.	150 − 79 = ?	800 − 673 = ?
9.	4 × 26 = ?	606 − 598 = ?
0.	60 × 42 = ?	30 × 319 = ?

— Solve each of these sentences.

1. $n - 16 = 38$ $\qquad 75 - n = 25$

2. $28 + n = 40$ $\qquad n \times 9 = 72$

— Read these numerals.

3. 456,125 600,024

4. 1,728,357 5,000,600

5. 9,206,100 15,001,100

Written

— Copy, find the answers, and check.

	a	b	c
1.	1 22 2 $458.47 245.68 7.34 + 26.75	1 $173 143 892 + .69	22 $9.89 2.98 .17 + 10.35 2339
2.	$4,000 − 697	$785.36 − 695.27	$60.00 − 21.07
3.	$7.06 × 67	$685 × 300	$30.80 × 59
4.	$6\overline{)522}$	$7\overline{)1{,}489}$	$9\overline{)\$10.89}$

5. 175 yd. is equivalent to ? ft. 1277

6. Six yards of nylon net cost $5.88. What is the cost of one yard of net?

7. How many weeks are there in 60 days? How many days are left over?

8. Ted has saved 75¢ a week for 5 weeks. How much more does he need to buy a book costing $4.50?

9. Three girls gave a party. The ice cream cost $1.50; the cookies cost 64¢; and the soft drinks cost $1.25. The girls plan to share the expenses equally. What is each girl's share of the cost?

10. Write numerals for the first 10 multiples of 2; 4; 5; 7; 10.

SELF-HELP TEST 3

Solve these sentences.

1. $7 \times 24 = (7 \times n) + (7 \times 4)$ [49]

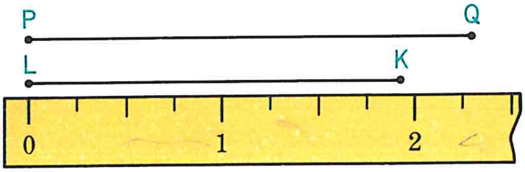
2. $45 + 45 + 45 = n \times 45$ [46]

3. $(18 \times 27) \times 43 = n \times (27 \times 43)$ [47]

4. How many sides does a pentagon have? a triangle? a quadrilateral?

How many vertices does each figure named above have? [77]

5. Tell how long each line segment shown below is to the nearest inch; to the nearest half inch. [83–85]



6. How many times is 10 used as a factor to get 10,000,000,000? [4]

7. From the set of whole numbers, list the solution set for each of the following sentences. [18]

$3 \times n > 12 \qquad 45 \div n = 9$

8. The length of a diameter of a circle is 16 feet. What is the length of a radius of that circle? [91]

9. List the set whose members are the first ten multiples of 9. [48]

10. On a number line, point Y is a greater distance to the right of zero than point X. Which point, Y or X, represents the greater number? [9]

SELF-HELP TEST 4

Copy and find the answers.

1. $\begin{array}{r} 459 \\ \times\ 8 \\ \hline \end{array}$ [50]

2. $\begin{array}{r} 335 \\ \times\ 712 \\ \hline \end{array}$ [55]

3. $8\overline{)59}$ [64–65]

4. $5\overline{)32}$ [64–65]

5. $\begin{array}{r} 2{,}321 \\ +\ 6{,}899 \\ \hline \end{array}$ [25]

6. $\begin{array}{r} 10{,}000 \\ -\ 7{,}482 \\ \hline \end{array}$ [34]

7. $572 \times 1{,}000 = \underline{\ ?\ }$ [51]

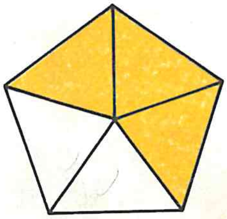
8. $\begin{array}{r} 3 \text{ ft. } 4 \text{ in.} \\ \times\ 4 \quad \\ \hline \end{array}$ [69]

9. Write numerals for the whole numbers that make the following number sentences true. [80–81]

$\frac{4}{8} = \frac{n}{2} \qquad \frac{3}{6} = \frac{2}{n}$

$\frac{2}{2} = n \qquad \frac{n}{8} = 1$

10. Write a fraction for the number that tells what part of the pentagonal region shown below is yellow. [80]



IT'S CHECK–UP TIME

— In each sentence below, n stands for any whole number. Find the solution sets.

1. $n \times 9 \leq 72$
2. $n \times 7 \leq 47$
3. $525 = (75 \times 7) + n$
4. $2,475 = (n \times 4) + 3$

— In each sentence below, n stands for any multiple of 10. Find the solution sets.

5. $n \times 8 \leq 657$
6. $n \times 6 \leq 439$

81 r1
8
648

— Find the greatest multiple of 100 that n can represent to make the sentence true.

7. $n \times 4 \leq 3,220$
8. $n \times 5 \leq 1,904$

— Copy, find the answers, and check.

	a	b
9.	$4\overline{)344}$	$6\overline{)329}$
10.	$7\overline{)989}$	$3\overline{)975}$
11.	$5\overline{)1,345}$	$8\overline{)6,750}$
12.	$9\overline{)6,048}$	$7\overline{)4,084}$
13.	$6\overline{)\$35.22}$	$4\overline{)\$18.96}$

— Copy and find the answers.

14. $3\overline{)12 \text{ ft. } 6 \text{ in.}}$
15. $6\overline{)7 \text{ lb. } 2 \text{ oz.}}$
16. $7\overline{)3 \text{ gal. } 2 \text{ qt.}}$

— Write only the answers.

17. $4 \times \$3.25 = \underline{\ ?\ }$
18. $8 \times \$5.50 = \underline{\ ?\ }$
19. Every ten days a bus makes 3 round trips between Cleveland, Ohio, and Memphis, Tennessee. It travels 4,590 miles in all. What is the distance between these two cities?
20. Tom saved 12¢ a day for 30 days. If he plans to share this money equally with his brother, how much money will each boy receive?

If you have time, try these —

— What do you think the symbol $\geq$ means? Find the solution set from the set of whole numbers for each of the following sentences.

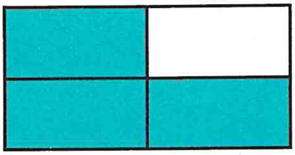
	a	b
21.	$n \times 4 \geq 12$	$n \times 5 \geq 43$
22.	$n \times 7 \geq 35$	$n \times 2 \geq 0$

Mathamusements

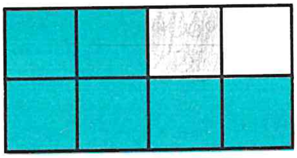
On one side of a piece of paper, write the sentence: "The sentence on the other side of this paper is false." On the other side, write: "The sentence on the other side of this paper is true." Is this second sentence true, or is it false?

CHAPTER 6

Equivalent fractions



1. The rectangular region shown above has been separated into ? smaller regions of the same size and shape. How many of these smaller regions are blue? What fractional number tells you the part of the rectangular region that is blue?

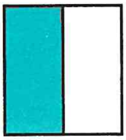


2. Shown above is the same rectangular region as in Ex. 1, now separated into ? smaller regions of the same size and shape. How many of these smaller regions are blue? What fractional number tells you the part of the rectangular region that is blue?

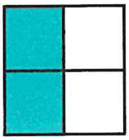
3. Has any more of the rectangular region been colored in the drawing for Ex. 2 than is colored in the drawing for Ex. 1? Then does the fraction $\frac{6}{8}$ name the same part of the whole region as the fraction $\frac{3}{4}$? We can show this idea by writing the following number sentence. $\frac{3}{4} = \frac{6}{8}$

4. Each drawing below shows the same rectangular region separated into a different number of smaller regions of the same size and shape. Write a fraction that names the part of the whole region that is blue in drawing A; in B; in C.

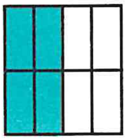
A



B



C



- Is the part of the region named by $\frac{2}{4}$ the same as the part named by $\frac{1}{2}$?
- The part named by $\frac{?}{8}$ in C is the same as the part named by $\frac{1}{2}$ or by $\frac{?}{4}$.
- Do $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ all name the same part of the rectangular region? Are these number sentences true?

$\frac{1}{2} = \frac{2}{4}$ $\quad$ $\frac{1}{2} = \frac{4}{8}$ $\quad$ $\frac{2}{4} = \frac{4}{8}$

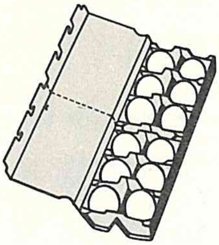
Fractions that name the same part of a whole (or the same part of a set), such as $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$, are called equivalent fractions.

---

5. Copy and complete each of the following sentences to show equivalent fractions.

$\frac{1}{2} = \frac{?}{4}$ $\quad$ 2 $\frac{?}{4} = \frac{4}{8}$ $\quad$ $\frac{3}{4} = \frac{?}{8}$

6. How many halves are there in 1 whole? How many fourths? sixths? eighths? Then are $\frac{2}{2}$, $\frac{4}{4}$, $\frac{6}{6}$, and $\frac{8}{8}$ equivalent fractions? Do they all name the whole number 1?



7. How many eggs are shown in this carton? How many eggs are there in $\frac{6}{12}$ of this set of eggs? in $\frac{3}{6}$ of this set? in $\frac{2}{4}$ of this set? in $\frac{1}{2}$?

Do $\frac{6}{12}$, $\frac{3}{6}$, $\frac{2}{4}$, and $\frac{1}{2}$ name the same part of the set of eggs? Then these fractions can be called _?_ fractions.

8. Multiply the numerator and the denominator of $\frac{1}{2}$ by 2. Is $\frac{2}{4}$ the result? Is $\frac{2}{4}$ equivalent to $\frac{1}{2}$?

9. Multiply the numerator and the lenominator of $\frac{1}{2}$ by 3. What is the esult? Is $\frac{3}{6}$ equivalent to $\frac{1}{2}$? to $\frac{2}{4}$?

10. Multiply the numerator and the lenominator of $\frac{1}{2}$ by 4; by 5; by 6. Do you think that each of the result-ng fractions is equivalent to $\frac{1}{2}$? Draw pictures to show your answer.

1. Study this set of fractions.

$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \ldots\right\}$$

Tell the numerator named in each. Tell the denominator. Is each numerator a multiple of the numerator f $\frac{1}{2}$? Is each denominator a multiple f the denominator of $\frac{1}{2}$?

Is this a set of equivalent fracons? What is the next member of e set?

12. Study this set of equivalent fractions. $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\right\}$

- Is the numerator named in each fraction a multiple of the numerator of $\frac{3}{4}$? Is the denominator named in each fraction a multiple of the denominator of $\frac{3}{4}$?
- What is the result if you multiply both the numerator and the denominator of $\frac{3}{4}$ by 2? by 3? by 4? What is the next member of the set shown above?
- What is the result if you divide both the numerator and the denominator of $\frac{12}{16}$ by 2? by 4?
- What is the result if you divide both the numerator and the denominator of $\frac{15}{20}$ by 5?
- Explain how you can find a fraction that shows a denominator of 20 and is equivalent to $\frac{3}{4}$; that shows a numerator of 3 and is equivalent to $\frac{18}{24}$.

★ Your discovery

Tell your discovery

What kind of fraction is the result when you multiply the numerator and the denominator of a fractional number by the same number? when you divide the numerator and the denominator by the same number?

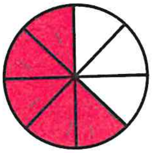
Use your discovery

Solve these sentences.

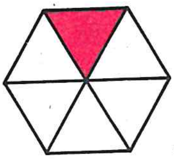
$\frac{2}{3} = \frac{\square}{12}$ $\frac{6}{9} = \frac{\square}{3}$ $\frac{3}{4} = \frac{27}{\square}$ $4/\frac{\overset{30}{120}}{150} = \frac{4}{\square}$

12

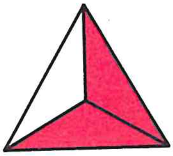
Practice with fractions



A



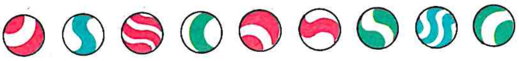
B



C

1. Write a fraction for the number that tells you the part of figure A that is red; the part of figure B that is red; the part of figure C that is red.

2. Name the numerator shown by each fraction you wrote in Ex. 1. Name the denominator shown by each.



3. How many marbles in the set shown above are red? green? blue?

- What fractional number tells you the part of the set that is red?
- Write a fraction for the number that tells you what part is green.
- Write a fraction for the number that tells you what part is blue.

4. Two or more fractions that name the same part of a set, or of a whole, are called $\underline{?}$ fractions.

— Solve each of these sentences.

	a	b
5.	$\frac{4 \times 3}{4 \times 5} = \frac{\triangle}{20}$	$\frac{6 \div 3}{9 \div 3} = \frac{\square}{3}$
6.	$\frac{2 \times 4}{2 \times 7} = \frac{\triangle}{\square}$	$\frac{8 \div 4}{12 \div 4} = \frac{\triangle}{\square}$

— Study each set of equivalent fractions given below. Then copy each set and write three more fractions that are members of the set.

7. $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots\}$

8. $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\}$

9. $\{\frac{18}{24}, \frac{15}{20}, \frac{12}{16}, \frac{?}{\_\_}, \frac{?}{\_\_}, \frac{?}{\_\_}\}$

10. Which of these number sentences is false?

$\frac{3}{5} \overset{5}{=} \frac{15}{25}$ $\qquad \frac{4}{9} \overset{4}{=} \frac{12}{36}$ $\qquad \frac{9}{27} \overset{9}{=} \frac{1}{3}$

— Solve each of these sentences.

	a	b	c
11.	$\frac{1}{2} \overset{4}{=} \frac{\square}{8}$	$\frac{6}{8_{2}} \overset{2}{=} \frac{\triangle}{4}$	$\frac{2}{3} \overset{7}{=} \frac{\square}{21}$
12.	$\frac{3}{4} \underset{4}{=} \frac{12}{\triangle}$	$\frac{7}{14} \overset{7}{\underset{7}{=}} \frac{1}{\square}$	$\frac{14}{16} \overset{2}{=} \frac{7}{\triangle}$
13.	$\frac{5}{8} \overset{2}{=} \frac{\square}{16}$	$\frac{9}{15} \overset{7}{=} \frac{3}{\triangle}$	$\frac{4}{7} \overset{3}{=} \frac{\square}{21}$
14.	$\frac{9}{16} \overset{2}{=} \frac{18}{\triangle}$	$\frac{6}{9} \overset{3}{=} \frac{\square}{3}$	$\frac{5}{15} = \frac{1}{\triangle}$

15. Barbara baked 10 cupcakes. She gave 5 of them to her brother. Her mother said she gave $\frac{5}{10}$ of them to her brother. Her father said she gave $\frac{1}{2}$ of them to her brother. Who was correct? Explain.

16. Bill bought a box of one dozen crayons. He gave $\frac{1}{2}$ of the set of crayons to his sister. How many did he give her?

If you need more practice, do Practice Set 45, page 337.

Keeping up in mathematics

Oral

— Tell the correct answers.

1. Does 598,321 increased by two thousand equal 600,321 or 618,321 or 798,321?

2. If 320 marbles are shared equally by 8 boys, will each boy receive 4, or 40, or 400 marbles?

3. Is 16 qt. the same amount as 32 gal., 4 gal., 8 gal., or 24 gal.?

4. Is 6 dozen eggs the same as 60 eggs, 62 eggs, or 72 eggs?

5. Which is another name for 1: $\frac{3}{5}$, $\frac{4}{5}$, or $\frac{5}{5}$?

6. To solve this number sentence, would you subtract, multiply, or divide? $\$1.75 + n = \$2.25$

7. Is 29 × $5.03 about $15.00, 150.00, or $1,500.00?

8. Will the quotient for $3\overline{)942}$ be amed by a numeral with 1 digit, 2 igits, or 3 digits?

9. Which of these fractions is equiv-lent to $\frac{3}{4}$? $\frac{5}{8}$ $\frac{12}{16}$ $\frac{4}{5}$

0. Which is the best estimate for ,600 ÷ 8? 200 300 400

1. Harry weighs 76 pounds. His ther weighs 184 pounds. Is the ifference between their weights out 90 pounds, about 100 pounds, about 110 pounds?

. Which of these fractions names e same number as $\frac{6}{10}$? $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$

Written

— Copy, find the answers, and check.

	a	b
1.	36 47 175 86 + 295	$7.55 .98 3.04 .62 + 4.98
2.	7,062 − 3,948	$805.03 − 28.65
3.	849 × 56	$7.06 × 48
4.	$4\overline{)348}$	$9\overline{)576}$
5.	$7\overline{)2,772}$	$8\overline{)5,735}$
6.	$9\overline{)5,319}$	$6\overline{)4,691}$

7. Write the decimal numeral for ninety-six thousand eighty-nine.

8. The length of a piece of rope is 144 yards. How many feet of rope is that?

9. Solve this number sentence.

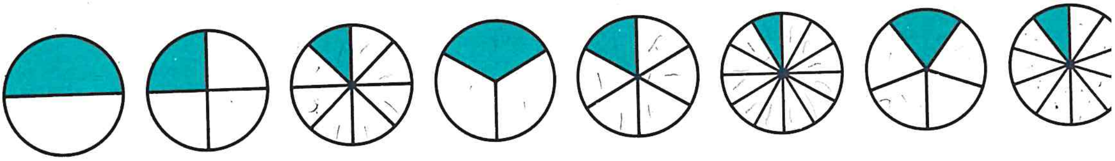
$$8 \times n = 1{,}968$$

40
18
58

100
58
43

10. Don bought 3 candy bars at 6¢ each and 2 magazines at 40¢ each. How much change did he receive from $1.00?

Comparing fractional numbers



1. The eight drawings above show the same circular region separated into different numbers of smaller regions of the same size. For each drawing, write a fraction for the number that tells what part of the circular region is blue.

— Use the drawings above to help you tell which number is greater.

	a	b	c
2.	$\frac{1}{2}$ or $\frac{1}{4}$	$\frac{1}{4}$ or $\frac{1}{3}$	$\frac{1}{6}$ or $\frac{1}{4}$
3.	$\frac{1}{6}$ or $\frac{1}{8}$	$\frac{1}{5}$ or $\frac{1}{2}$	$\frac{1}{8}$ or $\frac{1}{3}$
4.	$\frac{1}{5}$ or $\frac{1}{4}$	$\frac{1}{2}$ or $\frac{1}{12}$	$\frac{1}{5}$ or $\frac{1}{10}$

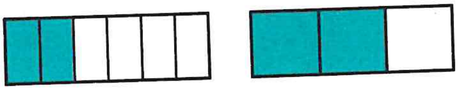
5. Both $\frac{1}{4}$ and $\frac{1}{8}$ have the same numerator but different ?.

- Which has the greater denominator?
- Which is the greater number, $\frac{1}{4}$ or $\frac{1}{8}$?

6. Do $\frac{1}{3}$ and $\frac{1}{5}$ have the same numerator? Which has the greater denominator? Which is the greater number, $\frac{1}{3}$ or $\frac{1}{5}$?

7. Both $\frac{1}{10}$ and $\frac{1}{12}$ have the same ? but different ?. Which has the greater denominator? Which is the greater number, $\frac{1}{10}$ or $\frac{1}{12}$?

8. Each drawing below shows the same rectangular region separated into a different number of smaller regions of the same size. For each drawing, write a fraction to name the part of the rectangular region that is blue.



- Both $\frac{2}{6}$ and $\frac{2}{3}$ have the same ?.
- Which has the greater denominator, $\frac{2}{6}$ or $\frac{2}{3}$?
- Is this sentence true? $\frac{2}{3} > \frac{2}{6}$

9. Which number is greater? Make drawings of regions if you need help.

$\frac{3}{4}$ or $\frac{3}{8}$ $\frac{5}{8}$ or $\frac{5}{6}$ $\frac{2}{5}$ or $\frac{2}{3}$ $\frac{3}{5}$ or $\frac{3}{10}$

Your discovery

Tell your discovery

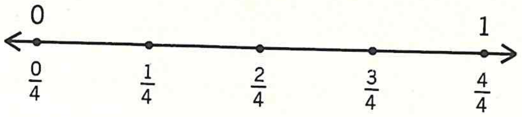
Tell what you have found about two fractional numbers that have the same numerator but different denominators.

Use your discovery

From each set shown below, tel which is the greatest number and which is the smallest number.

$\{\frac{1}{7}, \frac{1}{2}, \frac{1}{9}, \frac{1}{5}, \frac{1}{4}\}$ $\{\frac{40}{55}, \frac{40}{900}, \frac{40}{67}, \frac{40}{82}, \frac{40}{40}$

10. On the number line shown below, the segment between 0 and 1 has been separated into ? smaller segments of equal length.



- The length of the segment from 0 to $\frac{1}{4}$ is what part of the length of the segment from 0 to 1?
- What part of the length of the segment from 0 to 1 is the length of the segment from 0 to $\frac{3}{4}$? from 0 to $\frac{2}{4}$? from 0 to $\frac{4}{4}$?

11. On the number line shown above, is the fraction $\frac{1}{4}$ at the left or at the right of the fraction $\frac{2}{4}$? Then is this number sentence true? $\frac{1}{4} < \frac{2}{4}$ Explain.

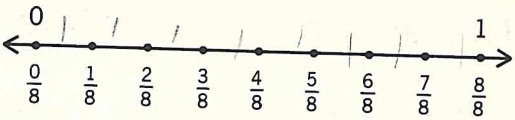
12. Explain how the number line above shows that each of these sentences is true.

$\frac{1}{4} < \frac{4}{4}$ $\frac{1}{4} < \frac{3}{4}$ $\frac{3}{4} > \frac{2}{4}$

13. Both $\frac{3}{4}$ and $\frac{2}{4}$ have the same denominator but different ?.

- Which has the smaller numerator, $\frac{3}{4}$ or $\frac{2}{4}$?
- Which is the smaller number, or $\frac{2}{4}$?

4. Do $\frac{3}{4}$ and $\frac{1}{4}$ have the same denominator? Which has the greater umerator? Which is the greater umber, $\frac{3}{4}$ or $\frac{1}{4}$?



15. On the number line shown above, the segment between 0 and 1 has been separated into ? smaller segments of equal length.

How does this number line show that each sentence below is true?

$\frac{1}{8} < \frac{3}{8}$ $\frac{4}{8} < \frac{7}{8}$ $\frac{3}{8} < \frac{5}{8}$

16. Do $\frac{3}{8}$ and $\frac{7}{8}$ have the same denominator? Which has the greater numerator? Which is the greater number, $\frac{3}{8}$ or $\frac{7}{8}$?

Your discovery

Tell your discovery

If two fractional numbers have the same denominator but different numerators, how can you tell which is the smaller fractional number? the greater fractional number?

Use your discovery

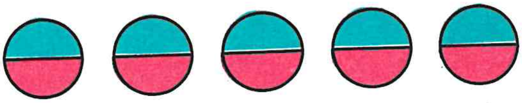
From each set shown below, tell which is the greatest number and which is the smallest number.

$\{\frac{3}{5}, \frac{2}{5}, \frac{4}{5}, \frac{1}{5}, \frac{5}{5}\}$ $\{\frac{14}{92}, \frac{60}{92}, \frac{5}{92}, \frac{82}{92}, \frac{1}{92}\}$

— Copy and replace each ● with > or < to make a true sentence.

	a	b	c
17.	$\frac{4}{5}$ ● $\frac{1}{5}$	$\frac{3}{7}$ ● $\frac{5}{7}$	$\frac{1}{9}$ ● $\frac{1}{8}$
18.	$\frac{3}{5}$ ● $\frac{3}{8}$	$\frac{2}{3}$ ● $\frac{1}{3}$	$\frac{4}{7}$ ● $\frac{4}{5}$
19.	$\frac{5}{8}$ ● $\frac{3}{8}$	$\frac{1}{6}$ ● $\frac{1}{5}$	$\frac{2}{4}$ ● $\frac{3}{4}$

Fractions that name whole numbers



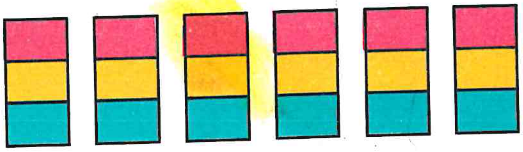
1. Each circular region shown above has been separated into $\underline{?}$ smaller regions of the same size and shape. What part of each circular region is red? What part is blue?

2. How many halves are there in 1 circular region? in 2? 3? 4? 5?

3. Write a fraction as a name for 2 halves; 3 halves; 4 halves; 5 halves.

4. Count the halves shown in the circular regions above. Begin: $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{?}{2}$, $\frac{?}{2}$, $\frac{?}{2}$, $\frac{?}{2}$, $\frac{?}{2}$, $\frac{?}{2}$, $\frac{?}{2}$.

5. What whole number is named by the fraction $\frac{2}{2}$? $\frac{4}{2}$? $\frac{6}{2}$? $\frac{8}{2}$? $\frac{10}{2}$?



6. Each card above is separated into $\underline{?}$ parts of the same size and shape. What part of each card is red? yellow? blue?

7. How many thirds are there in 1 whole card? in 2 whole cards? in 3? 4? 5? 6?

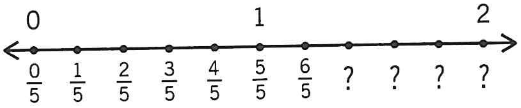
8. Count by thirds from $\frac{1}{3}$ to $\frac{18}{3}$.

9. Which fractions in the following set are names for whole numbers?

$$\left\{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}, \frac{9}{3}, \frac{10}{3}, \frac{11}{3}\right\}$$

10. What whole number is named by the fraction $\frac{12}{3}$? by $\frac{15}{3}$? by $\frac{18}{3}$?

11. Are $\frac{4}{2}$ and $\frac{6}{3}$ equivalent fractions? Do they both name a whole number? What whole number do they name?



12. Use the number line shown above to help you count from $\frac{0}{5}$ to $\frac{10}{5}$.

13. How many fifths are equal to the whole number 1? 2? 3? 4? 5?

— The fractions in each set below are names for whole numbers. For each set, tell the whole number named and write three more fractions that are members of the set.

	a	b
14.	$\left\{\frac{4}{2}, \frac{6}{3}, \frac{8}{4}, \frac{10}{5}, \cdots\right\}$	$\left\{\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \cdots\right\}$
15.	$\left\{\frac{8}{2}, \frac{12}{3}, \frac{16}{4}, \frac{20}{5}, \cdots\right\}$	$\left\{\frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \cdots\right\}$

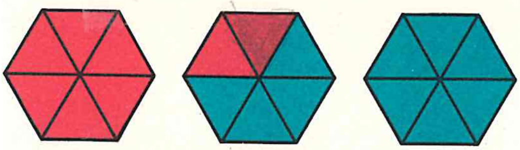
16. If the numerator shown by a fraction is a multiple of the denominator, does the fraction name a whole number?

17. Write four fractions that name each whole number from 0 to 10.

18. Do you think that all whole numbers are fractional numbers? Are all fractional numbers whole numbers?

Is the set of whole numbers a subset of the set of fractional numbers?

Naming fractional numbers greater than 1



1. Each hexagonal region shown above is separated into $\underline{?}$ smaller regions of the same size and shape. Each of these smaller regions is what part of a hexagonal region?

2. How many sixths of the hexagonal regions are red? Which of the following number sentences is true?

$\frac{8}{6} < 1 \qquad \frac{8}{6} = 1 \qquad \frac{8}{6} > 1$

3. Is one whole hexagonal region red? Is $\frac{2}{6}$ of another hexagonal region red also? Then is this number sentence true? $\frac{8}{6} = 1 + \frac{2}{6}$

4. How many sixths of the hexagonal regions shown above are blue?

- $\frac{10}{6}$ is $\underline{?}$ sixths greater than 1.
- Is $\frac{10}{6}$ equal to $1 + \frac{4}{6}$?

5. Another way to show $1 + \frac{4}{6}$ is to write $1\frac{4}{6}$. What is another way to show $1 + \frac{2}{6}$?

6. A numeral such as $1\frac{4}{6}$ or $1\frac{2}{6}$ is called a **mixed form** for the fracional number. We read $1\frac{4}{6}$ as "one and four sixths." How should you ead the numeral $1\frac{2}{6}$?

7. Read each of these numerals.

$\frac{3}{4} \qquad 1\frac{5}{8} \qquad 2\frac{1}{3} \qquad 4\frac{1}{8} \qquad 6\frac{4}{5} \qquad 8\frac{1}{2}$

8. How many fourths is equal to 3? Then is $\frac{15}{4}$ greater than 3?

- $\frac{15}{4}$ is $\underline{?}$ fourths greater than 3.
- Is the following number sentence true?

$$\frac{15}{4} = 3 + \frac{3}{4}$$

- Express $3 + \frac{3}{4}$ in mixed form.

— Express each of the numbers named below in mixed form.

	a	b	c
9.	$1 + \frac{2}{3}$	$3 + \frac{4}{5}$	$2 + \frac{5}{8}$
10.	$4 + \frac{1}{2}$	$6 + \frac{5}{8}$	$1 + \frac{1}{6}$
11.	$2 + \frac{3}{7}$	$1 + \frac{7}{9}$	$6 + \frac{2}{3}$

12. Count by eighths to 24 eighths this way: $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}, \frac{9}{8}, \frac{10}{8}$, and so on.

Now count by eighths to 3 this way: $0, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, 1, 1\frac{1}{8}, 1\frac{2}{8}, 1\frac{3}{8}$, and so on.

— Study each of the following sets of fractional numbers for a pattern. Then write numerals for the next three members of each set.

13. $\{\frac{0}{3}, \frac{2}{3}, \frac{4}{3}, \frac{6}{3}, \cdots\}$

14. $\{\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \cdots\}$

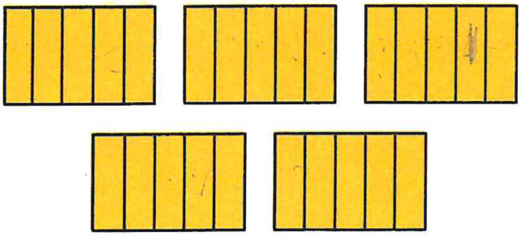
15. $\{\frac{0}{5}, \frac{3}{5}, \frac{6}{5}, \frac{9}{5}, \cdots\}$

16. $\{\frac{2}{5}, \frac{4}{5}, 1\frac{1}{5}, 1\frac{3}{5}, \cdots\}$

17. $\{\frac{0}{7}, \frac{3}{7}, \frac{6}{7}, 1\frac{2}{7}, 1\frac{5}{7}, \cdots\}$

18. $\{\frac{1}{9}, \frac{5}{9}, 1, 1\frac{4}{9}, \cdots\}$

Using a mixed form to name fractional numbers



1. Each of the cards shown above has been separated into $\underline{?}$ parts of the same size and shape. Each of these parts is what part of a whole card?

How many fifths are there in 1 whole card? in 2 whole cards? in 5 whole cards?

2. Count by fifths to $\frac{15}{5}$. Begin: $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}$, and so on.

3. Count by fifths to 3. Begin: $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, 1\frac{1}{5}$, and so on.

4. Use the cards to find the false number sentence.

$\frac{7}{5} = 1\frac{2}{5}$ $\quad \frac{11}{5} = 2\frac{1}{5}$ $\quad \frac{17}{5} = 3\frac{2}{5}$ $\quad \frac{8}{5} = 1\frac{4}{5}$

5. In doing the exercises above, did you discover that the same number of fifths can often be named two different ways? Does $3\frac{2}{5}$ name the same number as $\frac{17}{5}$?

- Use a mixed form to name $\frac{18}{5}$.
- Use a mixed form to name $\frac{19}{5}$.

6. What whole number is named by $\frac{20}{5}$? Then is $\frac{21}{5}$ greater than 4?

- $\frac{21}{5}$ is $\underline{?}$ fifth greater than 4.
- Use a mixed form to name $\frac{21}{5}$.
- Use a mixed form to name $\frac{22}{5}$.

7. What whole number is named by $\frac{5}{5}$? $\frac{10}{5}$? $\frac{15}{5}$? $\frac{20}{5}$? $\frac{25}{5}$?

8. Is $\frac{7}{5}$ greater than $\frac{5}{5}$ and less than $\frac{10}{5}$? Then is $\frac{7}{5}$ greater than 1 and less than 2?

9. Between what two whole numbers is $\frac{12}{5}$? $\frac{18}{5}$? $\frac{24}{5}$?

10. How many fifths greater than $\frac{5}{5}$ is $\frac{7}{5}$? Then does $\frac{7}{5}$ name the same number as $\frac{5}{5} + \frac{2}{5}$, or $1 + \frac{2}{5}$? Use a mixed form to name this number.

11. How many fifths greater than $\frac{10}{5}$ is $\frac{13}{5}$? Then does $\frac{13}{5}$ name the same number as $\frac{10}{5} + \frac{3}{5}$, or $2 + \frac{3}{5}$? Use a mixed form to name $\frac{13}{5}$.

12. How many fifths greater than $\frac{15}{5}$ is $\frac{19}{5}$? So $\frac{19}{5}$ names the same number as $3 + \frac{?}{5}$, or $3\frac{?}{5}$.

13. How many fifths greater than $\frac{20}{5}$ is $\frac{24}{5}$? So $\frac{24}{5}$ names the same number as $4 + \frac{?}{5}$, or $4\frac{?}{5}$.

— Copy and replace *n* with a mixed form for the fractional number that makes the sentence true.

	a	b	c
14.	$\frac{6}{5} = n$	$\frac{8}{5} = n$	$\frac{11}{5} = n$
15.	$\frac{16}{5} = n$	$\frac{22}{5} = n$	$\frac{19}{5} = n$
16.	$\frac{9}{2} = n$	$\frac{14}{3} = n$	$\frac{24}{5} = n$
17.	$\frac{27}{4} = n$	$\frac{31}{6} = n$	$\frac{37}{8} = n$

Be your own teacher

You have not yet learned how to solve the following problems. But if you think carefully of what you have discovered about fractions and fractional numbers, you should be able to solve some of these problems. See how many you can solve. You may wish to use the drawings on pages 114–22 to help you.

1. Jane had a piece of ribbon $2\frac{3}{4}$ yards long. She cut off two pieces, each $\frac{1}{4}$ yard long. How much ribbon did she have left?

2. Robbie has two rocks. One rock weighs $\frac{3}{8}$ pound and the other rock weighs $\frac{7}{8}$ pound. Together, how much do the rocks weigh?

3. Nancy bought $3\frac{1}{2}$ yards of material. How much less than 4 yards s this?

4. Jerry weighs $74\frac{3}{4}$ pounds. How much must he gain to weigh 75 pounds?

5. George rides his bicycle $1\frac{1}{4}$ miles to school each morning. To make a round trip, he must ride $\underline{\ ?\ }$ miles.

6. Amy has a piece of material $8\frac{1}{2}$ nches wide. How much must she cut off to have a piece $7\frac{3}{4}$ inches wide?

7. Don has two pet ducks. One duck weighs $3\frac{1}{2}$ pounds and the other weighs $2\frac{1}{4}$ pounds. Together, how much do the two ducks weigh?

8. How many days will 2 pounds of dog food last if $\frac{1}{4}$ pound is used each day?

9. Judy has 8 sticks of butter. Each stick weighs $\frac{1}{4}$ pound. How many pounds of butter does Judy have?

10. Each day Charley's dog eats $\frac{1}{2}$ pound of dog food. In one week his dog will eat $\underline{\ ?\ }$ pounds of dog food.

11. Mary is making cookies. The recipe calls for $\frac{3}{4}$ cup of sugar. How much sugar should Mary use if she is doubling the recipe?

12. Sally is $55\frac{3}{4}$ inches tall. How much less than 5 feet tall is she?

13. Henry weighs 74 pounds with his shoes on. If his shoes weigh $1\frac{3}{8}$ pounds, how much does Henry weigh without his shoes?

14. Dennis has a sheet of paper $6\frac{1}{2}$ inches wide. If he separates it into two columns of equal width, how wide will each column be?

15. Last weekend, Chris and his father went fishing. Chris caught a fish that weighed $3\frac{5}{8}$ pounds. His father caught a fish that weighed $2\frac{1}{2}$ pounds. Chris's fish weighed $\underline{\ ?\ }$ pounds more than his father's. Together, the two fish weighed $\underline{\ ?\ }$ pounds.

Inquiry into geometry

Paperfolding

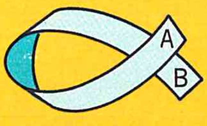
At one time you probably learned how to fold a piece of paper to make a hat, a sailboat, an airplane, or a drinking cup. But do you know how to fold paper to make a model of a pentagon or a model of a hexagon?

A pentagon

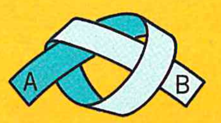
Cut a rectangular-shaped strip of paper about $1\frac{1}{2}$ inches wide and 15 inches long. Mark both sides of one end A and both sides of the other end B.

▶ Form a loop with the strip by bringing end A over across end B as shown in figure I below. Then bring end A under end B and through the loop as shown in figure II.

I

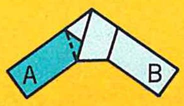


II

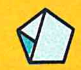


▶ Pull ends A and B outward until your model looks like figure III below. Press the model tightly to crease the edges. Then trim off the A and B ends of the strip so your model looks like figure IV. The edges of the model represent a pentagon.

III



IV

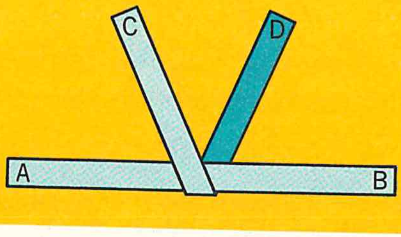


A hexagon

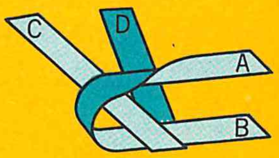
Cut two rectangular-shaped strips of paper about $1\frac{1}{2}$ inches wide and 2 feet long. Mark the ends of one strip A and B. Mark the ends of the other strip C and D.

▶ Loop end C under and then over across strip AB as shown in figure I below. Then loop end A over on top of ends C and D as shown in figure II.

I



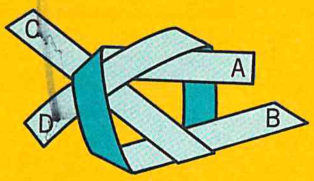
II



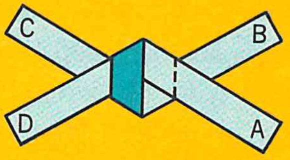
▶ Bring end D over and across ends A and C and through the loop made by strip AB, as shown in figure III.

Bring end A over across end B. Pull ends A and B together to the right and ends C and D together to the left and crease the edges as shown in figure IV. Trim off ends A, B, C, and D to obtain your model of a hexagon.

III



IV



For further inquiry

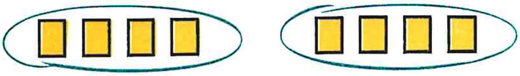
Boats, birds, helmets, and tents, as well as geometric figures, can be made by folding paper. The art of paperfolding is called origami. See if you can find a book in the library about origami.

Fractional parts of sets

1. Suppose you have a set of 6 members and separate it into 2 subsets with the same number of members in each subset. To find how many members there are in each of these subsets, do you add, subtract, multiply, or divide?

2. How many cards are in the whole set shown below? The set of cards has been separated into 2 subsets with $\underline{?}$ cards in each subset.

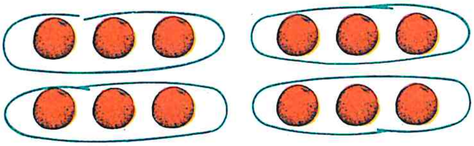
$$8 \div 2 = \underline{?}$$



Does each subset contain $\frac{1}{2}$ of the set of cards? Then the following number sentence is also true.

$$\tfrac{1}{2} \text{ of } 8 = 4$$

3. How many oranges are there in this entire set of oranges?

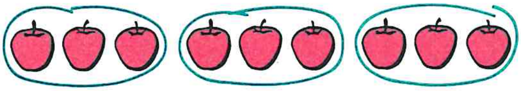


- The set of oranges has been separated into $\underline{?}$ subsets with the same number of oranges in each. How many oranges are in each subset?

$$12 \div 4 = \underline{?}$$

- Does each subset contain $\frac{1}{4}$ of the set of oranges? Then is the following number sentence true?

$$\tfrac{1}{4} \text{ of } 12 = 3$$



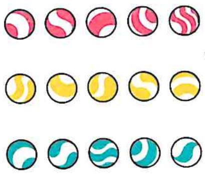
4. How many apples are there in the whole set shown above?

- This set of apples has been separated into $\underline{?}$ subsets with $\underline{?}$ apples in each.

$$9 \div 3 = \underline{?}$$

- How many apples are there in $\frac{1}{3}$ of the set of apples shown above?

$$\tfrac{1}{3} \text{ of } 9 = \underline{?}$$



5. How many marbles are there in the set shown at the right? Use this picture to help you solve the following number sentences.

$15 \div 5 = □$ $\frac{1}{5}$ of 15 = □
$15 \div 3 = △$ $\frac{1}{3}$ of 15 = △

— Solve these pairs of sentences.

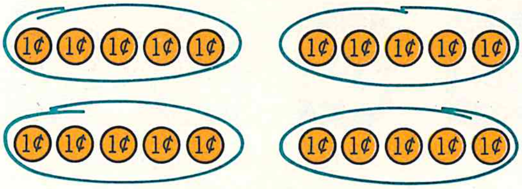
6. $24 \div 4 = n$ $\frac{1}{4}$ of $24 = n$
7. $18 \div 6 = □$ $\frac{1}{6}$ of 18 = □
8. $35 \div 5 = △$ $\frac{1}{5}$ of 35 = △

9. Explain how you would find what $\frac{1}{8}$ of 40 is. $\frac{1}{8}$ of $40 = \underline{?}$

— Solve these open sentences.

a b
10. $\frac{1}{2}$ of 16 = △ $\frac{1}{3}$ of 27 = □
11. $\frac{1}{8}$ of $32 = n$ $\frac{1}{6}$ of 36 = △
12. $\frac{1}{9}$ of 45 = □ $\frac{1}{4}$ of $16 = n$

13. How many pennies are in the entire set shown below?



This set of pennies has been separated into $\underline{?}$ subsets with $\underline{?}$ pennies in each subset.

- What is the number of pennies in $\frac{1}{4}$ of 20 pennies? $\frac{1}{4}$ of $20 = \underline{?}$
- What is the number of pennies in $\frac{2}{4}$ of 20 pennies? $\frac{2}{4}$ of $20 = \underline{?}$
  Is $\frac{2}{4}$ of 20 pennies 2 times as many as $\frac{1}{4}$ of 20 pennies?
- What is the number of pennies in $\frac{3}{4}$ of 20 pennies? $\frac{3}{4}$ of $20 = \underline{?}$
  Is $\frac{3}{4}$ of 20 pennies 3 times as many as $\frac{1}{4}$ of 20 pennies?

14. Steve has a collection of 30 marbles. One sixth of the set of marbles is red. How many marbles are red? $\frac{1}{6}$ of $30 = \underline{?}$

15. Two sixths of Steve's set of 30 marbles is blue. How many marbles are blue?

Since $\frac{1}{6}$ of 30 is 5, then $\frac{2}{6}$ of 30 is 2×5, or $\underline{?}$. $\frac{2}{6}$ of $30 = \underline{?}$

16. Three sixths of the set of 30 marbles that Steve has is white. How many marbles are white?

Since $\frac{1}{6}$ of 30 is 5, then $\frac{3}{6}$ of 30 is $\underline{?} \times 5$, or $\underline{?}$. $\frac{3}{6}$ of $30 = \underline{?}$

17. Is this number sentence true? $\frac{4}{6}$ of $30 = 4 \times (\frac{1}{6}$ of $30)$ Explain.

— Solve these sentences.

	a	b
18.	$\frac{1}{3}$ of $15 = \square$	$\frac{2}{3}$ of $15 = \triangle$
19.	$\frac{1}{6}$ of $24 = n$	$\frac{5}{6}$ of $24 = \square$
20.	$\frac{1}{8}$ of $48 = \triangle$ 6	$\frac{3}{8}$ of $48 = n$
21.	$\frac{3}{4}$ of $16 = \square$	$\frac{5}{8}$ of $56 = \triangle$
22.	$\frac{2}{3}$ of $9 = n$	$\frac{1}{7}$ of $49 = \square$
23.	$\frac{3}{5}$ of $40 = \triangle$	$\frac{1}{2}$ of $10 = n$
24.	$\frac{5}{6}$ of $36 = \square$	$\frac{3}{8}$ of $16 = \triangle$
25.	$\frac{4}{5}$ of $35 = n$	$\frac{1}{4}$ of $20 = \square$
26.	$\frac{3}{4}$ of $20 = \triangle$	$\frac{4}{7}$ of $21 = n$
27.	$\frac{5}{9}$ of $36 = \square$	$\frac{7}{8}$ of $64 = \triangle$

28. Mary's mother baked 24 cupcakes. After school, Mary and her friends ate $\frac{2}{3}$ of the cupcakes. How many cupcakes did they eat?

56

29. Edward has 48 baseball trading cards. Five eighths of the set of cards show players of the National League. How many of Edward's cards show players of the National League?

3×9 273

30. Ruth baked 36 cupcakes. She put chocolate icing on $\frac{3}{4}$ of them and vanilla icing on the rest. How many cupcakes had vanilla icing?

Fractional parts of amounts of money

1. Joe and his three brothers are saving pennies to buy some equipment for a camping trip. If each of the jars shown in the picture contains the same number of pennies, what fractional number tells what part of the set of pennies is contained in each jar? What else do you need to know before you can tell how many pennies there are in each jar?

- Suppose the four jars contain 160 pennies. Then each jar contains $\frac{1}{4}$ of 160, or ?, pennies. $\frac{1}{4}$ of 160 = ?
- Suppose the four jars contain 248 pennies. Which of the following shows the amount of money that each jar contains: $9.92; $1.24; or $.62? $\frac{1}{4}$ of 248 = ?

— Solve each of the following sentences.

	a	b	c	d
2.	$\frac{1}{3}$ of 18¢ = n	$\frac{2}{3}$ of 18¢ = n	$\frac{1}{8}$ of 16¢ = n	$\frac{5}{8}$ of 16¢ = n
3.	$\frac{1}{5}$ of $.35 = n	$\frac{4}{5}$ of $.35 = n	$\frac{1}{4}$ of $2.00 = n	$\frac{3}{4}$ of $2.00 = n
4.	$\frac{1}{5}$ of 80¢ = n	$\frac{2}{5}$ of 80¢ = n	$\frac{1}{10}$ of 70¢ = n	$\frac{7}{10}$ of 70¢ = n
5.	$\frac{1}{8}$ of $1.60 = n	$\frac{5}{8}$ of $1.60 = n	$\frac{1}{3}$ of $2.40 = n	$\frac{2}{3}$ of $2.40 = n
6.	$\frac{3}{8}$ of 40¢ = n	$\frac{3}{4}$ of 64¢ = n	$\frac{5}{6}$ of 48¢ = n	$\frac{5}{8}$ of 40¢ = n
7.	$\frac{2}{3}$ of $.75 = n	$\frac{5}{8}$ of $.56 = n	$\frac{5}{12}$ of $.36 = n	$\frac{5}{6}$ of $.90 = n
8.	$\frac{2}{5}$ of $1.35 = n	$\frac{2}{3}$ of $2.46 = n	$\frac{2}{7}$ of $1.54 = n	$\frac{9}{10}$ of $1.20 = n

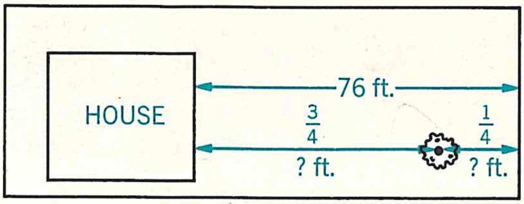
9. How much will $\frac{3}{4}$ pound of candy cost if 1 pound of candy costs 36¢?

16

10. Helen wishes to buy a new blouse. The cost of the blouse is $2.94. She already has saved $\frac{2}{3}$ of this amount. How much money has Helen saved for the blouse?

11. Fred's father has agreed to pay $\frac{3}{4}$ of the cost of a model boat. Fred will pay the rest. The model boat costs $3.28. How much will Fred have to pay? How much will his father have to pay?

Fractional parts of measures



1. The distance from Bill's house to the fence at the end of his back yard is 76 feet. At a point $\frac{3}{4}$ of the distance from the house to the fence, Bill wishes to plant a tree.

- How many feet from the fence will the tree be located?

$\frac{1}{4}$ of 76 = ?

- How many feet from the house will the tree be located?

$\frac{3}{4}$ of 76 = ?

2. Mary has a hair ribbon that is 1 foot long. Joan has a hair ribbon $\frac{2}{3}$ as long. How many inches make 1 foot? What is $\frac{2}{3}$ of 12? Then how many inches long is Joan's ribbon?

3. One contractor says he can build a house in 32 days. Another contractor says he can build the same house in $\frac{7}{8}$ of that time. How many days will it take the second contractor to build the house? $\frac{7}{8}$ of 32 = ?

4. Joan's cooky recipe calls for $\frac{3}{8}$ cup of butter. If 16 tablespoons make 1 cup, how many tablespoons of butter should Joan use?

5. Mr. Romano's boat is 24 feet long. Mr. Jayson's is $\frac{3}{4}$ as long. How long is Mr. Jayson's boat?

6. Margaret weighs $\frac{7}{8}$ as much as her brother. If Margaret's brother weighs 96 pounds, how much does Margaret weigh?

7. Betty can walk to school in 52 minutes. It takes Bob $\frac{3}{4}$ as long to walk to school. How long does it take Bob to walk to school?

— Do **a**, **b**, and **c** in Ex. 8. Then do **a**, **b**, and **c** in Ex. 9, and so on.

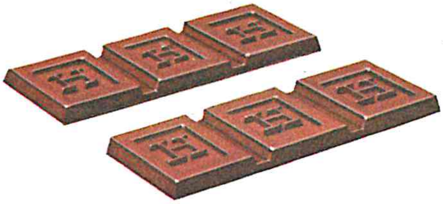
	a	b	c
8.	1 ft. makes ? in.	$\frac{1}{4}$ ft. makes ? in.	$\frac{3}{4}$ ft. makes ? in.
9.	1 yd. makes ? in.	$\frac{1}{4}$ yd. makes ? in.	$\frac{3}{4}$ yd. makes ? in.
10.	1 hr. makes ? min.	$\frac{1}{3}$ hr. makes ? min.	$\frac{2}{3}$ hr. makes ? min.
11.	1 gal. makes ? qt.	$\frac{1}{4}$ gal. makes ? qt.	$\frac{3}{4}$ gal. makes ? qt.
12.	1 lb. makes ? oz.	$\frac{1}{8}$ lb. makes ? oz.	$\frac{3}{8}$ lb. makes ? oz.
13.	1 day makes ? hr.	$\frac{1}{6}$ day makes ? hr.	$\frac{5}{6}$ day makes ? hr.
14.	1 yd. makes ? ft.	$\frac{1}{3}$ yd. makes ? ft.	$\frac{2}{3}$ yd. makes ? ft.

Fractions as names for quotients

1. Amy had 2 candy bars to share equally with Jane and Mary.

- When you separate a set into subsets of the same size, do you add, subtract, multiply, or divide to find the number in each subset?
- To find how much candy each girl should receive, do we need to solve this number sentence?

$$2 \div 3 = n$$

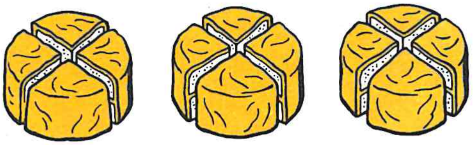


2. Amy separated each candy bar into 3 smaller pieces of the same size and shape.

- Each of these smaller pieces is $\underline{?}$ third of a whole candy bar.
- Each girl took one of the smaller pieces of the first bar and one of the smaller pieces of the second bar. Each girl took $\underline{?}$ thirds of a whole candy bar.
- Is this number sentence true?

$$2 \div 3 = \frac{2}{3}$$

A numeral such as $\frac{2}{3}$ shows the division of 2 by 3. We call the number $\frac{2}{3}$ the quotient of the numerator 2 divided by the denominator 3.



3. Four boys wished to share equally 3 cakes. One boy cut the cakes into fourths as shown above.

- Each boy took $\frac{1}{4}$ of the first cake, $\frac{1}{4}$ of the second cake, and $\frac{1}{4}$ of the third cake, or $\underline{?}$ fourths in all.
- Is this number sentence true?

$$3 \div 4 = \frac{3}{4}$$

4. Make a drawing to show how 4 candy bars would be shared equally by 5 boys.

$$4 \div 5 = \underline{?}$$

5. Draw pictures to show how 2 small pies would be shared equally by 3 boys; by 4 boys.

$$2 \div 3 = \underline{?} \qquad 2 \div 4 = \underline{?}$$

— Solve each sentence.

	a	b
6.	$3 \div 5 = n$	$1 \div 6 = n$
7.	$4 \div 7 = n$	$2 \div 5 = n$
8.	$1 \div 4 = n$	$3 \div 8 = n$

— Copy and complete each of the following sentences.

9. $\frac{2}{3}$ is another name for $2 \div \underline{?}$.

10. $\frac{3}{4}$ is the quotient of $3 \div \underline{?}$.

11. $\frac{2}{5}$ is another name for $\underline{?} \div 5$.

12. $\frac{4}{7}$ is the quotient of $\underline{?} \div 7$.

Using a mixed form to name a quotient

1. You have discovered that $\frac{2}{3}$ is ne way to express the quotient of $\underline{\ ?\ } \div 3$. In the same way, $\frac{17}{3}$ is ne way to express the quotient of $7 \div \underline{\ ?\ }$.

— Copy and complete each of these umber sentences.

a b

2. $\frac{5}{9} = 5 \div \underline{\ ?\ }$ $\frac{7}{3} = 7 \div \underline{\ ?\ }$

3. $\frac{6}{5} = \underline{\ ?\ } \div 5$ $\frac{14}{3} = \underline{\ ?\ } \div 3$

4. $\frac{25}{8} = 25 \div \underline{\ ?\ }$ $\frac{32}{6} = \underline{\ ?\ } \div 6$

5. $\frac{54}{7} = \underline{\ ?\ } \div 7$ $\frac{27}{4} = 27 \div \underline{\ ?\ }$

— Copy and replace n with a fraction to make a true sentence.

6. $15 \div 4 = n$ $32 \div 7 = n$

7. $28 \div 5 = n$ $56 \div 9 = n$

8. $51 \div 8 = n$ $25 \div 4 = n$

9. $43 \div 6 = n$ $71 \div 8 = n$

0. Can $\frac{17}{4}$ be expressed in mixed rm? Which of the following is nother name for $\frac{17}{4}$?

$1\frac{1}{4}$ $2\frac{1}{4}$ $3\frac{1}{4}$ $4\frac{1}{4}$

1. Use a mixed form to name $\frac{8}{3}$. Is one name for $8 \div 3$? Then is $2\frac{2}{3}$ so a name for $8 \div 3$? Explain.

2. In the set of numerals shown bew, two of the members are names r the quotient of $18 \div 5$. Which o are they?

$\{1\frac{3}{5}, \frac{18}{5}, \frac{5}{18}, 3\frac{3}{5}\}$

— Beside each number named in the first column, a set of numerals is shown. From each set, select two members that name the number shown in the first column.

13. $25 \div 6$ $\{\frac{25}{6}, \frac{6}{25}, 4\frac{1}{6}, 6\frac{1}{4}\}$

14. $14 \div 3$ $\{\frac{3}{14}, \frac{14}{3}, 2\frac{2}{3}, 4\frac{2}{3}\}$

15. $32 \div 5$ 6 $\{6\frac{2}{5}, 5\frac{5}{6}, \frac{32}{5}, \frac{5}{32}\}$

16. $45 \div 8$ $\{\frac{8}{45}, 5\frac{5}{8}, 8\frac{5}{8}, \frac{45}{8}\}$

— Name the quotient for each of the following, first using a fraction and then using a mixed form.

	a	b	c
17.	$3\overline{)5}$	$5\overline{)8}$	$3\overline{)4}$
18.	$2\overline{)7}$	$4\overline{)9}$	$7\overline{)15}$
19.	$3\overline{)22}$	$8\overline{)45}$	$6\overline{)35}$
20.	$7\overline{)52}$	$9\overline{)25}$	$4\overline{)37}$
21.	$9\overline{)41}$	$5\overline{)26}$	$8\overline{)13}$

22. Seventeen small pies are to be shared equally by 10 boys.

Bill said, "Each boy will receive $\frac{17}{10}$ pies." Harry said, "Each boy will receive $1\frac{7}{10}$ pies." Who is correct, Bill or Harry? Explain.

23. Twelve cupcakes are to be shared equally by 5 boys. How many cupcakes will each boy receive? Express your answer two ways: as a fraction and in mixed form.

PROBLEM TEST 3

1. Joe wants to saw a board into 2 pieces of equal length. If the board is 3 feet 2 inches long, how long should each piece be?

2. Six boys plan to share equally the cost of a tent. The tent costs $35.70. How much money should each boy pay?

3. Jean bought a pen for 75¢ and a book for $1.35. How much change did she receive from a 5-dollar bill?

4. Ben and his brother together saved $.45 a week for 12 weeks. If they share the money equally, how much money will each boy have?

5. Marie bought 6 yards of material at 98¢ a yard and a spool of thread for 27¢. How much money did she spend all together?

6. Mr. Thompson is going to make a 2,200-mile trip. If he drives 453 miles each day for 4 days, how far will he have to drive on the fifth day in order to complete his trip?

7. Ground beef sells for 76¢ a pound. How much will $\frac{3}{4}$ pound of ground beef cost?

8. Carl can jump 6 feet. His sister can jump $\frac{2}{3}$ as far. How far can his sister jump? 2

9. Three eighths of the set of pupils in a fifth-grade class are girls. There are 32 pupils in the class. How many are girls?

10. If 5 boys share equally 9 cupcakes, how many will each boy have?

Write your score on your Problem Test Record.

Mathamusements

One Saturday at 3:00, Mr. Bunch told his son Don that he would have to finish his mathematics homework before dinner. His homework was to solve 24 problems. Don thought he would have plenty of time, so he went out to play baseball with his friends. After $1\frac{1}{4}$ hours, he came home and had a snack. It took him $\frac{1}{4}$ hour to eat. What time was it then?

Don then did $\frac{1}{2}$ of the problems in $\frac{3}{4}$ hour. During the next hour he did only $\frac{1}{3}$ of the remaining problems because he was playing with his dog. How late was it then? How many examples did Don still have to do?

After Don spent $\frac{1}{6}$ hour walking his dog, he began to finish the problems. He did $\frac{1}{4}$ of the remaining problems, taking 10 minutes to do each one, and the rest in 5 minutes each. Then he went to eat dinner. What time did Don start dinner?

T'S CHECK–UP TIME

– Write numerals for the whole umbers that make these number entences true.

	a	b
1.	$\frac{3}{3} = n$	$\frac{25}{5} = n$
2.	$\frac{16}{8} = n$	$3 \div n = \frac{3}{10}$
3.	$\frac{8}{5} = \frac{n}{15}$	$\frac{20}{n} = \frac{5}{12}$
4.	$\frac{15}{7} = 2\frac{n}{7}$	$\frac{n}{8} = 4\frac{1}{8}$

– Copy each sentence and replace ne ● with =, >, or < to make the entence true.

5.	$\frac{1}{4}$ ● $\frac{3}{4}$	$\frac{2}{3}$ ● $\frac{2}{6}$
6.	$\frac{2}{5}$ ● $\frac{4}{10}$	$\frac{3}{4}$ ● $\frac{7}{8}$
7.	$1\frac{3}{5}$ ● $\frac{9}{5}$	$\frac{10}{3}$ ● $3\frac{1}{3}$
8.	$24 \div 7$ ● $\frac{23}{7}$	$2\frac{5}{8}$ ● $20 \div 8$

- Solve each of the following num-er sentences.

.	$4 \div 5 = n$	$7 \div 8 = n$
.	$\frac{4}{5}$ of $30 = n$	$\frac{2}{3}$ of $27 = n$
.	$\frac{5}{8}$ of $48 = n$	$\frac{5}{12}$ of $36 = n$

- Copy and complete the following ntences.

. $\frac{1}{3}$ of \$3.63 = ?

. $\frac{2}{5}$ of \$4.45 = ?

— Write numerals for the next two members of each set.

14. $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \dots\}$

15. $\{\frac{1}{6}, \frac{3}{6}, \frac{5}{6}, \frac{7}{6}, \frac{9}{6}, \dots\}$

16. $\{\frac{1}{8}, \frac{4}{8}, \frac{7}{8}, 1\frac{2}{8}, 1\frac{5}{8}, \dots\}$

— Copy and complete the following sentences.

17. $\frac{5}{8}$ lb. makes ? oz.

18. $\frac{5}{9}$ yd. makes ? in.

19. Three boys plan to share equally 5 doughnuts. How many doughnuts will each boy receive? Name your answer in two ways.

20. Janet and Marie plan to share the cost of a 12-dollar record album. Janet will pay $\frac{2}{3}$ of the cost and Marie will pay the rest. How much money should Janet pay? How much should Marie pay?

If you have time, try these —

— You know that $1 + \frac{5}{6}$ equals $1\frac{5}{6}$, or $\frac{11}{6}$. Since $\frac{6}{6}$ equals 1, then $\frac{6}{6} + \frac{5}{6}$ equals $1\frac{5}{6}$, or $\frac{11}{6}$. Write two numerals to complete each of the following number sentences.

	a	b
21.	$\frac{4}{4} + \frac{3}{4} =$?	$\frac{15}{3} + \frac{2}{3} =$?
22.	$\frac{30}{5} + \frac{4}{5} =$?	$\frac{18}{6} + \frac{1}{6} =$?

Dividing by tens

1. The set shown below is the set of multiples of what number?

$$\{0, 10, 20, 30, \cdots\}$$

Name the first ten members of this set.

2. Name the first ten multiples of 20; of 30; 40; 50; 60; 70; 80; 90.

3. Solve this sentence.

$$6 \times 10 = \triangle$$

Now use your answer to solve this sentence. $60 \div 10 = \square$

4. Suppose you wish to solve this sentence. $120 \div 30 = n$ Think: 12 tens ÷ 3 tens is 4, so the solution set for the sentence is $\{4\}$.

— Solve the number sentence in column **a**. Then use your answer to solve the sentence in column **b**.

	a	b
5.	$8 \times 10 = \triangle$	$80 \div 10 = \square$
6.	$6 \times 20 = \triangle$	$120 \div 20 = \square$
7.	$\square \times 60 = 180$	$180 \div 60 = \square$
8.	$\square \times 40 = 200$	$200 \div 40 = \square$

— Find the quotient for each.

	a	b	c
9.	$30\overline{)60}$	$50\overline{)150}$	$20\overline{)80}$
10.	$70\overline{)210}$	$10\overline{)90}$	$40\overline{)160}$
11.	$60\overline{)240}$	$90\overline{)540}$	$80\overline{)560}$
12.	$40\overline{)320}$	$70\overline{)420}$	$30\overline{)240}$

13. Find the greatest whole number that makes this sentence true. $n \times 40 \leq 244$ Think: What is the greatest number that makes this sentence true? $n \times 4$ tens ≤ 24 tens So the greatest whole number that n stands for is ?.

— Find the greatest whole number that makes the sentence true.

	a	b
14.	$n \times 10 \leq 45$	$n \times 20 \leq 104$
15.	$n \times 60 \leq 428$	$n \times 30 \leq 100$

16. To divide 192 by 30, think: 19 tens ÷ 3 tens is about ?. So the greatest whole number of 30's in 192 is ?. Where is this number shown in the example?

$$\begin{array}{r} 6 \text{ r } ?? \\ 30\overline{)192} \\ 6 \times 30 \rightarrow \underline{180} \\ ?? \end{array}$$

What is the remainder when you subtract 6 thirties from 192? Copy and complete the example.

— Copy and find the answers.

	a	b	c
17.	$10\overline{)45}$	$20\overline{)104}$	$60\overline{)428}$
18.	$30\overline{)100}$	$40\overline{)214}$	$80\overline{)575}$

Dividing greater numbers by tens

1. How can you use sentence **a** to help you find the greatest multiple of 10 that makes sentence **b** true?

a. $n \times 2$ tens ≤ 6 hundreds **b.** $n \times 20 \leq 640$

Does 3 tens, or 30, make sentence **a** true? Does 30 also make sentence **b** true? Try replacing n with 40 in both sentences to see if 30 s the greatest multiple of 10 that makes the sentences true.

2. Suppose you wish to find the greatest multiple of 10 that makes this sentence true. $n \times 30 \leq 1{,}965$ Think: $n \times 3$ tens ≤ 19 hundreds. Since 6 tens makes this sentence true, then 60 must make the first sentence true. Try 70 to see if 60 is the greatest multiple of 10 that makes the first sentence true.

– For each of the following sentences, find the greatest multiple of 10 that makes the sentence true.

	a	b	c	d
3.	$n \times 10 \leq 310$	$n \times 40 \leq 2{,}800$	$n \times 70 \leq 2{,}310$	$n \times 20 \leq 1{,}700$
4.	$n \times 50 \leq 1{,}200$	$n \times 30 \leq 1{,}860$	$n \times 80 \leq 1{,}520$	$n \times 60 \leq 3{,}870$

5. Is the quotient for $643 \div 20$ as much as 10? s much as 100? Is it between 10 and 100?

- To find if the quotient is 20-some, 30-some, r 40-some, think: 6 hundreds $\div$ 2 tens is 3 tens, r 30; so $643 \div 20$ is _?_-some. Write 30 above 43 with 3 in the tens place.
- What is the remainder when you subtract 0 twenties from 643? Then how many more 0's are there in 643?
- What is the remainder? Copy and complete he example.

$$\begin{array}{rr} & \boxed{?}\boxed{?}\ \text{r}\ \boxed{?} \\ & 2 \\ & 30 \\ & 20\overline{)643} \\ 30 \times 20 \longrightarrow & \underline{600} \\ & 43 \\ 2 \times 20 \longrightarrow & \underline{40} \\ & \boxed{?} \end{array}$$

- Use this number sentence to check your answer. $643 = (32 \times 20) + 3$

- Copy, find the answers, and check.

	a	b	c	d	e	f
6.	$10\overline{)310}$	$40\overline{)2{,}840}$	$70\overline{)2{,}310}$	$50\overline{)1{,}200}$	$30\overline{)1{,}860}$	$90\overline{)5{,}545}$
7.	$20\overline{)705}$	$60\overline{)843}$	$80\overline{)2{,}000}$	$90\overline{)1{,}456}$	$40\overline{)2{,}253}$	$70\overline{)4{,}765}$
8.	$70\overline{)1{,}511}$	$90\overline{)7{,}430}$	$30\overline{)972}$	$60\overline{)5{,}738}$	$20\overline{)1{,}915}$	$50\overline{)2{,}748}$

Solving inequalities

1. Look at sentences **a** and **b**.

a. $n \times 40 \leq 138$ **b.** $n \times 42 \leq 138$

- To find the greatest whole number that makes sentence **a** true, think of 138 as about 13 tens. Then find the greatest whole number that makes this sentence true. $n \times 4$ tens ≤ 13 tens Is this number also the greatest whole number that makes sentence **a** true?

- To find the greatest whole number that makes sentence **b** true, think of 42 as about 4 tens and 138 as about 13 tens. Then find the greatest whole number that makes this sentence true. $n \times 4$ tens ≤ 13 tens Is this number the greatest whole number that makes sentence **b** true?

- How do sentences **a** and **b** differ? How are they alike? To find the greatest whole number that makes the sentences true, can you think of both of them this way?

$n \times 4$ tens ≤ 13 tens

— For each of the following, find the greatest whole number that makes the sentence true.

	a	b
2.	$\triangle \times 30 \leq 172$	$\square \times 32 \leq 172$
3.	$\triangle \times 60 \leq 429$	$\square \times 61 \leq 429$
4.	$\triangle \times 20 \leq 95$	$\square \times 23 \leq 95$
5.	$\triangle \times 80 \leq 620$	$\square \times 81 \leq 620$
6.	$\triangle \times 50 \leq 246$	$\square \times 54 \leq 246$
7.	$\triangle \times 40 \leq 138$	$\square \times 43 \leq 138$

8. Explain how rounding the given factor to the nearest 10 in each sentence in column **b**, Ex. 2–7, can help you find the greatest whole number that makes the sentence true.

— Find the greatest whole number that makes the sentence true.

	a	b
9.	$n \times 61 \leq 320$	$n \times 41 \leq 152$
10.	$n \times 64 \leq 458$	$n \times 52 \leq 160$
11.	$n \times 83 \leq 523$	$n \times 94 \leq 248$

12. Round the given factor in this number sentence to the nearest 10. $\square \times 54 \leq 320$ Now find the greatest whole number that makes this sentence true. $\triangle \times 50 \leq 320$ Does the same whole number also make the first sentence true? What is the next smaller whole number? Does this number make the first sentence true?

Even though rounding the given factor to the nearest 10 does not always give you the number you want for the original sentence, it does help you find that number quickly. Explain how it helps.

— Find the greatest whole number that makes the sentence true.

13.	$n \times 23 \leq 139$	$n \times 23 \leq 82$
14.	$n \times 74 \leq 565$	$n \times 41 \leq 122$
15.	$n \times 32 \leq 235$	$n \times 94 \leq 186$
16.	$n \times 53 \leq 440$	$n \times 81 \leq 240$

Dividing by tens and ones

1. To find the quotient for 172 ÷ 32, find the greatest whole number of 32's in 172. Think: 17 tens ÷ 3 tens is about 5. Where is this number shown in the example below? What is the remainder when you subtract 5 thirty-twos from 172?

$$\begin{array}{r} 5\text{ r}12 \\ 32\overline{)172} \\ 5 \times 32 \rightarrow \underline{160} \\ 12 \end{array}$$

Use this number sentence to check your answer. $172 = (5 \times 32) + 12$

2. What is 61 rounded to the nearest 10? Are there as many as 6 sixties in 429? as many as 7? as many as 8?

$$\begin{array}{r} ?\text{ r }? \\ 61\overline{)429} \\ n \times 61 \rightarrow \underline{???} \\ ? \end{array}$$

Copy and complete the example.

– Copy and divide. Write a true entence to check each answer.

	a	b	c
3.	$61\overline{)320}$	$41\overline{)152}$	$32\overline{)160}$
4.	$64\overline{)458}$	$72\overline{)288}$	$52\overline{)160}$
5.	$83\overline{)523}$	$91\overline{)275}$	$44\overline{)264}$

6. Suppose you wish to find what 132 ÷ 24 is. What is 24 rounded to the nearest 10? Think: 13 tens ÷ 2 tens is about 6.

What is the product of 6 and 24? Is the following number sentence true? $6 \times 24 \leq 132$ Then is 5 the greatest whole number of 24's in 132?

In the example below, should n be replaced by 5 or by 6? Why? Copy and complete the example. It is not necessary to copy the part printed in red. Write a true number sentence to check your answer.

$$\begin{array}{r} ?\text{ r }?? \\ 24\overline{)132} \\ n \times 24 \rightarrow \underline{???} \\ ?? \end{array}$$

7. What is the greatest whole number of 60's in 246? What is the greatest whole number of 63's in 246? Then 246 ÷ 63 is $\underline{\ ?\ }$ r $\underline{\ ?\ }$.

$$63\overline{)246}$$

— Copy and divide. Write a true sentence to check each answer.

	a	b	c
8.	$23\overline{)82}$	$74\overline{)565}$	$41\overline{)122}$
9.	$32\overline{)214}$	$91\overline{)725}$	$53\overline{)440}$
10.	$23\overline{)187}$	$64\overline{)435}$	$81\overline{)648}$
11.	$73\overline{)290}$	$52\overline{)308}$	$44\overline{)346}$

Dividing greater numbers by tens and ones

1. Suppose you want to find the greatest multiple of 10 that makes this sentence true. $n \times 23 \leq 552$ Think: 23 is about 2 tens and 552 is about 5 hundreds; 2 tens × 2 tens ≤ 5 hundreds; so n might represent 2 tens, or 20. Is 20 the greatest multiple of 10 that makes this sentence true? $n \times 23 \leq 552$ Then the quotient for 23)552 is ? -some.

— Find the greatest multiple of 10 that makes the sentence in column **a** true. Then complete the matching sentence in column **b.**

	a	b
2.	$n \times 14 \leq 392$	The quotient for 14)392 is ? -some.
3.	$n \times 32 \leq 1{,}644$	The quotient for 32)1,644 is ? -some.
4.	$n \times 83 \leq 4{,}316$	The quotient for 83)4,316 is ? -some.

5. Study the example at the right.

- To find the greatest number of tens in the quotient, think: 42 is about 4 tens, and 1,525 is about 15 hundreds; 15 hundreds ÷ 4 tens is about 3 tens. The quotient is 30-some.
- What is the remainder when you subtract 30 forty-twos from 1,525?
- What is the greatest whole number of 42's in 265? What is the remainder when you subtract 6 forty-twos from 265?
- Copy and complete the example.
- Use this number sentence to check.

$(36 \times 42) + \underline{?} = 1{,}525$

$$\begin{array}{rr}
 & 36 \text{ r } ?? \\
 & 6 \\
 & 30 \\
 & 42\overline{)1{,}525} \\
30 \times 42 \rightarrow & 1{,}260 \\
 & 265 \\
6 \times 42 \longrightarrow & 252 \\
 & ??
\end{array}$$

— Copy and divide. Write a true number sentence to check.

	a	b	c	d	e	f
6.	31)2,573	24)1,776	63)4,914	41)3,584	22)1,381	74)5,123
7.	43)1,945	62)2,542	93)7,103	32)900	64)1,617	81)2,857
8.	84)7,812	44)2,552	34)2,138	12)1,148	21)1,875	53)1,537

If you need more practice, do Practice Set 40, page 336.

Estimating quotients and dividing

1. Round the given factor in sentence **a** below to the nearest 10. Is the result the same as the given factor in sentence **b**?

a. $\square \times 39 \leq 212$ **b.** $\triangle \times 40 \leq 212$

What is the greatest whole number that makes sentence **b** true? Does 5 also make sentence **a** true? Does 6? The greatest whole number that makes sentence **a** true is ?.

2. Round the given factor in sentence **a** to the nearest 10.

a. $\square \times 47 \leq 384$ **b.** $\triangle \times 50 \leq 384$

What is the greatest whole number that makes sentence **b** true? Does 7 also make sentence **a** true? Does 8? Does 9? Then the greatest whole number that makes sentence **a** true is ?.

3. The greatest whole number that makes sentence **b** true is 4.

. $\square \times 77 \leq 325$ **b.** $\triangle \times 80 \leq 325$

Does 4 also make sentence **a** true? Does 5? The greatest whole num-er that makes sentence **a** true is ?.

4. What is the greatest whole num-er that makes sentence **b** true?

$\square \times 18 \leq 115$ **b.** $\triangle \times 20 \leq 115$

oes this number also make sentence true? Does 6 also make sentence **a** ue? Does 7? Then the greatest hole number that makes sentence **a** ue is ?.

5. Explain how rounding the given factor to the nearest 10 can help you find the greatest whole number that makes this number sentence true. $n \times 27 \leq 152$

— For each of the following sentences, find the greatest whole number that makes the sentence true.

	a	b
6.	$n \times 19 \leq 110$	$n \times 98 \leq 620$
7.	$n \times 47 \leq 351$	$n \times 39 \leq 175$
8.	$n \times 28 \leq 256$	$n \times 87 \leq 276$
9.	$n \times 59 \leq 312$	$n \times 68 \leq 275$

10. To divide 137 by 27, first find the greatest whole number of 27's in 137. Where is this number shown in the example below? What is the remainder when you subtract 5 twenty-sevens from 137? Copy and complete the example.

$$\begin{array}{r} 5 \text{ r } ? \\ 27\overline{)137} \\ 5 \times 27 \rightarrow \underline{135} \\ ? \end{array}$$

— Copy and find the answers.

	a	b	c
11.	$19\overline{)110}$	$38\overline{)246}$	$98\overline{)497}$
12.	$47\overline{)351}$	$29\overline{)214}$	$57\overline{)237}$
13.	$58\overline{)184}$	$67\overline{)536}$	$49\overline{)196}$
14.	$79\overline{)316}$	$18\overline{)144}$	$69\overline{)473}$

Dividing greater numbers by tens and ones

1. To estimate the quotient for 898 ÷ 28, find the greatest multiple of 10 that makes this sentence true. $n \times 28 \leq 898$ Think: ? × 3 tens ≤ 8 hundreds; so n might represent 20. But since 898 is close to 900, and 28 is not as great as 30, n could represent 3 tens, or 30. Check to find out which one it is. $20 \times 28 =$? $30 \times 28 =$? Will the quotient be 20-some or 30-some?

2. Is 2,632 greater than 10 × 47? What is the greatest multiple of 10 that makes this number sentence true? $n \times 47 \leq 2{,}632$ Then will the quotient for 2,632 ÷ 47 be 50-some or 60-some?

— Find the greatest multiple of 10 that makes each sentence in column **a** true. Then complete the matching sentence in column **b**.

	a	b
3.	$n \times 59 \leq 1{,}357$	The quotient for 1,357 ÷ 59 is ?-some.
4.	$n \times 87 \leq 3{,}659$	The quotient for 3,659 ÷ 87 is ?-some.
5.	$n \times 38 \leq 3{,}324$	The quotient for 3,324 ÷ 38 is ?-some.

6. Study the example at the right.

- To estimate the quotient, think: 29 hundreds ÷ 4 tens is about 7 tens, or ?. So 2,964 ÷ 38 may be 70-some. Could it be as much as 80-some?
- What is the remainder when you subtract 70 thirty-eights from 2,964?
- What is the greatest whole number of 38's in 304? What is the remainder when you subtract 8 thirty-eights from 304?
- Copy and complete the example. Check by multiplying.

$$\begin{array}{r} ??\\ \hline 8\\ 70\\ 38\overline{)2{,}964}\\ 70 \times 38 \rightarrow \underline{2{,}660}\\ 304\\ 8 \times 38 \rightarrow \underline{304}\\ ?\end{array}$$

— Copy and divide. Write a number sentence to check each answer.

	a	b	c	d	e	f
7.	17)646	69)4,416	78)1,404	29)2,853	88)3,102	97)6,887
8.	77)7,095	48)1,697	28)1,260	99)8,217	38)3,283	58)4,814
9.	49)2,891	27)2,100	18)1,149	57)2,397	69)2,700	89)2,319

If you need more practice, do Practice Set 41, page 336.

iven factors with numerals that end in 5

1. Suppose you wish to find the reatest whole number that makes entence **a** true.

a. $n \times 35 \le 294$

he given factor, 35, is between what wo multiples of 10?

What is the greatest whole number hat makes sentence **b** true? that akes sentence **c** true?

. $\triangle \times 30 \le 294$ **c.** $\square \times 40 \le 294$

Then the greatest whole number hat makes sentence **a** true is either , 8, or 9. Why? Which one of hese three numbers is it?

2. Between what two multiples of 0 is the given factor in sentence **a**?

a. $n \times 85 \le 326$

What is the greatest whole number hat makes sentence **b** true? that akes sentence **c** true?

. $\triangle \times 80 \le 326$ **c.** $\square \times 90 \le 326$

Then the greatest whole number hat makes sentence **a** true is either _ or $\underline{?}$. Which is it?

3. Between what two multiples of 0 is the given factor in sentence **a**?

a. $n \times 65 \le 235$

What is the greatest whole number at makes sentence **b** true? that akes sentence **c** true?

. $\triangle \times 60 \le 235$ **c.** $\square \times 70 \le 235$

Then the greatest whole number at makes sentence **a** true is $\underline{?}$.

4. Explain how you can use sentence **b** and sentence **c** to help you find the greatest whole number that makes sentence **a** true.

a. $n \times 25 \le 165$

b. $\triangle \times 20 \le 165$ **c.** $\square \times 30 \le 165$

— For each of the following sentences, find the greatest whole number that makes the sentence true.

	a	b
5.	$n \times 35 \le 216$	$n \times 55 \le 368$
6.	$n \times 85 \le 483$	$n \times 45 \le 183$
7.	$n \times 25 \le 206$	$n \times 65 \le 475$

8. To divide 368 by 75, first find the greatest whole number of 75's in 368. Where is this number shown in the example? What is the remainder? Copy and complete the example.

```
               4 r ??
            75)368
4 × 75 →       300
               ??
```

Use this sentence to check your answer. $(4 \times 75) + \underline{?} = 368$

— Copy, divide, and check.

	a	b	c
9.	65)260	15)105	45)99
10.	25)225	75)665	95)842
11.	45)405	65)364	35)140
12.	15)142	85)543	55)400

Given factors with numerals that end in 6

1. To find the greatest whole number that makes sentence **a** true, first find the greatest whole number that makes sentence **b** true; that makes sentence **c** true.

a. $n \times 26 \le 198$

b. $\triangle \times 20 \le 198$ **c.** $\square \times 30 \le 198$

Then the greatest whole number that makes sentence **a** true is either 6, 7, 8, or 9. Why? Which number is it?

2. What is the greatest whole number that makes sentence **b** true? that makes sentence **c** true?

a. $n \times 46 \le 156$

b. $\triangle \times 40 \le 156$ **c.** $\square \times 50 \le 156$

Then what is the greatest whole number that makes sentence **a** true? Why?

3. The greatest whole number that makes sentence **b** true is ?. The greatest whole number that makes sentence **c** true is ?.

a. $n \times 76 \le 296$

b. $\triangle \times 70 \le 296$ **c.** $\square \times 80 \le 296$

Use these two numbers to help you find the greatest whole number that makes sentence **a** true.

4. Explain how you can use sentences **b** and **c** to help you find the greatest whole number that makes sentence **a** true.

a. $n \times 56 \le 285$

b. $\triangle \times 50 \le 285$ **c.** $\square \times 60 \le 285$

— For each of the following sentences, find the greatest whole number that makes the sentence true.

	a	b
5.	$n \times 36 \le 112$	$n \times 46 \le 368$
6.	$n \times 86 \le 600$	$n \times 56 \le 298$
7.	$n \times 96 \le 192$	$n \times 76 \le 378$
8.	$n \times 26 \le 198$	$n \times 66 \le 335$

9. To divide 258 by 46, first find the greatest whole number of 46's in 258. Where is this number shown in the example below? What is the remainder when you subtract 5 forty-sixes from 258? Copy and complete the example.

$$\begin{array}{r} 5 \text{ r } ?? \\ 46\overline{)258} \\ 5 \times 46 \rightarrow \underline{230} \\ ?? \end{array}$$

Use this number sentence to check your answer. $(5 \times 46) + ? = 258$

— Copy, divide, and check.

	a	b	c
10.	$26\overline{)130}$	$56\overline{)378}$	$36\overline{)144}$
11.	$76\overline{)171}$	$16\overline{)112}$	$46\overline{)434}$
12.	$46\overline{)322}$	$86\overline{)516}$	$26\overline{)234}$
13.	$76\overline{)584}$	$66\overline{)536}$	$96\overline{)847}$
14.	$16\overline{)61}$	$96\overline{)409}$	$66\overline{)220}$
15.	$56\overline{)280}$	$86\overline{)805}$	$36\overline{)312}$

Dividing greater numbers

1. Is 1,465 greater than 10×35? Then is the quotient for $1{,}465 \div 35$ greater than 10? Is the quotient greater than 20? than 30? 40? 50? We can say the quotient for $1{,}465 \div 35$ is $\underline{?}$-some.

2. Is 3,472 greater than 10×56? What is the greatest multiple of 10 that makes this number sentence true? $n \times 56 \le 3{,}472$ Then will the quotient for $3{,}472 \div 56$ be 50-some, 60-some, or 70-some?

— Find the greatest multiple of 10 that makes each sentence in column **a** true. Then complete the matching sentence in column **b**.

	a	b
3.	$n \times 75 \le 3{,}215$ 30	The quotient for $3{,}215 \div 75$ is $\underline{?}$-some.
4.	$n \times 46 \le 1{,}015$ 20	The quotient for $1{,}015 \div 46$ is $\underline{?}$-some.
5.	$n \times 65 \le 1{,}270$ 10	The quotient for $1{,}270 \div 65$ is $\underline{?}$-some.
6.	$n \times 26 \le 2{,}171$ 80	The quotient for $2{,}171 \div 26$ is $\underline{?}$-some.
7.	$n \times 85 \le 3{,}350$ 30	The quotient for $3{,}350 \div 85$ is $\underline{?}$-some.

8. To estimate the quotient for $76\overline{)4{,}762}$, think: 47 hundreds ÷ 8 tens is about 50, and 47 hundreds ÷ 7 tens is about 60. Is the quotient 50-some or 60-some?

- What is the remainder when you subtract 60 seventy-sixes from 4,762?
- How many more 76's are there in 4,762? Where is this number shown in the example?
- Copy and complete the example. Check.

$$4{,}762 = (62 \times 76) + \underline{?}$$

$$\begin{array}{rr}
 & ??\ \text{r}\ ?? \\
 & 2 \\
 & 60 \\
 & 76\overline{)4{,}762} \\
60 \times 76 \rightarrow & 4{,}560 \\
 & 202 \\
2 \times 76 \longrightarrow & 152 \\
 & ??
\end{array}$$

— Copy and divide. Write a number sentence to check each answer.

	a	b	c	d	e	f
9.	$36\overline{)864}$	$25\overline{)1{,}321}$	$46\overline{)1{,}555}$	$55\overline{)4{,}015}$	$16\overline{)569}$	$35\overline{)2{,}361}$
10.	$86\overline{)1{,}537}$	$75\overline{)3{,}380}$	$26\overline{)1{,}732}$	$65\overline{)4{,}355}$	$45\overline{)3{,}735}$	$96\overline{)7{,}296}$
11.	$56\overline{)5{,}252}$	$95\overline{)2{,}000}$	$76\overline{)1{,}150}$	$85\overline{)4{,}000}$	$66\overline{)1{,}878}$	$75\overline{)6{,}695}$

If you need more practice, do Practice Set 42, page 337.

Practice in dividing

— Solve each of the following open sentences.

	a	b	c
1.	$241 = (26 \times 9) + \triangle$	$262 = (43 \times 6) + \triangle$	$294 = (5 \times 58) + \triangle$
2.	$160 = (7 \times 22) + \triangle$	$2{,}073 = (32 \times 64) + \triangle$	$3{,}222 = (76 \times 42) + \triangle$
3.	$187 = (\square \times 7) + 5$	$333 = (\square \times 4) + 1$	$142 = (\square \times 28) + 2$
4.	$380 = (\square \times 31) + 8$	$1{,}894 = (\square \times 54) + 4$	$1{,}789 = (\square \times 81) + 7$

— Copy and divide. Write a true number sentence to check each answer.

	a	b	c	d	e	f
5.	$96\overline{)355}$	$73\overline{)335}$	$88\overline{)222}$	$75\overline{)541}$	$54\overline{)340}$	$87\overline{)635}$
6.	$71\overline{)525}$	$82\overline{)246}$	$54\overline{)330}$	$93\overline{)465}$	$65\overline{)332}$	$95\overline{)855}$
7.	$32\overline{)768}$	$41\overline{)944}$	$22\overline{)567}$	$53\overline{)645}$	$81\overline{)945}$	$74\overline{)658}$
8.	$61\overline{)2{,}748}$	$34\overline{)2{,}074}$	$59\overline{)1{,}892}$	$43\overline{)3{,}495}$	$30\overline{)2{,}880}$	$94\overline{)8{,}272}$
9.	$36\overline{)1{,}479}$	$65\overline{)2{,}289}$	$56\overline{)3{,}976}$	$84\overline{)2{,}496}$	$91\overline{)4{,}682}$	$28\overline{)1{,}782}$
10.	$25\overline{)1{,}342}$	$47\overline{)3{,}950}$	$12\overline{)348}$	$76\overline{)4{,}266}$	$38\overline{)2{,}926}$	$36\overline{)3{,}420}$

11. If you solve sentence **a** at the right, will the solution set also be the solution set for sentence **b**? Explain.

What is the solution set for sentence **a**? for sentence **b**? for this open sentence? $8 \times n = 288$

a. $288 \div 8 = n$
b. $n \times 8 = 288$

— Solve each of these open sentences.

	a	b	c	d
12.	$n \times 48 = 144$	$n \times 7 = 434$	$27 \times n = 216$	$4 \times n = 144$
13.	$n \times 81 = 729$	$65 \times n = 1{,}755$	$71 \times n = 2{,}698$	$n \times 28 = 1{,}456$

— Find the greatest whole number that makes the sentence true.

14.	$n \times 8 \leq 48$	$n \times 6 \leq 48$	$n \times 5 \leq 97$	$n \times 3 \leq 132$
15.	$n \times 32 \leq 675$	$n \times 43 \leq 1{,}590$	$n \times 62 \leq 3{,}844$	$n \times 85 \leq 2{,}725$

If you need more practice, do Practice Set 43, page 337.

Using division to solve problems

1. Henry wished to arrange 398 boxes in stacks so there would be 15 boxes in each stack. How many stacks of 15 boxes could he have?

- Henry wrote this number sentence to help him solve the problem.

398 = (□ × 15) + △

Does □ stand for the number of stacks? What does △ stand for?

- Dividing, Henry found he could have as many as 26 stacks of boxes with 15 boxes in each stack. Explain.
- He also found he would have one stack with fewer than 15 boxes in it. Why? How many boxes would be in this smaller stack?

2. Twenty-one girl scouts plan to sell 384 boxes of cookies. The scout leader wishes to give each girl the same number of boxes to sell. How many boxes of cookies should she give each girl?

- Explain how you can use this number sentence to help you solve the problem.

384 = (□ × 21) + △

- How many boxes are left over?

3. Write a number sentence to help you find how many sets of 35 marbles you could make up from 800 marbles. How many marbles would be left over?

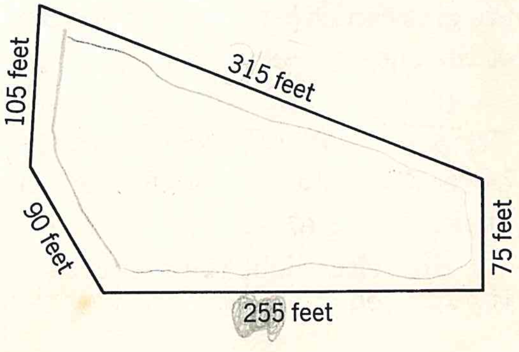
4. There are 200 members in the Southgate School cheering club who plan to travel by bus to the next basketball game. If 1 bus can carry 42 people, how many buses will the cheering club need?

5. Joe plans to arrange 192 cans of vegetables on a shelf in stacks with the same number of cans in each stack. Will he have any cans left over if he arranges them in stacks of 20? in stacks of 15? in stacks of 12? of 6?

6. Each row of seats in a large theater can accommodate 32 people. How many rows can be filled by 688 people? by 720 people? by 900 people? by 1,000 people?

Think twice!

7. Mr. Fedora is going to put up a fence around the field shown in the drawing below. Every 15 feet, he has to put in a post to hold the fence up. How many posts will he need?



PROBLEM-SOLVING HELP

Solving problems involving money

1. One Saturday, 6 boys collected empty soda bottles and received $2.30 when they took the bottles back to the store. If the boys share this money equally, how much will each boy receive?

- You can think of $2.30 as ? cents and divide as you would if the dividend were 230.

- Is the example at the right correct as far as the work is shown? Does the 38 tell the number of bottles, the number of boys, or the number of cents?

$$\begin{array}{rr}
 & 38 \text{ r } ? \\
 & \overline{8} \\
 & 30 \\
 & 6\overline{)\$2.30} \\
30 \times 6 \rightarrow & 1\,80 \\
 & \overline{50} \\
8 \times 6 \longrightarrow & 48 \\
 & \overline{?}
\end{array}$$

Copy and complete the example, writing in a dollar sign and a cents point to show that the answer stands for an amount of money.

- What is the remainder? Does the remainder tell a number of boys or a number of cents? Explain.

Write a cents sign in your answer after the numeral for the remainder.

2. Five girls plan to share equally the cost of a book of sewing instructions. If the book costs $3.84, what is each girl's share?

- Which of the following expressions shows the answer you should get after dividing?

$.75 r9¢ $.76 r4¢

- Does the remainder mean that the girls can or cannot share the cost of the book equally? Explain.

- If 4 girls pay $.76 each and 1 girl pays $.80, will they have the correct amount of money for the book?

- How else might the 5 girls share the cost of the book?

3. Is it possible for 12 boys to share equally the cost of a $10.98 football? Explain.

4. George wishes to buy a small transistor radio for $14.95. He plans to save the same amount of money each week.

How much money must George save each week in order to buy the radio in 6 weeks? in 8 weeks? in 16 weeks?

5. Mary saved the same amount of money every week for one year. She saved a total of $18.20. How many weeks are there in one year? How much money did Mary save each week?

6. Jim wants to buy a book of adventure stories. The book costs $5.98. Jim plans to save $.35 each week for the book. How many weeks will it take him to save $5.98?

- You can think of $.35 as ? cents and $5.98 as ? cents. Then divide as you would divide 598 by 35.
- Study the example at the right. Is the work correct as far as it is shown? Does 17 stand for an amount of money or a number of weeks?
- What is the remainder? Does the remainder stand for an amount of money or a number of weeks?

$$\begin{array}{rr} & 17 \text{ r } ? \\ \cline{2-2} & 7 \\ & 10 \\ & \$.35\overline{)\$5.98} \\ 10 \times 35 \rightarrow & 3\;50 \\ \cline{2-2} & 2\;48 \\ 7 \times 35 \rightarrow & 2\;45 \\ \cline{2-2} & ? \end{array}$$

- Copy and complete the example. Write a cents sign in your answer after the numeral for the remainder.
- It will take Jim ? weeks to save $5.98 if during the last week he adds ? cents to what he normally saves each week.

7. June plans to save quarters until she has saved $8.50. How many quarters will she have to save?

8. To raise money for team uniforms, the Little League fathers sold booster buttons for 75¢ each. One week they made $39.75. How many buttons did they sell that week?

9. Billy wishes to buy some paperback books for his older brother's birthday gift. The books cost 40¢ each. How many can Billy buy if he has $2.50 to spend?

10. Dues for the model-builders club are 35¢. The club treasurer has collected $5.95 in dues. If every member has paid his dues, how many members are there in the club?

11. Ann has saved $3.50 to buy dress patterns. How many patterns can she buy if they cost 48¢ each?

12. Tickets to the fifth-grade class play cost 60¢ each. How many tickets did the class sell if they collected $29.40 in ticket money?

13. The science club is going to buy notebooks for the members. The notebooks they want cost 79¢ each. How many can they buy if they have $11.84 to spend?

14. John wants to buy 8 yards of rope. He has $3.00. The rope costs 46¢ a yard. Can he buy 8 yards of rope? How many yards can he buy with the $3.00?

Keeping up in mathematics

Oral

1. If you replace the digit 6 in the numeral 860,000 with the digit 0, the resulting numeral names a number $\underline{?}$ less than 860,000.

2. Is $469 \div 52$ about 7, about 8, or about 9?

3. Is 6×29 about 120, about 180, or about 110?

4. Read each of these numerals.

5,505 1,100,100 110,010

— Use a mixed form or a numeral for a whole number to rename each number shown below.

	a	b	c	d
5.	$\frac{8}{5}$	$\frac{10}{5}$	$\frac{7}{3}$	$\frac{9}{2}$
6.	$\frac{12}{4}$	$\frac{13}{6}$	$\frac{6}{6}$	$\frac{24}{3}$
7.	$\frac{19}{6}$	$\frac{20}{4}$	$\frac{17}{8}$	$\frac{17}{4}$

8. To find the difference between \$1.25 and \$.75, you should $\underline{?}$. The difference is $\underline{?}$.

9. Which names the greater number, $\frac{1}{2}$ or $\frac{1}{8}$? $\frac{2}{6}$ or $\frac{5}{6}$? $\frac{3}{4}$ or $\frac{4}{3}$?

10. Study the set of fractions shown below for a pattern. Then name three other members of this set.

$$\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \cdots\right\}$$

11. Since 24×33 equals 792, what does $792 \div 24$ equal? Is $792 \div 25$ greater or less than 33? Explain.

12. How many yards make 24 feet?

Written

1. Find the sum.

$275 + 42 + 348 + 64 = \underline{?}$

2. Find the total value of \$2.73, \$2.08, \$.85, \$.37, and \$5.95.

3. Subtract 2,604 from 6,503.

4. What is the difference between \$32.50 and \$12.78?

5. What is the product of 875 and 790? of 790 and 875?

6. How much is 76 times \$5.82?

— Copy, find the answers, and check.

	a	b	c
7.	$7\overline{)5{,}080}$	$8\overline{)3{,}296}$	$9\overline{)5{,}683}$
8.	$45\overline{)572}$	$31\overline{)220}$	$26\overline{)1{,}892}$
9.	$63\overline{)1{,}395}$	$52\overline{)4{,}260}$	$19\overline{)287}$

10. This week the school nurse found that Debbie is 56 inches tall and weighs 90 pounds. Last year Debbie was $55\frac{1}{4}$ inches tall and weighed $85\frac{1}{2}$ pounds. Find Debbie's gain in height and weight.

— Copy and compute.

	a	b
11.	3 hr. 30 min. + 1 hr. 48 min.	5 ft. 7 in. − 2 ft. 9 in.
12.	9 gal. 3 qt. × 6	$2\overline{)6 \text{ yd. } 2 \text{ ft.}}$

A shorter way to show division

A

$$\begin{array}{r} \underline{24}\text{ r1} \\ 4 \\ 20 \\ 32\overline{)769} \\ 20 \times 32 \rightarrow \underline{640} \\ 129 \\ 4 \times 32 \rightarrow \underline{128} \\ 1 \end{array}$$

B

$$\begin{array}{r} 24\text{ r1} \\ 32\overline{)769} \\ 20 \times 32 \rightarrow \underline{640} \\ 129 \\ 4 \times 32 \rightarrow \underline{128} \\ 1 \end{array}$$

1. You know how to show the division of 769 by 32 as shown in example A above. Explain the way the division is done in A.

In B you see another way to show the division of 769 by 32.

- What is the greatest multiple of 10 that makes this sentence true? $n \times 3$ tens $\leq$ 7 hundreds Where is this number shown in the quotient for A? Does the 2 in the tens place of the quotient for B show 2 tens, or 20?
- What is the greatest whole number of 32's in 129? Where is this number shown in the quotient for A? Does the 4 in the ones place of the quotient for B show the same number?
- In A, the quotient is found by adding 20 and 4, with the sum shown on top. In B, does 24 already show 20 + 4?

2. The example at the right for 2,495 ÷ 53 s incomplete. Copy and complete it using the way shown above in example B.

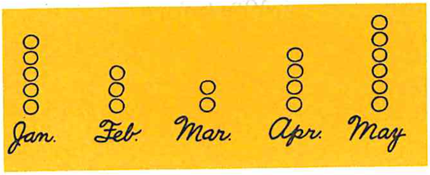
$$\begin{array}{r} 4\boxed{?} \\ 53\overline{)2,495} \\ 40 \times 53 \rightarrow \underline{2,120} \end{array}$$

– Use the shorter way to show division for each example below.

	a	b	c	d	e	f
3.	$9\overline{)552}$	$8\overline{)398}$	$3\overline{)255}$	$6\overline{)326}$	$5\overline{)383}$	$7\overline{)651}$
4.	$42\overline{)1,578}$	$60\overline{)5,050}$	$55\overline{)2,655}$	$24\overline{)1,752}$	$94\overline{)5,838}$	$86\overline{)3,526}$
5.	$27\overline{)2,472}$	$81\overline{)5,206}$	$36\overline{)2,452}$	$56\overline{)1,297}$	$47\overline{)2,773}$	$58\overline{)5,394}$
6.	$38\overline{)1,749}$	$72\overline{)2,035}$	$95\overline{)9,401}$	$18\overline{)1,746}$	$74\overline{)6,068}$	$43\overline{)3,314}$

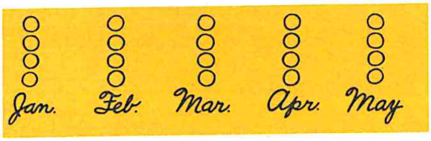
The meaning of average

1. Andy is saving dimes. He drew the following chart to show the number of dimes he saved each month for 5 months.



- How many dimes did Andy save during each month named above?
- How many dimes did Andy save all together?

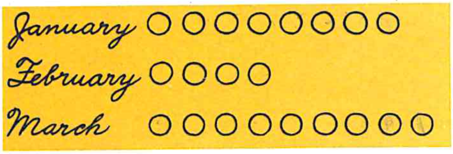
2. Andy also drew the chart shown below.



- What is the total number of dimes shown in this chart? Compare this number with the total number of dimes shown in the first chart.
- Did Andy show 5 sets of dimes in his first chart? In his second chart, he showed _?_ sets of dimes with _?_ members in each set.

When sets of 5, 3, 2, 4, and 6 are arranged in 5 sets with the same number in each, the number of members in each set is called the **average**.

3. Andy saved an average of _?_ dimes a month.



4. The chart above shows the number of dimes Alice saved each month for 3 months.

- Alice saved a total of _?_ dimes during the 3 months.
- To find the average, she must separate these dimes into _?_ sets with the same number in each set.
- Alice saved an average of _?_ dimes a month.

5. Andy (Ex. 1–3) saved an average of _?_ dimes a month. Alice (Ex. 4) saved an average of _?_ dimes a month. Suppose each continues to save the same average number of dimes a month. Who will have saved the greater number of dimes at the end of the year? Explain.

6. Harry saved dimes for 4 months. He saved 6 dimes the first month, 8 dimes the second, 3 dimes the third, and 7 dimes the fourth. He saved an average of _?_ dimes a month.

7. Who is saving the greatest average number of dimes a month, Andy, Alice, or Harry? If each continues to save the same average number of dimes a month, who will have saved the greatest number of dimes at the end of 1 year? 2 years? Explain.

Finding the average

Each week the fifth-grade class is given a practice test in mathematics. There are 10 exercises in each test. Jean and Helen are comparing their test scores for the past 4 weeks.

Weeks	Jean	Helen
1st	9	10
2nd	10	9
3rd	8	3
4th	9	10

1. What is the total number of exercises Jean answered correctly in the 4 tests? What is 36 ÷ 4? Does Jean have the same total score as someone who scored 9 on each test for 4 weeks? Then, Jean's average score is 9.

2. What is the total number of exercises Helen answered correctly in the 4 tests? What is 32 ÷ 4? Helen has the same score as someone who scored ? on each test for 4 weeks. Helen's average score is ?.

Your discovery

9

Tell your discovery

Tell how you can use addition and division to find the average of a set of numbers.

Use your discovery

Find the average of {16, 39, 44}.

3. Look at the scores shown in the chart above Ex. 1. On how many tests did Jean receive the higher score? On how many tests did Helen receive the higher score? Does this tell you who did better work on these 4 tests?

4. Who has the greater average score for the 4 tests, Jean or Helen? Does this tell you who did better work on these 4 tests? Explain.

5. Helen said that one low score pulled her average down. What did she mean?

6. What would Helen's average have been if she had made a score of 7 on the third week's test?

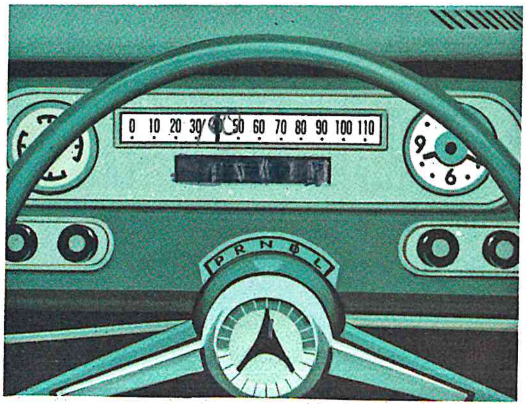
7. Serena said that she had an average of 7 exercises correct on the 4 tests. How many exercises did she solve correctly of the 40 given during the 4 weeks?

8. Martha had a total of 40 exercises correct in 4 tests. This is an average of ? exercises correct on each test.

— Find the average of each set.

	a	b
9.	{6, 2, 12, 8}	{5, 0, 9, 6}
10.	{7, 8, 0, 10, 5}	{3, 2, 4}
11.	{8, 9, 7, 7, 9, 8}	{4, 2}
12.	{1, 2, 4, 2, 1}	{0, 5, 10}

Finding the average rate



1. Don often watches the speedometer when he rides in the car with his father. When the pointer is at 40, Don knows they are traveling at a rate of 40 miles per hour (40 mph). If they continue at this rate, how far will they travel in 2 hours? 3 hours? 4 hours?

2. At what rate are Don and his father traveling when the pointer is at 50? halfway between 40 and 50?

3. One day Don noticed that he and his father had traveled 84 miles in 2 hours. Don told his father, "We have been traveling at an *average rate* of 42 miles per hour." How did Don get the 42?

Do you think the pointer on the speedometer was at 42 all the time? Explain.

4. At the end of 3 hours, Don noticed that they had traveled 132 miles. This is an average rate of ? miles per hour.

5. The dashboard of a car has an odometer, which tells the number of miles the car has gone, and a clock. Don used the odometer and the clock to make this record showing the distance traveled each hour during a 4-hour trip.

1st hour	48 miles
2nd hour	52 miles
3rd hour	46 miles
4th hour	50 miles

- The total distance traveled during the 4 hours was ? miles.
- To find the average rate, Don should divide ? by ?.
- The average rate was ? miles per hour.

6. Shown below is a record of another trip. Find the average rate for this trip.

1st hour	38 miles
2nd hour	44 miles
3rd hour	48 miles
4th hour	46 miles
5th hour	54 miles

During which hours was the speed above the average rate? below the average rate? equal to the average rate?

SELF–HELP TEST 5

1. Tell which fractions below are equivalent fractions. **[114–15]**

$\frac{3}{4}$ $\frac{1}{4}$ $\frac{6}{8}$ $\frac{3}{8}$ $\frac{12}{16}$ $\frac{9}{12}$

2. In each sentence below, n stands for any multiple of 10. Find the solution set for each sentence. **[98]**

$n \times 9 \leq 342$ $n \times 6 \leq 473$

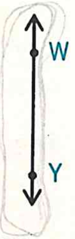
3. In the numeral 6,754,321, what does the 4 stand for? the 7? the 6? **[5]**

4. Which of the following names the figure at the right?

$\overline{WY}$ $\overleftrightarrow{YW}$ $\overrightarrow{WY}$ $\overrightarrow{YW}$ **[74]**

5. Copy and solve these sentences. **[25, 29, 33]**

$n + 56 = 341$ $n - 34 = 89$



— Study each set shown below for a pattern. Then write numerals for the next five members of each set. **[121]**

6. $\{\frac{1}{3}, 1, 1\frac{2}{3}, \ldots\}$

7. $\{\frac{1}{7}, \frac{3}{7}, \frac{5}{7}, \frac{7}{7}, \ldots\}$

8. Copy each sentence below and replace the ● with >, <, or = to make a true sentence. **[118–21]**

$\frac{2}{3}$ ● $\frac{2}{5}$ $\frac{1}{12}$ ● $\frac{1}{14}$ $\frac{5}{6}$ ● $\frac{6}{5}$

9. Write a mixed form for the fractional number that makes each sentence true. **[121–22]**

$10 + \frac{2}{3} = n$ $\frac{17}{6} = n$

10. Is the product of 4,327 and 7,984 an even number or is it an odd number? **[12]**

SELF–HELP TEST 6

— Copy and find the answers.

1. $6\overline{)378}$ **[101]**

2. $4\overline{)725}$ **[104]**

3. $8\overline{)3{,}752}$ **[104]**

4. $5\overline{)4{,}431}$ **[104]**

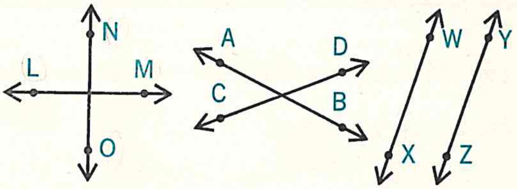
5. $3 \div 4 =$? **[130]**

6. $7 \div 5 =$? **[130–31]**

7. $\frac{3}{4}$ of 36 = ? **[126–27]**

8. $\frac{3}{5}$ of 475 = ? **[126–27]**

9. Name the lines shown below that are parallel; perpendicular. **[94]**



10. Copy and complete.

- $\frac{7}{9}$ yd. makes ? in. **[129]**
- $\frac{3}{8}$ gal. makes ? pt. **[129]**
- $\frac{1}{10}$ T. makes ? lb. **[129]**

IT'S CHECK–UP TIME

— For each sentence below, find the greatest whole number that makes the sentence true.

1. $n \times 60 \leq 375$
2. $n \times 32 \leq 210$
3. $n \times 79 \leq 398$

— For each sentence below, find the greatest multiple of 10 that makes the sentence true.

4. $n \times 82 \leq 1{,}886$
5. $n \times 78 \leq 3{,}905$
6. $n \times 56 \leq 2{,}647$

— Write numerals for the whole numbers that make these sentences true.

7. $634 = (24 \times 26) + n$
8. $2{,}418 = (n \times 37) + 13$

— Copy, find the answers, and check.

9. $35\overline{)280}$
10. $66\overline{)483}$
11. $17\overline{)104}$
12. $49\overline{)4{,}593}$
13. $26\overline{)1{,}742}$
14. $37\overline{)\$23.31}$

— Find the average of the numbers in each set.

15. {23, 18, 64, 37, 43}
16. {117, 234, 188, 165}

17. Frost School has 768 pupils. There are 24 classes in the school. What is the average number of pupils per class?

18. If Stan saves 65¢ a week, how long will it take him to save $25.00?

19. Mr. Ramon drove 220 miles on Monday, 375 miles on Tuesday, and 287 miles on Wednesday. He drove an average of _?_ miles per day.

⑳. Fifteen girls plan to share equally the cost of a radio for their club. The radio costs $14.25. How much money should each girl pay?

If you have time, try these —

21. Tom said he and his father made a 3-hour car trip driving at an average rate of 45 miles per hour. They traveled 52 miles the first hour and 37 miles the second hour. How far did they travel the third hour?

22. 226,885 ÷ 312 = _?_

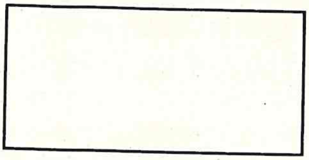
Mathamusements

The members of a set of numbers are all different counting numbers arranged in order. The average of the first two numbers in the set is 2. The average of the first 3 numbers in the set is 3. The average of the first 4 numbers in the set is 4. There is no end to the numbers in the set or to the pattern of averages.

What is the set of numbers?

CHAPTER 8

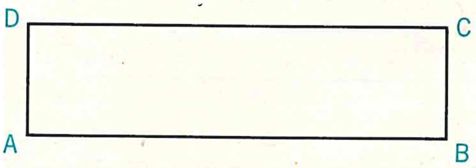
Rectangles and squares



1. Why is the figure shown above alled a quadrilateral?

Use a model of a right angle to est the four angles of this quadriateral. D What do you discover?

If all four angles of a quadrilateral re right angles, the quadrilateral is alled a **rectangle.**



2. Is the closed path shown above rectangle? How can you tell?

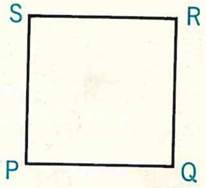
Name the four vertices of this gure. We can call this closed path ectangle $ABCD$.

3. Which pairs of sides of rectangle BCD have the same length?

Line segments AB and DC are alled *opposite sides* of the rectangle. ame another pair of opposite sides rectangle $ABCD$.

Does side AB appear to be parallel side DC? What other sides apear to be parallel to each other?

The opposite sides of a rectangle are parallel and are equal in length to each other.



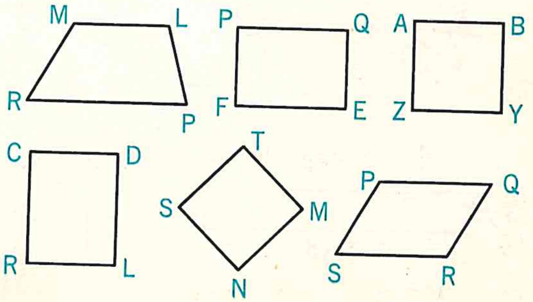
4. Use your right-angle model to test the four angles of this quadrilateral. Is the figure a rectangle? Sides SR and PQ are parallel. Name two other parallel sides.

5. Measure the four sides of rectangle $PQRS$. Are the opposite sides equal in length? Are all four sides equal in length?

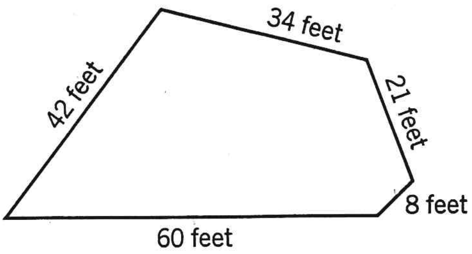
A rectangle that has all four sides equal in length is called a **square.**

6. Is every square a rectangle? Is every rectangle a square? Explain.

7. Use four letters to name each of the quadrilaterals shown below. Tell which of these quadrilaterals are rectangles. Tell which are squares.



Perimeter



1. The drawing above represents a school playground and shows the length of each side of the playground. How many sides does the playground have? What do we call a polygon with five sides?

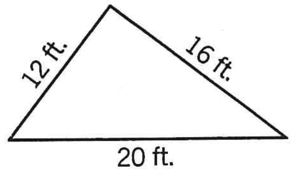
2. Workmen plan to put a fence around the playground shown above. How many feet of fence are needed?

To find the answer, should you find the sum of the lengths of the sides of the playground? Why?

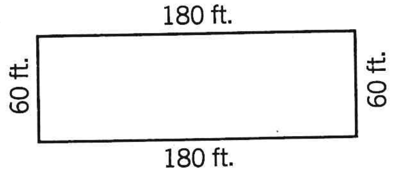
42 + 60 + 8 + 21 + 34 = ?

The sum of the lengths of the sides of a polygon is called the perimeter of the polygon.

3. A flower garden is shaped like the figure shown below. How many sides does the garden have? What is the length of each side? What is the perimeter of the garden?



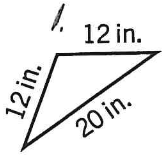
4. The lot on which a gas station is located is shaped like the figure shown below.



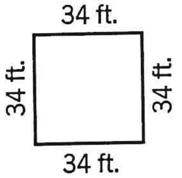
What is the length of the lot? What is the width? Find the perimeter of the lot.

5. Find the perimeter of each polygon shown below.

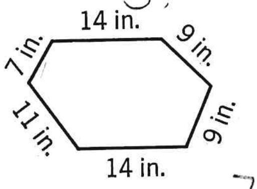
1.



2.



3.



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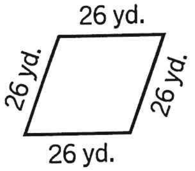
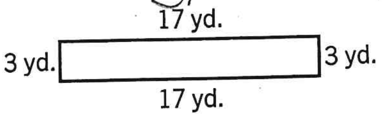
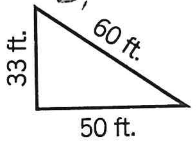


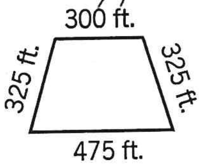
5.



6,



7.



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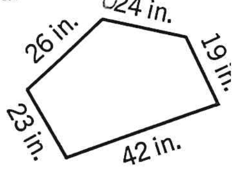
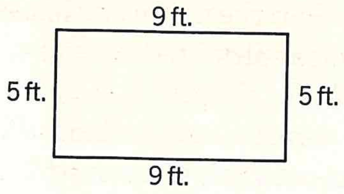


Perimeters of rectangles



1. The figure shown above represents a rectangle. What is the length of this rectangle? What is the width?

2. If P stands for the perimeter of the rectangle shown above, can you use this sentence to find P?

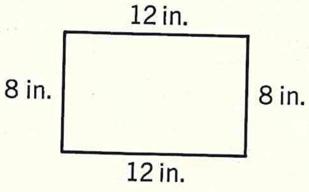
$$P = (9 + 5) + (9 + 5)$$

- What property of addition would you use to rewrite the sentence like his? $P = (9 + 9) + (5 + 5)$
- Does 2×9 name the same number as $9 + 9$? Does 2×5 name he same number as $5 + 5$? Does $(2 \times 9) + (2 \times 5)$ name the same umber as $(9 + 9) + (5 + 5)$?
- What number is 2×9? 2×5? What is the sum of these products? hen the perimeter of the rectangle hown above is ? feet.

3. Can you use this number sennce to find the perimeter of the ctangle shown below?

$$P = (2 \times 12) + (2 \times 8)$$

What is the perimeter?



4. Explain how the distributive property helps you to know that the following number sentence is true.

$$(2 \times 12) + (2 \times 8) = 2 \times (12 + 8)$$

Then can you use this sentence to find the perimeter of the rectangle in Ex. 3? $P = 2 \times (12 + 8)$

5. Why can the following number sentence be used to find the perimeter of a rectangle that is 6 feet long and 8 feet wide? $P = 2 \times (6 + 8)$

To find the perimeter of any rectangle, can you add the length and width and multiply the sum by 2?

6. Suppose l stands for the length of a rectangle and w stands for the width. Explain why this sentence tells you how to find the perimeter of the rectangle.

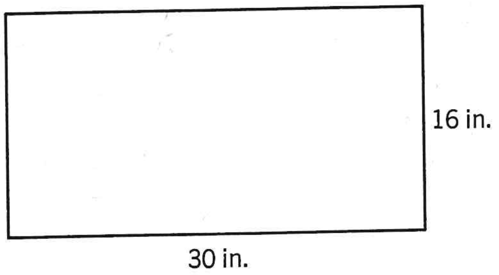
$$P = 2 \times (l + w)$$

We call a sentence such as the one shown above a **formula**. The formula $P = 2 \times (l + w)$ can help you find the perimeter of any rectangle. Explain.

7. Find the perimeter of each of the rectangles whose measurements are given in the table below.

LENGTH	WIDTH
4 feet	2 feet
9 inches	6 inches
10 feet	7 feet
22 inches	18 inches
54 feet	21 feet

Perimeters of squares

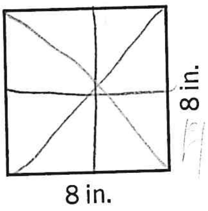


1. The length and width of a rectangle are called the *dimensions* of the rectangle. What are the dimensions of the rectangle shown above? What is its perimeter?

2. Another way to express the dimensions of the rectangle shown above is to write 30 *in. by* 16 *in.*, or simply 30″ × 16″. The symbol ″ means inch or inches; the symbol ′ means foot or feet.

What is the perimeter of a rectangle whose dimensions are 24″ by 14″? 52′ × 38′?

3. The drawing at the right represents a special kind of rectangle. What name do we give to this kind of rectangle? What is its perimeter?



- Can you use this sentence to find the perimeter? $P = 2 \times (8 + 8)$
- Are all four sides of a square equal in length? Then can you also use this sentence to find the perimeter? $P = 4 \times 8$ Explain.

4. To find the perimeter of a square, can you multiply the length of one side by 4? Why?

If *s* stands for the length of one side of a square, then this is the formula for the perimeter of a square.

$$P = 4 \times s$$

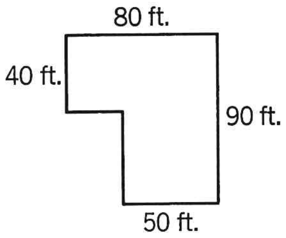
5. Find the perimeter of a square whose side measures 7″; 16″; 32′; 100′; 360′.

6. The dimensions of a square are 9″ by 9″. The dimensions of a rectangle are 12″ by 6″. Which figure has the greater perimeter?

7. How many feet of fencing are required to enclose a rectangular-shaped field 80′ by 120′? How many feet are required to enclose a square field 40′ by 40′?

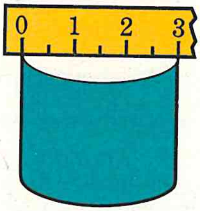
8. Jerry's back yard is a rectangular-shaped lot that is 60 feet long and 45 feet wide. It is completely surrounded by a fence. How many feet of fence were required to surround the lot?

9. Find the perimeter of the plot of ground represented by the drawing below.

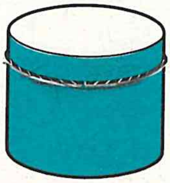


Experimenting with models of circles

1. John wanted to discover if there is a way to find the distance around a circle without measuring that distance. He experimented with the top of a tin can, which he saw was shaped like a circle.

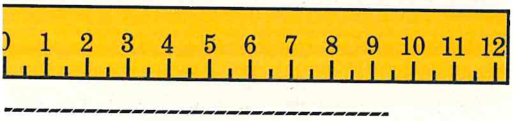


Using his ruler, John first found that a diameter of the top of the can was 3 inches in length.



Next, he cut a piece of string so that, when wrapped around the can once, the ends met but did not overlap.

He then placed this string along a uler and found that it was about $9\frac{1}{2}$ nches in length.



- Is the distance around the can reater than 3 times the length of diameter? 3 × 3 = ?
- Is the distance around the can ess than 4 times the length of a iameter? 4 × 3 = ?
- Which is a better estimate for he distance around the top of the an, 3 times the length of a diamter or 4 times the length of a iameter?

2. John conducted the same experiment with other tin cans with circular tops of different sizes. He made the chart below to show his results.

CAN	LENGTH OF DIAMETER IN INCHES	DISTANCE AROUND TOP IN INCHES
A	4	$12\frac{1}{2}$
B	2	$6\frac{1}{4}$
C	8	25

- Multiply the length of each diameter by 3; by 4.
- Is the distance around the top of each can closer to 3 times or to 4 times the length of a diameter?

3. Using the results of John's experiment, complete this sentence: If the length of a diameter of a circle is 5 inches, the distance around the circle is about ? × 5, or ?, inches.

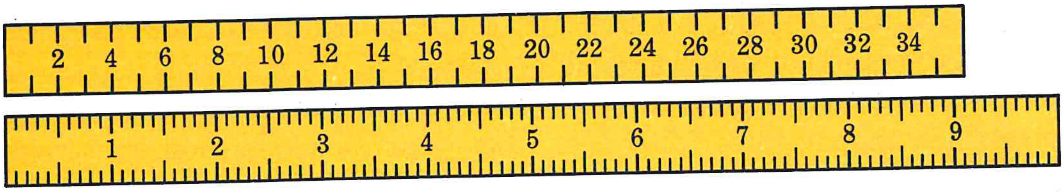
— The lengths of diameters of six circles are given below. Estimate the distance around each circle.

	a	b	c
4.	3 feet	5 inches	9 yards
5.	7 inches	12 feet	15 yards

Think twice!

6. The length of a radius of John's bicycle tire is 12 inches. About how many inches is the distance around the tire?

The metric system of measures



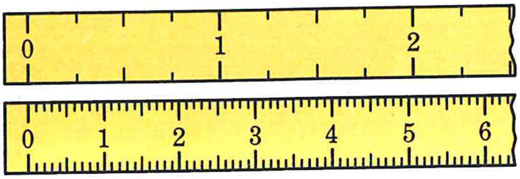
1. One of the two rulers pictured above is called a *yardstick.* The length of the stick is one yard. How many inches make a yard?

2. The other stick pictured above is called a *meterstick.* The length of this stick is one **meter.** The meter is a basic unit of measure in the **metric system of measures.** Which is longer, a yardstick or a meterstick? Which is the greater unit, the yard or the meter?

3. The metric system of measures is based on multiples of ten. Another unit of measure in the metric system is the **decimeter.** It takes 10 decimeters to make 1 meter.

Another unit in the metric system is the **centimeter.** It takes 10 centimeters to make 1 decimeter. How many centimeters make 1 meter?

4. Shown below are an inch ruler and a centimeter ruler. About how many centimeters are equivalent to 1 inch? 2 inches?



10 centimeters (cm) make 1 decimeter (dm).

10 decimeters (dm) make 1 meter (m).

5. Look at the chart shown above. Explain why we can say the metric system is based on multiples of 10.

6. Why is our system of numeration called a *decimal* system? Does our system of measures have a standard unit 10 inches long? 100 inches? 10 feet? 100 feet? Is our system of measures a decimal system?

Can we call the metric system of measures a decimal system? Why?

Copy and complete.

7. 10 cm make ? dm
8. 10 dm make ? m
9. 100 cm make ? m
10. 1,000 cm make ? m
11. 10 m make ? dm
12. 10 dm make ? cm
13. 10 m make ? cm
14. 500 cm make ? m
15. 6 m make ? dm
16. 18 dm make ? cm
17. 4 m make ? cm

8. A unit that is greater than the neter is the **dekameter**. It takes 10 neters to make 1 dekameter. How nany decimeters make 1 dekameter? 100 How many centimeters make 1 dekaneter? 1000

9. Two other units of measure are he **hectometer** and the **kilometer**. t takes 10 dekameters to make 1 ectometer and 10 hectometers to nake 1 kilometer. How many meers make 1 hectometer? 100 1 kiloneter? 1000

0 meters (m) make 1 dekameter (dkm).
0 dekameters (dkm) make 1 hectometer (hm).
0 hectometers (hm) make 1 kilometer (km).

— Copy and complete.

0. 100 m make ? hm
1. 1,000 m make ? hm
2. ? hm make 1 km
3. ? dkm make 1 km
4. 10 km make ? hm
5. 10 km make ? dkm
6. 10 km make ? m
7. ? m make 5 km
8. ? dkm make 20 hm
9. 12 km make ? hm
0. 8 km make ? m
1. 100,000 m make ? dkm
2. 100,000 m make ? km

33. How many centimeters make one decimeter? Then is 16 centimeters the same as 1 decimeter 6 centimeters?

34. How many meters make one dekameter? Then 32 m is the same as 3 dkm ? m.

35. Is this sentence true: 58 dm make 5 m 8 dm? Copy and complete this sentence: 74 hm make 7 km ? 4 hm.

— Copy and complete each of the following sentences.

36. 18 cm make 1 dm ? cm
37. 43 m make 4 dkm ? m
38. 76 hm make 7 km ? hm
39. 54 dm make ? m 4 dm
40. 29 cm make ? dm 9 cm
41. 85 m make ? dkm 5 m
42. ? cm make 1 dm 4 cm
43. ? m make 3 dkm 6 m
44. ? hm make 9 km 2 hm

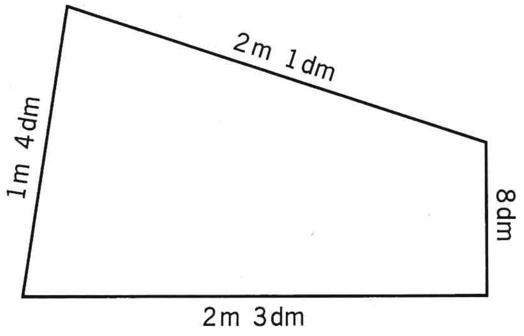
45. How many centimeters make 1 meter? 1 decimeter? Then is this sentence true: 143 cm make 1 m 4 dm 3 cm?

46. It takes ? m to make 1 hm; ? m to make 1 dkm. Thus, 345 m make 3 hm 4 dkm ? m.

— Copy and complete.

47. 172 cm make 1 m 7 dm ? cm
48. 384 m make ? hm 8 dkm 4 m
49. 529 dkm make 5 km ? hm 9 dkm
50. ? cm make 4 m 6 dm 3 cm

Perimeters using metric units



1. The drawing above represents a plot of ground around which Henry's father wishes to place a fence. What is the perimeter of the plot?

- To find the perimeter, Henry added the measures of the sides as shown below.

2 m 3 dm
8 dm
2 m 1 dm
+ 1 m 4 dm
5 m 16 dm

- Did Henry find the sum of the measures expressed in decimeters and then the sum of the measures expressed in meters? How many decimeters make 1 meter? Then, 5 m 16 dm is the same as 6 m _?_ dm.

2. Find the perimeter of a four-sided plot where the lengths of the sides are: 2 m 5 dm, 1 m 4 dm, 3 m 2 dm, and 1 m 6 dm.

3. Use Henry's method to find the perimeter of a rectangle whose dimensions are 2 m 7 dm by 1 m 5 dm.

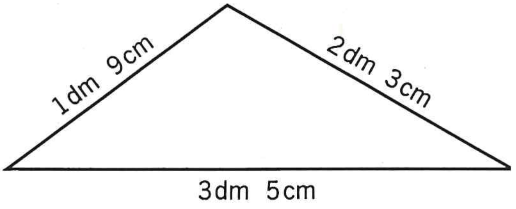
4. To find the perimeter of the plot shown in Ex. 1, Marie changed all the measurements to decimeters before she added.

2 m 3 dm make 23 dm
8 dm make 8 dm
2 m 1 dm make 21 dm
+ 1 m 4 dm make 14 dm
66 dm

Did Marie add correctly? How many decimeters make 1 meter? Then 66 dm is the same as _?_ m 6 dm. Is this the same answer Henry found for the perimeter?

5. Use Marie's method to find the perimeter of the plot described in Ex. 2.

6. Use either Henry's or Marie's method to find the perimeter of the triangle shown below.



7. The measurements of the sides of a triangle are: 1 km 6 hm, 2 km 2 hm, and 3 km 9 hm. Find the perimeter of the triangle.

8. Find the perimeter of a triangle whose sides measure 3 dkm 6 m, 2 dkm 2 m, and 3 dkm 5 m.

Review of four operations

– Write the numerals that are missing from charts A and B below.

A

	ADDEND	ADDEND	SUM
1.	27	18	?
2.	39	?	72
3.	46	?	100
4.	?	234	1,000
5.	?	87	87
6.	198	?	800
7.	344	?	345
8.	1,645	2,896	?
9.	?	7,340	7,340
10.	2,531	?	8,000

B

	FACTOR	FACTOR	PRODUCT
1.	20	17	?
2.	30	24	?
3.	?	8	16
4.	9	?	72
5.	?	12	84
6.	27	?	162
7.	?	52	52
8.	48	?	0
9.	59	78	?
10.	?	38	646

– The pair of numbers named in the first column of charts C and D s related by some operation to the number named in the second olumn. The third column shows the operation. Copy and complete.

C

	NUMBER PAIR	RESULT	OPERATION
1.	15, 5	20	?
2.	15, 5	?	–
3.	15, 5	75	?
4.	15, 5	?	÷
5.	28, 32	?	–
6.	480, 6	?	÷
7.	96, 52	148	?
8.	39, 3	13	?
9.	18, 9	?	×
10.	52, 0	52	?

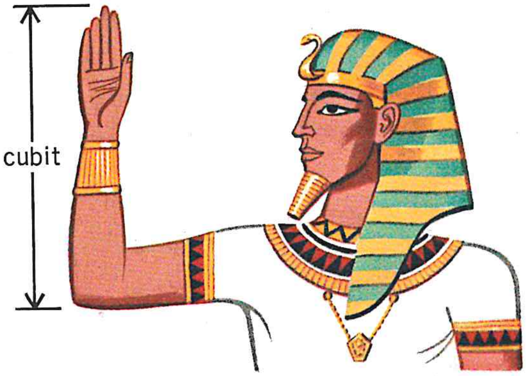
D

	NUMBER PAIR	RESULT	OPERATION
1.	279, 9	?	÷
2.	320, 150	170	?
3.	19, ?	5	–
4.	581, ?	923	+
5.	256, 32	?	÷
6.	96, ?	12	÷
7.	371, 42	?	×
8.	785, 515	1,300	?
9.	49, ?	49	×
10.	86, 0	0	?

Inquiry into history

Early units of measure

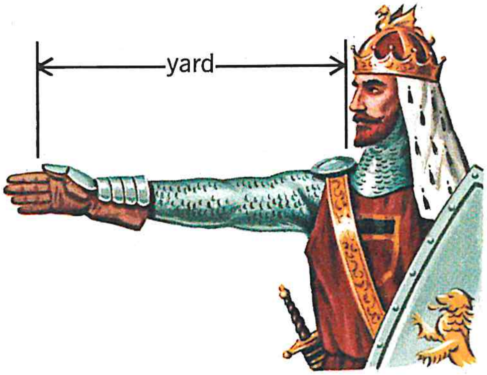
Man did not always use standard units such as our present-day inch, foot, and mile to measure distance. In earlier times, the length of an object was often measured by comparing it with the length of a part of the human body. Can you think of some difficulties that might occur through the use of such "body units"?



▶ The ancient Egyptians and other people of earlier times used a **cubit** to measure length. One cubit was the length between a person's elbow and the tips of his fingers.

Compare the length of your forearm and fingers with that of your father's forearm and fingers.

Would you rather use your arm or your father's arm as a cubit unit if you were buying 3 cubits of material? if you were selling the material?



▶ A **yard** originally was the distance from the tip of a person's nose to the end of his thumb when his arm and hand were stretched out.

Use your outstretched arm to find how many "yards" wide your room is. If your teacher used her arm as a unit, would her measurement of the room be the same as yours? Explain.

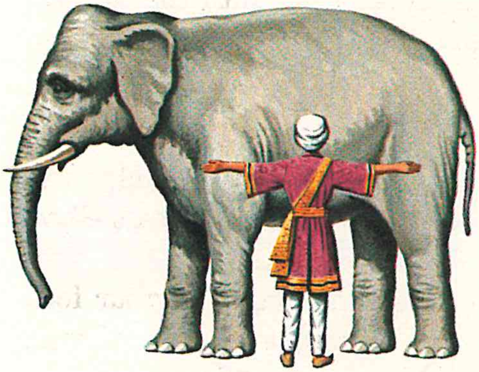
In the twelfth century, King Henry I of England made a law that the "yard" was the length from the tip of his nose to the end of his thumb when his arm and hand were stretched out. Why do you suppose he made such a law?

▶ The width of a person's middle finger was also used as a unit for measuring length. This unit was called a **digit.**

Use your middle finger to tell how many digits wide your mathematics book is. Ask your teacher or your father to do the same with his finger. How do the results compare? Why?

▶ The length of a man's **foot** was a very natural and common unit used in earlier times. Suppose you used your foot and your father used his foot to measure the length of a room. Who would say the room is a greater number of units long, you or your father? Why?

▶ Today, a **fathom** is a unit of measure equivalent to 6 feet. In earlier times a fathom was the distance from the finger tips of one hand to the finger tips of the other hand when a man's arms and hands were outstretched.



Ask a classmate to stretch out his arms. Then measure the distance between his finger tips with a tape measure. What is the difference in inches between his fathom and a standard fathom used today?

▶ A **hand** is an early unit of measure that was the width of a man's hand. Compare the width of your hand with that of your father's hand.

Even today, horses are often measured and their heights reported as so many hands high. However, today's hand is usually considered to be equivalent to 4 inches.

For further inquiry

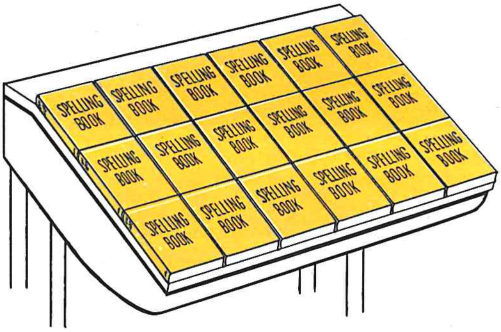
Listed below are other uncommon units of measure. Use a dictionary to find out something about these units and where they might be used today.

link	chain	furlong	bolt	skein	hank
gill	noggin	pottle	hogshead	cable	league

The measure of a region — area

1. As you know, a closed path and the set of points enclosed by the path are called a region. Explain why the flat surface of a desk top might make you think of a region.

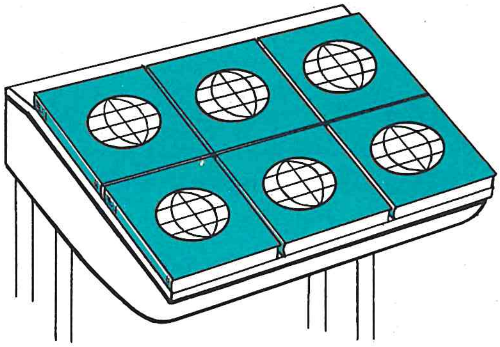
2. Amy thought of the top of her desk as a region. She placed spelling books on her desk until the top was covered like this.



- How many spelling books did Amy place in each row across her desk?
- How many rows were needed to cover the surface of the desk?
- Explain why 6 × 3 names the number of spelling books needed to cover the surface of the desk.
- If you think of the surface of a spelling book as a unit of measure, then 6 × 3, or _?_, is a measure of the region represented by the surface of Amy's desk.

The measure of a region is called the area of the region.

3. Sue covered the top of Amy's desk with geography books like this.



- Explain why 3 × 2 names the number of geography books needed to cover the top of Amy's desk.
- If you use the surface of a geography book as a unit of measure, the area of Amy's desk top is _?_.

4. Mary covered Amy's desk top with 2 rows of reading books with 4 books in each row. Using a reading book as a unit, the area is _?_.

5. In Ex. 2–4, each girl measured the same desk top. Did each give the same measure for the desk top? Explain.

6. Tom used sheets of paper from his notebook to measure the teacher's desk top. He said the area is 16.

Fred used sheets of paper from his notebook to measure the same desk top. He said the area is 12.

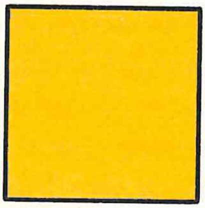
Explain how each boy could be correct even though they gave different measures.

Standard units of area

1. To measure a line segment, we use another segment as a unit. To measure an angle, we use another ? as a unit. What do you think is used as a unit to measure a region?

2. In your earlier work with measures, you discovered reasons why we use *standard* units of measure. What is one reason?

A model of a standard unit of measure used to find area is shown below.



Use your ruler to measure the ength of each side of this region. Does the figure have four right ngles? What special type of quadilateral region is shown in the drawng?

A standard unit of area is a square egion whose sides are 1 inch in ength. This unit is called a **square nch** (**sq. in.**).

3. Draw a model of a square inch. ut out the model you drew.

Draw a rectangle whose dimen- ons are 5 in. by 7 in. Use your odel of the square inch to find the ea of this rectangular region.

4. On a piece of newspaper, draw a model of a square whose sides are 1 foot in length. Cut out the model. The area of this square region is called 1 square foot. A **square foot** (**sq. ft.**) is another standard unit of area.

Use your model of a square foot to estimate the area of your desk top. About how many times does the model fit across the top of the desk in a row? About how many rows are there?

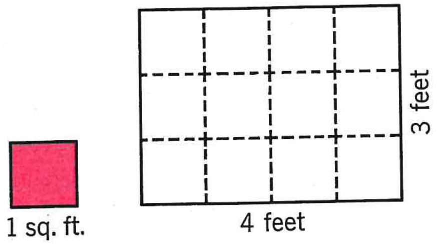
The area of your desk top in square feet is about ?.

5. A rectangular-shaped table top is 8 feet long and 4 feet wide. How many square feet are there in one row across the table top? How many rows of square feet are there? Then the area of the table top in square feet is ?.

6. Another standard unit of area is called a **square yard** (**sq. yd.**). This is a square region whose sides are 1 yard in length.

- A piece of material that is 1 yard wide and 3 yards long contains ? square yards of material.
- How many square yards of material are there in a piece that is 2 yards wide and 3 yards long? 2 yards wide and 6 yards long? 4 yards wide and 4 yards long?

Area of rectangular regions



1. The model at the left in the drawing above represents 1 square foot. The model at the right represents a rectangular region 4 feet long and 3 feet wide.

- How many square feet are there in one row across the rectangular region? What dimension of the rectangular region tells you the number of square feet in one row?
- How many rows of 4 square feet are there in the rectangular region? What dimension of the rectangular region tells you the number of rows of 4 square feet?
- Does 4 × 3 tell you how many square feet there are in this rectangular region?

2. A rectangular region is 7 feet long and 2 feet wide. Find the area.

- How many square feet are there in one row across this rectangular region? What is the length of the region?
- How many rows of 7 square feet are there in the rectangular region? What is the width of the region?
- What number is 7 × 2? What is the area?

Your discovery

Tell your discovery

If the dimensions of a rectangular region are expressed in the same unit, how can you quickly find the area of the rectangular region?

Use your discovery

Find the area of the rectangular region whose dimensions are 3 feet by 6 feet; 8 inches by 12 inches; 50 yards by 90 yards.

3. If we use l to stand for the length of a rectangular region, w to stand for the width, and A to stand for the area, is the following sentence a formula for finding the area of any rectangular region? Explain.

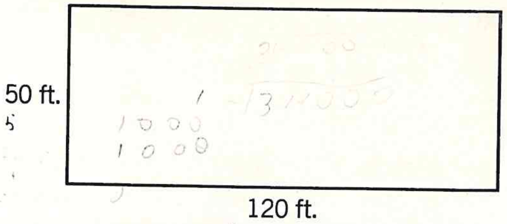
$$A = l \times w$$

Tell why it is necessary to express the length and width in the same unit before you can use this formula to find the area.

4. Find the area of the rectangular regions whose dimensions are given in the table below.

LENGTH	WIDTH
13 inches	9 inches
28 feet	15 feet
32 yards	18 yards
12 inches	12 inches
3 feet	3 feet

Finding areas



1. The drawing above represents a plot of ground to be used for a city park. To buy enough grass seed, the landscape gardeners need to know the area of the plot.

- What is the length of the plot? What is the width? Then the area is ? square feet.
- If each bag of grass seed is enough for 1,000 square feet, how many bags of seed should the gardeners buy?

2. Bob and his father are going to cover the floor of their recreation room with vinyl tiles. Each tile represents a square region, 1 foot on each side.

If the dimensions of the floor are 12 feet by 15 feet, how many tiles should Bob and his father buy?

3. Joan's mother would like to cover the wall above her stove and sink with plastic wall tile. The dimensions of this region are 8 feet by 3 feet.

If one box of tiles will cover 6 square feet, how many boxes does Joan's mother need?

4. You know that 1 foot square is a square region that measures 1 foot on each side. How many inches make 1 foot? What is the area of a square region that measures 12 inches by 12 inches? Then how many square inches make 1 square foot?

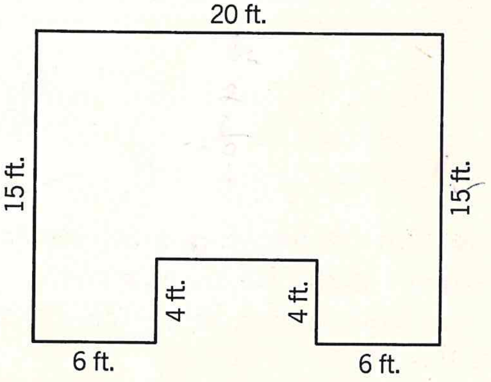
5. The dimensions of 1 yard square are 1 yard by 1 yard. Another way to express these dimensions is ? feet by ? feet. How many square feet make 1 square yard?

6. Which of the following tells the number of square inches in 1 square yard?

108 72 1,296 144 81

7. The dimensions of a rectangular-shaped flower bed are 6 feet by 9 feet. Find the area of the flower bed in square feet; in square yards; in square inches.

8. Find the area of the region shown in this drawing.



16
16

31
16
14

Keeping up in mathematics

Oral

1. If you replace the 4 with a 0 in 740,000, the result names a number how much less than the number named originally?

$$\boxed{\begin{array}{r}8\\45\overline{)330}\end{array}}$$

2. How can you tell if 8 is the correct quotient for this example?

3. Will the quotient for 1,350 ÷ 43 be 30-some, 40-some, or 50-some?

4. Study each set given below for a pattern. Then name three more members of each set.

$$\{\tfrac{1}{2}, \tfrac{2}{4}, \tfrac{3}{6}, \tfrac{4}{8}, \cdots\}$$

$$\{\tfrac{0}{3}, \tfrac{2}{3}, \tfrac{4}{3}, \tfrac{6}{3}, \cdots\}$$

$$\{\tfrac{0}{5}, \tfrac{2}{5}, \tfrac{4}{5}, 1\tfrac{1}{5}, 1\tfrac{3}{5}, \cdots\}$$

5. Which is the greater number: $\frac{3}{4}$ or $\frac{2}{4}$? $\frac{1}{2}$ or $\frac{1}{3}$? $\frac{8}{5}$ or $1\frac{4}{5}$?

6. If each of two addends is doubled, is the sum of the two addends 2 times as much, 4 times as much, or 8 times as much as the original sum?

7. Which will make the quotient twice as much, doubling the divisor or doubling the dividend?

8. Tell the length of a diameter of a circle if the length of a radius of that circle is 5 inches; 6 feet; $2\frac{1}{2}$ inches; 3 feet 4 inches.

Written

— Copy and compute.

	a	b	c
1.	433 386 854 + 795	5,629 7,234 8,917 + 436	37,259 7,484 9,318 27,465 6,528 + 52,347
2.	618 − 429	3,760 − 1,423	52,649 − 25,967
3.	384 × 28	6,912 × 305	82,560 × 25
4.	$9\overline{)819}$	$76\overline{)296}$	$82\overline{)1,725}$

5. Find the average for this set of scores.

$$\{66, 72, 62, 60, 69, 61\}$$

6. The weights of three bags of apples are 3 lb. 5 oz., 8 lb. 12 oz., and 6 lb. 14 oz.

Find the total weight of the bags of apples.

— Solve each of the following number sentences.

	a	b
7.	$15 \times n = 135$	$n + 16 = 38$
8.	$n \div 12 = 82$	$184 - n = 76$
9.	$54 + n = 54$	$n \times 95 = 95$

The mile as a unit of measure

1. Find an object in the classroom hat is about 1 foot in length. Now ind an object that is about 10 times s long as the first object.

Can you think of any object that s 100 times as long as the first obect? 1,000 times as long? 5,000 imes as long?

It is difficult to think of any object hat is 5,000 feet in length. But that s about the length of a standard nit of measure that you often use, he mile.

5,280 feet make 1 mile (mi.).

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2. What is 5,280 feet rounded to he nearest thousand feet?

Many homes have front yards 50 eet wide. In one mile, there can be bout $\underline{\ ?\ }$ lots this wide.

3. The length of a football field is 00 feet. In one mile, about how any football fields can be located nd to end?

4. The distance a train travels in oing from St. Louis to Washington, .C., is about 980 miles. About how any feet is this distance?

5. The airline distance between St. ouis and Washington, D.C., is bout 712 miles. About how many et is this?

6. A jetliner traveling between New York City and Chicago flew at an altitude (height) of 30,000 feet. About how many miles high was the plane flying?

— Many boys and girls can walk a mile in about 15 minutes. Use that length of time to answer Ex. 7–10.

7. About how many miles can Jim walk in an hour?

8. Dan walked from his home to school. It took him $\frac{1}{2}$ hour.

Dan could say that the school is about $\underline{\ ?\ }$ miles from his house.

9. It takes Tom a little more than 7 minutes to walk from his home to school. Is the distance between Tom's home and the school greater than or less than one mile?

About what part of a mile is the distance Tom walks to school?

10. The flight deck of the aircraft carrier U.S.S. *Forrestal* is 1,039 feet in length. About how long would it take a boy to walk from one end of the carrier to the other?

U.S. NAVY

PROBLEM-SOLVING HELP

Time — rate — distance problems

1. If a bus travels at an average rate of 42 miles an hour, what distance will it travel in 1 hour? in 2 hours? in 3 hours?

2. At an average rate of 60 miles an hour, what distance will a train travel in 3 hours? in 5 hours? in 24 hours?

3. How far does a jetliner travel in 3 hours at an average rate of 600 miles an hour?

4. Suppose d stands for the distance, r stands for the average rate of travel, and t stands for the number of hours, or time. Explain how you can use the following formula to find the distance when you know the rate and the time.

$$d = r \times t$$

Use the formula to help you find how far a boy can walk in 5 hours at an average rate of 4 miles an hour.

5. During a period of 5 hours, Mr. Jessup drove at an average rate of 40 miles an hour. How far did he drive during this period?

6. Walking at an average rate of 4 miles an hour, how far can a boy walk in 3 hours?

At an average rate of 2 miles an hour, a boy can walk just ___?___ miles in 3 hours.

7. If a bus travels at an average rate of 40 miles an hour, how many hours will it take the bus to travel 200 miles?

- Which do you have to find: the distance, the rate, or the time?
- What is the distance? What is the rate?

$$\boxed{\begin{array}{c} d = r \times t \\ 200 = 40 \times t \end{array}}$$

- Do you add, subtract, multiply, or divide to solve this sentence? $200 = 40 \times t$
- Solve the sentence.

8. If a train travels at an average rate of 65 miles an hour, how many hours will it take the train to travel 130 miles? 195 miles? 585 miles?

9. If a small plane traveled at an average rate of 75 miles an hour, how many hours would it take the plane to go 225 miles?

10. A bus made a trip of 266 miles in 7 hours. Find its average rate of travel.

$$7\overline{)266}^{\,38} \quad 21 \quad 56$$

- Explain how the formula is used to get the sentence shown below it in the box.

$$\boxed{\begin{array}{c} d = r \times t \\ 266 = r \times 7 \end{array}}$$

- Solve the sentence.

11. A train makes a 342-mile trip in 6 hours. That is an average rate of ___?___ miles an hour.

12. At what average rate must a car travel to go 280 miles in 8 hours?

Graphing time, rate, and distance

Elaine made the following table to show how the distance increases vith the number of hours when a car travels at an average rate of 40 niles an hour.

Number of hours	1	2	3	4	5
Number of miles	40	80	120	160	200

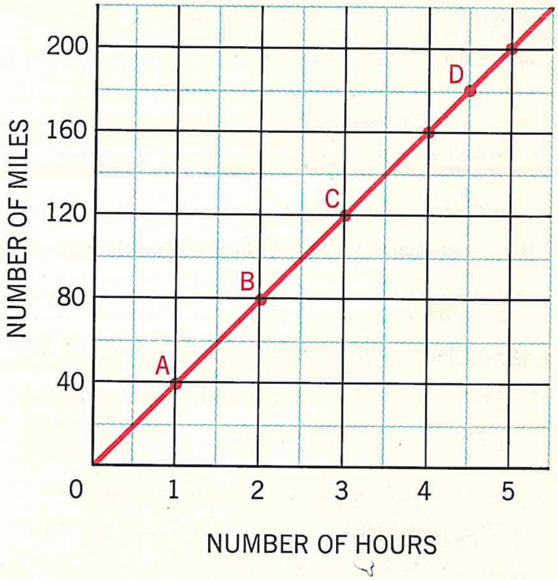
1. Does each column of the table show a pair of numbers? What pair of numbers is shown in the first column? in the second column?

2. We can show the pair of numbers named in the first column by vriting (1, 40). The second number pair would be shown this way: 2, 80). In a similar way, show each of the other three number pairs named in the table.

3. The number pair (2, 80) tells us that in 2 hours the car travels 30 miles. What does each of the following number pairs tell you about the number of hours and the number of miles traveled?

(6, 240) (8, 320) (10, 400) (12, 480)

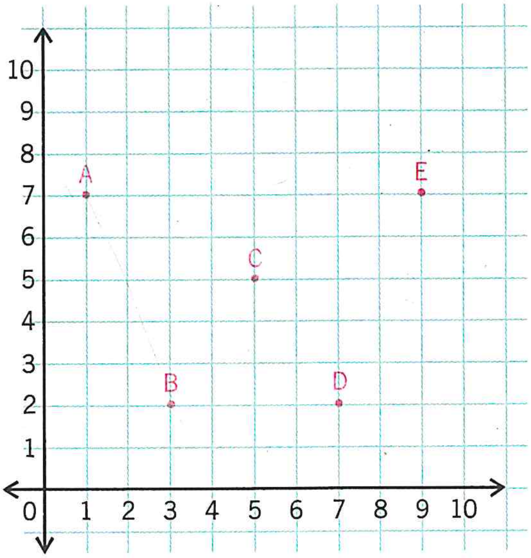
4. Shown at the right is a **graph** Elaine made to illustrate the infor-nation given in the table above.



- What does the scale at the left f the graph tell you? What does the cale below the graph tell you?
- Four points have been marked on he graph and named. Does each oint stand for a number pair? Vhat number pair is represented by oint *A*? by point *B*? point *C*? oint *D*?
- Use this graph to tell how far the ar travels in 2 hours; in 4 hours; in ½ hours; in 3½ hours.
- Copy the graph and extend it to include 8 hours and 320 miles. Ise your graph to tell how far the car travels in 6 hours; in 7 hours; ½ hours; 8 hours.

Graphing number pairs

1. Bill and Barry discovered a game they could play using graph paper and number pairs. The boys first made a drawing of two lines on the graph paper, one parallel to the bottom edge of the paper and the other parallel to the left edge of the paper. Where the two lines intersected, they wrote the numeral for zero and marked off two scales using numerals for whole numbers as shown in the drawing at the right.



- Barry said, "Point *A* is over 1 and up 7." Bill then began at 0 on the bottom scale, counted 1 space over to the right, and then 7 spaces up from this point. Here he marked a dot and the letter *A*. Find point *A* in the drawing.

- "Point *B*," said Barry, "is over 3 and up 2." Find point *B* in the drawing and explain why Bill marked it where he did.

- Barry said that point *C* is over 5 and up 5. Did Bill locate point *C* in the drawing correctly? Explain.

- The instructions for the next point were, "Point *D*: over 7, up 2." Begin at 0 in the drawing and count over 7 spaces and then up 2 spaces. Is this where Bill marked a dot for point *D*?

- Find point *E* in the drawing and explain how the instructions "over 9, up 7" tell you how to locate this point.

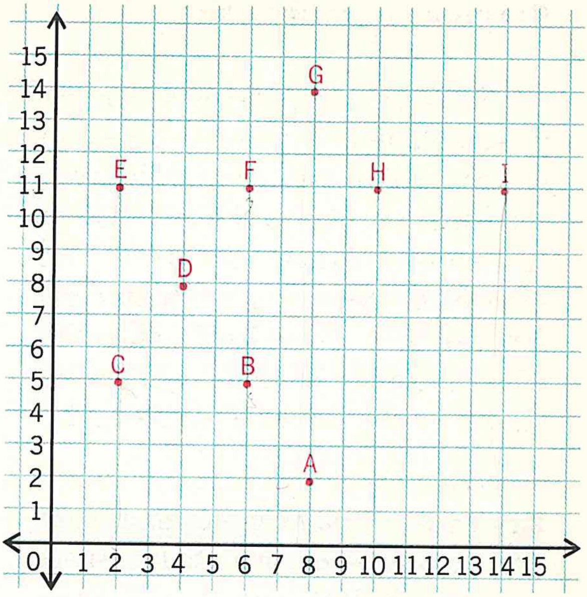
- The last part of the game was to connect the dots with line segments beginning with point *A* and following in order the letters of the alphabet; that is, to join point *A* to point *B*, *B* to *C*, *C* to *D*, and *D* to *E*.

- On graph paper, copy the graph shown above and join the points with line segments in this way. What letter of the alphabet does the final drawing show?

2. Bill wrote the instructions for a drawing Barry was to make. But nstead of writing, "Point *A*: over 8, ıp 2," he simply wrote, "*A* (8, 2)."

- What does the first number in 8, 2) tell Barry to do? What does he second number tell him to do? ind point *A* in the drawing at the ight. Did Barry correctly locate his point?

- Does *B* (6, 5) mean that point *B* s located over 6 and up 5? What loes *C* (2, 5) mean? Did Barry corectly locate points *B* and *C* in the rawing?



- What point in the drawing is located by the number pair (4, 8)? Vhat point is located by the number pair (2, 11)? by the number air (6, 11)?

- What pair of numbers did Bill use to tell Barry how to locate oint *G*? point *H*? point *I*?

- Copy the drawing shown above on graph paper. Carefully mark he points already shown. Then locate and mark the points for the ollowing number pairs: *J* (12, 8); *K* (14, 5); *L* (10, 5).

- Join the points in alphabetical order with line segments. Finish our drawing by joining point *L* to point *A*. Your work is correct if our drawing shows a star with six points.

3. On a piece of graph paper, locate and label a point for each of e following number pairs. Then, beginning with point *A*, join the oints with line segments in this order: *A* to *B*, *B* to *C*, *C* to *D*, and forth. The last segment should join points *P* and *A*.

A (4, 3)	*B* (2, 5)	*C* (5, 5)	*D* (5, 6)
E (7, 6)	*F* (7, 8)	*G* (8, 8)	*H* (8, 6)
I (9, 6)	*J* (9, 8)	*K* (10, 8)	*L* (10, 6)
M (13, 6)	*N* (13, 5)	*O* (16, 5)	*P* (13, 3)

A broken-line graph

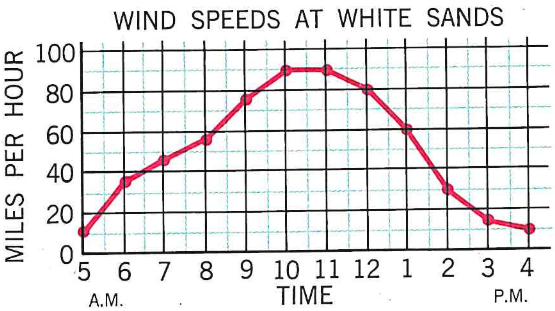
After a hurricane hit White Sands, Mr. Murphy's fifth-grade class visited the weather station to find out about winds. The weather observer told the class that any winds that blow faster than 75 miles per hour are classed as hurricane winds. He had measured the wind speed once each hour and had made this table to show his measurements.

WIND SPEEDS DURING STORM

5 A.M.	10 mph	11 A.M.	90 mph
6 A.M.	35 mph	12 NOON	80 mph
7 A.M.	45 mph	1 P.M.	60 mph
8 A.M.	55 mph	2 P.M.	30 mph
9 A.M.	75 mph	3 P.M.	15 mph
10 A.M.	90 mph	4 P.M.	10 mph

Using this table, the class drew the **broken-line graph** shown below. They thought of the hour and the measure of the wind speed at that hour as a number pair.

After locating a point on the graph for each number pair, they joined these points with line segments.



1. Between what two hours in the morning did the weather station first record wind speeds at hurricane strength?

2. Between what two hours in the afternoon did the weather station first record wind speeds at less than hurricane strength?

3. Between what two times of the day does the graph show the wind speed increasing?

4. Between 5 A.M. and 6 A.M. the wind speed increased 25 mph. Did it increase as many miles per hour during any other hour? Then was the rate of increase the greatest between 5 A.M. and 6 A.M.?

5. Between what two times of the day does the graph show the wind speed decreasing?

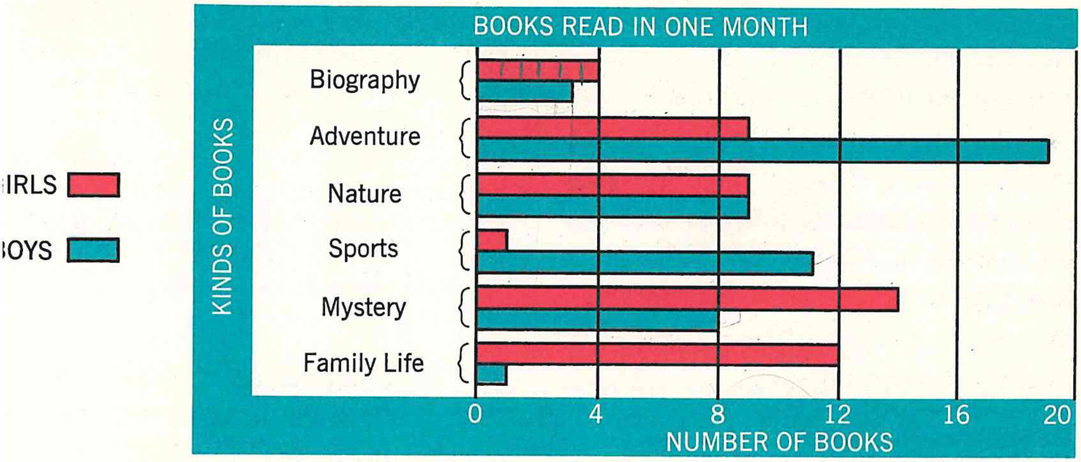
6. During what hour did the wind speed decrease at the greatest rate?

During what hour did the wind speed decrease at the least rate?

7. Estimate the speed of the wind at 6:30; at 7:30; at 12:30. Why do you think the graph does not show the exact wind speed at these times?

8. Which is easier to use to tell the rate of change in wind speed: the table of facts or the broken-line graph? Why?

A bar graph



The pupils of the fifth grade of Kennedy School made a list of the inds of books they read last month. Then the boys and the girls ade separate records of the number of each kind of book read. They ut these facts together and represented them on the **bar graph** hown above.

1. What kinds of books were read? Vhere is this shown on the graph?

2. The red bar marked Biography xtends to 4. This means that the irls in the class read ? biographies ast month. How many biographies id the boys in the class read?

3. What kind of book was read ost often by the girls? by the boys?

4. What kind of book was read ast often by the boys? by the irls?

5. Both boys and girls read the ame number of ? stories. How any of these stories were read by e class as a whole?

6. Bar graphs are often used to make comparisons quickly.

Can you tell quickly, without counting or adding, which kind of book was read most often by the class as a whole? least often by the class as a whole?

7. The fifth-grade class is going to sell books at the school fair. They are using the bar graph to help them decide what kinds of books to sell.

- Should they have more adventure books or more sports books for sale? more mystery stories or more nature stories?
- What kind of book should they have the least number of for sale?

PROBLEM TEST 4

1. The earth completes one rotation every 24 hours. How many times does it rotate in 1,272 hours?

2. Forty-seven people contributed to a fresh-air fund. The amount collected was $44.65. What was the average amount contributed by each person?

3. The art club has $13.06 in its treasury. The club wants to buy some special pencils that cost 45¢ each. How many pencils can it buy?

4. Martha took 7 history tests during the year. Her scores were {84, 97, 75, 89, 87, 94, 90}. What was her average score for the 7 tests?

5. On his bicycle, Evan rode 5 miles the first hour, 3 miles the second hour, and 4 miles the third hour. What was his average rate of travel?

6. A square-shaped field is 275 feet long. How many feet of fence are needed to enclose the field?

7. A rectangular-shaped table top measures 18 inches by 36 inches. To cover it with tiles that are 1 inch square, how many tiles would you need?

8. Running at an average rate of 250 meters per minute, how far can a boy run in 12 minutes?

9. Each of three boys saved 25¢ a week for 6 weeks. How much money did the boys save all together?

10. A train travels at an average rate of 53 miles per hour. How long does it take the train to travel 954 miles?

Write your score on your Problem Test Record.

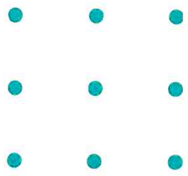
Mathamusements

- What is the perimeter of the largest square you can form using 3 yardsticks and a foot ruler, each of which is 1 inch wide? The tools must lie flat on the floor. Is there a smallest square you can form using these tools?

- Try to draw a picture of 4 triangles and 2 squares using just 8 line segments; using just 7 segments.

- Can you draw 13 dots in 6 rows with 3 dots in each row?

- Copy these 9 dots on a piece of paper and try to connect them using just 4 line segments. Do not lift your pencil.



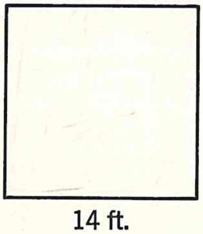
IT'S CHECK-UP TIME

1. Find the perimeter of a rectangle whose dimensions are 12′ by 6′.

2. Find the area of a rectangular region whose dimensions are 35 inches by 50 inches.

3. Find the perimeter of the square shown at the right.

4. Find the area of the square region shown at the right.



5. Is the distance around a circle whose diameter is 12 inches about 36 inches or is it about 48 inches?

6. The measurement of each side of a pentagon is 4 feet 8 inches. What is the perimeter of the pentagon?

7. The measurements of the sides of a triangle are 2 m 3 dm, 2 m 6 dm, and 3 m 4 dm. Find the perimeter of the triangle.

— Copy and complete.

8. 346 cm make 3 m 4 dm ? cm

9. ? m make 7 hm 2 dkm 5 m

10. 481 dm make 4 dkm ? m 1 dm

11. 1 mi. 200 ft. make ? ft.

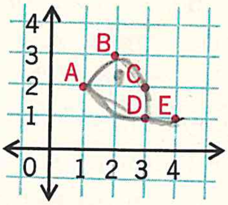
12. 10,600 ft. make 2 mi. ? ft.

13. 1 sq. ft. makes ? sq. in.

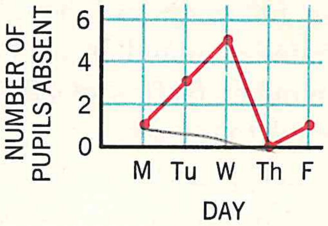
— For each sentence below, give two operation signs that can replace the ● to make the sentence true.

14. 37 ● 0 = 37 **15.** 45 ● 1 = 45

16. What pair of numbers can be used to locate point *A* on the graph shown below? point *B*? *C*? *D*? *E*?



17. The graph shown below tells the number of pupils who were absent from the fifth-grade class each day for a week. On what day were the most pupils absent? the fewest?



18. What is the average number of pupils absent each day during the week shown on the graph above?

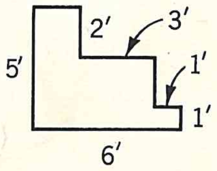
19. How long will it take a train traveling at an average rate of 65 miles an hour to go 390 miles?

20. How far will a plane fly in 3 hours if it flies at an average rate of 120 miles an hour?

If you have time, try these —

21. Find the perimeter of this figure.

22. Find the area of this region.



CHAPTER 9

Prime and composite numbers

1. Is 20 the product of 4 and 5? Then 4 and 5 are factors of 20. Are 1 and 20 factors of 20? Name two other whole-number factors of 20.

Does the following set show every number found in all pairs of whole-number factors of 20?

{1, 2, 4, 5, 10, 20}

2. Is 2 a factor of 16? Is 4? Name as many other whole-number factors of 16 as you can.

3. The following set should show every number found in all pairs of whole-number factors of 32. Copy and complete the set.

{1, 2, 4, 8, _?_, _?_}

4. Does the following set show all the whole-number factors of 36? If not, what factors are not shown?

{1, 2, 3, 4, 6, 18, 36}

— For each number shown in the first column below, an incomplete set of whole-number factors is given. Copy and complete each set.

5. 6 {1, 2, 3, _?_}

6. 25 {1, _?_, _?_}

7. 40 {1, 2, 4, 5, _?_, _?_, _?_, _?_}

8. Find all the whole-number factors for each number named below.

24 3 9 15 7 11 14

9. What whole number is a factor of every number named in Ex. 8?

10. Is 1 a factor of every whole number? Is every whole number a factor of itself? Explain.

11. Name each number in Ex. 8 that has only two whole-number factors, 1 and the number itself.

Any whole number greater than 1 that has only one pair of whole-number factors, 1 and the number itself, is called a prime number.

12. Which of the numbers named in Ex. 8 are prime numbers?

13. Is 1 a prime number? Is 2? 4? 5? Why is 0 not a prime number?

14. Which of the numbers named in Ex. 8 are not prime numbers?

Any whole number greater than 1 that is not a prime number is called a composite number.

15. Which whole numbers from 2 through 25 are prime numbers and which are composite numbers?

16. For each composite number you named in Ex. 15, list all the whole-number factors.

Common factors

1. The first set shown below is the set of whole-number factors of 12. The second set shown is the set of whole-number factors of 16. What numbers are members of both sets?

$\{1, 2, 3, 4, 6, 12\}$
$\{1, 2, 4, 8, 16\}$

Since 2 is a factor of 12 and a factor of 16, we say 2 is a **common factor** of 12 and 16. Name two other common factors of 12 and 16.

2. List the set of whole-number factors of 4. List the set of whole-number factors of 6. What numbers are members of both sets? Then what are the common factors of 4 and 6?

— Name all the common factors of each pair of numbers shown below.

	a	b	c
3.	8 and 12	15 and 24	10 and 12
4.	6 and 9	12 and 20	16 and 24
5.	6 and 8	16 and 32	12 and 18

6. In Ex. 3a, what numbers did you say are common factors of 8 and 12? Which of these numbers is the **greatest common factor** of 8 and 12?

Find the greatest common factor of each pair of numbers given in Ex. 3–5.

7. What is the greatest common factor of a pair of prime numbers? Explain.

8. Is 1 a common factor of every pair of whole numbers? Explain.

— Which of the following pairs of whole numbers have no common factor other than 1?

	a	b	c
9.	3 and 5	6 and 4	12 and 15
10.	6 and 21	7 and 15	9 and 6

11. Name the common factors of 4 and 8; of 6 and 18; of 5 and 25.

Name the greatest common factor of 4 and 8; of 6 and 18; of 5 and 25. In each, is the greatest common factor one of the numbers in the pair?

— For which of these pairs of numbers is the greatest common factor also one of the numbers in the pair?

12.	2 and 4	6 and 9	7 and 21
13.	4 and 6	3 and 12	9 and 18

★ Your discovery

Tell your discovery

Think of any pair of whole numbers where one of the numbers is a multiple of the other. What can you say about the greatest common factor of this pair of numbers?

Use your discovery

What is the greatest common factor of 25 and 100? of 7 and 28? of 13 and 39? of 42 and 84?

Naming fractional numbers in lowest terms

On page 115 you discovered that, if the numerator and the denominator shown by a fraction are divided by the same number, the fraction for the result is equivalent to the fraction for the original number.

— Solve each of these sentences.

	a	b
1.	$\frac{6 \div 2}{8 \div 2} = \frac{3}{n}$	$\frac{12 \div 3}{15 \div 3} = \frac{n}{5}$
2.	$\frac{8 \div n}{12 \div 4} = \frac{2}{3}$	$\frac{5 \div 5}{10 \div n} = \frac{1}{2}$
3.	$\frac{6 \div n}{9 \div n} = \frac{2}{3}$	$\frac{14 \div n}{21 \div n} = \frac{2}{3}$

4. Shown below are two sets of equivalent fractions. For each set, tell by what number you divide the numerator and denominator of each of the last four members to obtain the first member.

- $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}\right\}$
- $\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}\right\}$

5. The numerator and denominator are often called simply the **terms**. What common factors do the terms of $\frac{4}{8}$ have? What is the greatest common factor of 4 and 8?

Divide the numerator and denominator of $\frac{4}{8}$ by the greatest common factor of 4 and 8. What is the result? Are $\frac{1}{2}$ and $\frac{4}{8}$ equivalent fractions? Do the terms of $\frac{1}{2}$ have any common factor greater than 1? Then to name $\frac{4}{8}$ *in lowest terms*, we would write $\frac{1}{2}$.

6. What is the greatest common factor of the terms of $\frac{6}{9}$? Divide both terms of $\frac{6}{9}$ by this number. What is the result? Do $\frac{6}{9}$ and $\frac{2}{3}$ name the same number?

Do the terms of $\frac{2}{3}$ have any common factor greater than 1? Then $\frac{6}{9}$ named in lowest terms is $\frac{2}{3}$.

— For each fractional number named below, find the greatest common factor of the terms. Then divide the numerator and denominator by this common factor to name the fractional number in lowest terms.

	a	b	c	d	e
7.	$\frac{6}{12}$	$\frac{8}{10}$	$\frac{5}{15}$	$\frac{12}{24}$	$\frac{16}{24}$ 4
8.	$\frac{7}{14}$	$\frac{9}{15}$	$\frac{8}{16}$	$\frac{11}{22}$	$\frac{9}{27}$

9. Two fractions equivalent to $\frac{16}{12}$ are $\frac{8}{?}$ and $\frac{?}{3}$. Which of these three fractions names the number in lowest terms?

The mixed form for $\frac{16}{12}$ is $1\frac{?}{12}$. Do $1\frac{2}{6}$ and $1\frac{1}{3}$ also name this number? Explain.

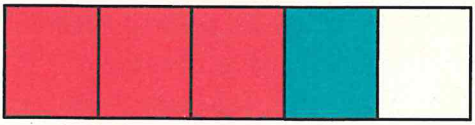
Which mixed form do you think names the number in lowest terms, $1\frac{4}{12}$, $1\frac{2}{6}$, or $1\frac{1}{3}$? Why?

You can name $\frac{16}{12}$ in lowest terms by writing either $\frac{4}{3}$ or $1\frac{1}{3}$.

— Name each of the following numbers in lowest terms two ways.

10.	$\frac{8}{6}$	$\frac{15}{10}$	$\frac{6}{4}$	$\frac{\overset{3}{24}}{16}$ 2	$\frac{18}{4}$
11.	$\frac{9}{6}$	$\frac{12}{8}$	$\frac{15}{6}$	$\frac{21}{14}$	$\frac{24}{15}$

Addition of fractional numbers



1. The rectangular region shown above has been separated into ? smaller regions of the same size and shape. What fractional number tells the part of the rectangular region each of the smaller regions is?

2. How many fifths of the rectangular region shown above are red? How many fifths are blue? How many fifths are either red or blue?

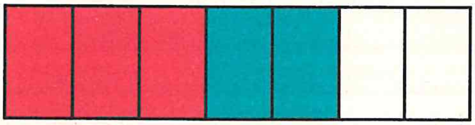
- Is the following sentence true?

  3 fifths + 1 fifth = 4 fifths

- Is this number sentence true?

  $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

3. What part of the rectangular region shown below is red? What part is blue?



What fraction can you use to replace n to make this number sentence true? $\frac{3}{7} + \frac{2}{7} = n$

— Use the two regions shown above to help you solve these sentences.

	a	b
4.	$\frac{2}{5} + \frac{2}{5} = n$	$\frac{1}{5} + \frac{2}{5} = n$
5.	$\frac{2}{7} + \frac{4}{7} = n$	$\frac{5}{7} + \frac{1}{7} = n$

6. Two or more fractions that show the same denominator, such as $\frac{2}{5}$, $\frac{1}{5}$, and $\frac{3}{5}$, are called **like fractions.** Two or more fractions that do not show the same denominator, such as $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{4}{7}$, are called **unlike fractions.**

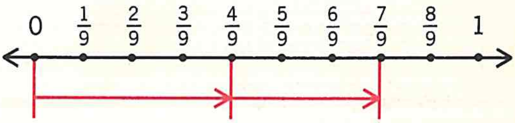
Which pairs below are like fractions? Which are unlike fractions?

$\frac{1}{3}, \frac{5}{3}$ $\quad$ $\frac{2}{7}, \frac{2}{9}$ $\quad$ $\frac{2}{5}, \frac{1}{10}$

$\frac{1}{2}, \frac{2}{4}$ $\quad$ $\frac{3}{4}, \frac{1}{4}$ $\quad$ $\frac{6}{12}, \frac{3}{12}$

7. If fractions are like fractions, they have a **common denominator.** Name the common denominator for each pair of addends in Ex. 4 and 5.

8. On the number line pictured below, the arrows show the addition of $\frac{4}{9}$ and $\frac{3}{9}$. Are $\frac{4}{9}$ and $\frac{3}{9}$ like fractions?



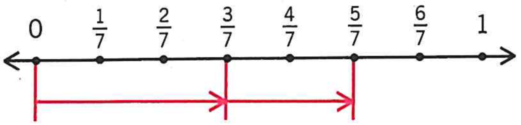
What fractional number is shown at the tip of the second arrow? What is the solution set for this sentence?

$\frac{4}{9} + \frac{3}{9} = n$

— Are the addends in the number sentences below named by like fractions? Draw a model of the number line to help you solve each sentence.

	a	b
9.	$\frac{3}{9} + \frac{5}{9} = n$	$\frac{2}{4} + \frac{1}{4} = n$
10.	$\frac{1}{5} + \frac{3}{5} = n$	$\frac{5}{8} + \frac{2}{8} = n$

Adding fractional numbers named by like fractions



1. The first arrow on the number line above shows a move of ? sevenths from 0. The second arrow shows a move of ? sevenths. What fraction names the point at the tip of the second arrow?

3 sevenths + 2 sevenths = ? sevenths

2. Are $\frac{3}{7}$ and $\frac{2}{7}$ like fractions? Why? Solve this sentence. $\frac{3}{7} + \frac{2}{7} = n$

3. Three ways to show the addition of $\frac{3}{7}$ and $\frac{2}{7}$ are given below. In each example, what is the sum?

A
3 sevenths
+ 2 sevenths
5 sevenths

B
$$\begin{array}{r} \frac{3}{7} \\ + \frac{2}{7} \\ \hline \frac{5}{7} \end{array}$$

C $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

- These examples show that to add $\frac{3}{7}$ and $\frac{2}{7}$, you need add only the ? to find the numerator of the sum. How does the denominator of the sum compare with the denominator of the addends?
- Write a number sentence similar to example C to show the addition of $\frac{5}{8}$ and $\frac{2}{8}$; $\frac{7}{10}$ and $\frac{2}{10}$; $\frac{2}{7}$ and $\frac{1}{7}$; $\frac{4}{9}$ and $\frac{1}{9}$.
- Tell how you add fractional numbers named by like fractions.

4. Suppose □, △, and ◇ represent any three counting numbers. Explain how the following sentence shows how to add fractional numbers named by like fractions.

$$\frac{\square}{\diamond} + \frac{\triangle}{\diamond} = \frac{\square + \triangle}{\diamond}$$

— Solve these number sentences.

	a	**b**
5.	$\frac{1}{4} + \frac{2}{4} = n$	$\frac{3}{10} + \frac{6}{10} = n$
6.	$\frac{3}{8} + \frac{2}{8} = n$	$\frac{5}{12} + \frac{6}{12} = n$
7.	$\frac{3}{14} + \frac{8}{14} = n$	$\frac{5}{16} + \frac{3}{16} = n$

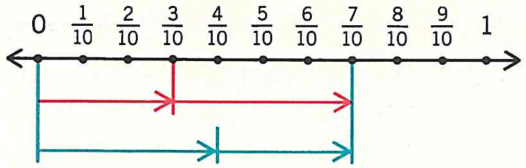
8. What is the sum of $\frac{2}{9}$ and $\frac{4}{9}$? Do the terms of your answer have any common factor greater than 1? Then to name the sum in lowest terms, would you write $\frac{6}{9}$ or $\frac{2}{3}$?

9. What is the sum of $\frac{5}{16}$ and $\frac{3}{16}$? To name your answer in lowest terms, would you write $\frac{8}{16}$, $\frac{4}{8}$, $\frac{2}{4}$, or $\frac{1}{2}$? Explain.

— Solve these number sentences. Name each answer in lowest terms.

10.	$\frac{1}{6} + \frac{2}{6} = n$	$\frac{5}{8} + \frac{1}{8} = n$
11.	$\frac{4}{9} + \frac{2}{9} = n$	$\frac{7}{5} + \frac{4}{5} = n$
12.	$\frac{1}{4} + \frac{1}{4} = n$	$\frac{5}{7} + \frac{2}{7} = n$
13.	$\frac{5}{8} + \frac{7}{8} = n$	$\frac{7}{12} + \frac{1}{12} = n$
14.	$\frac{3}{4} + \frac{1}{4} = n$	$\frac{2}{3} + \frac{4}{3} = n$

Properties of addition of fractional numbers



1. Explain how the red arrows show the solution set for this sentence. $\frac{3}{10} + \frac{4}{10} = n$ What is the solution set?

2. How do the blue arrows show the solution set for this sentence? $\frac{4}{10} + \frac{3}{10} = n$ What is the solution set?

3. Is this number sentence true?

$$\frac{3}{10} + \frac{4}{10} = \frac{4}{10} + \frac{3}{10}$$

4. What is the sum of $\frac{4}{5}$ and $\frac{3}{5}$? of $\frac{3}{5}$ and $\frac{4}{5}$? Is this sentence true?

$$\frac{4}{5} + \frac{3}{5} = \frac{3}{5} + \frac{4}{5}$$

— Solve these pairs of sentences.

5. $\frac{3}{8} + \frac{4}{8} = n$ $\quad\frac{4}{8} + \frac{3}{8} = n$

6. $\frac{1}{7} + \frac{3}{7} = n$ $\quad\frac{3}{7} + \frac{1}{7} = n$

7. $\frac{5}{4} + \frac{2}{4} = n$ $\quad\frac{2}{4} + \frac{5}{4} = n$

8. In Ex. 5–7, did it make any difference in what order you added?

If △, □, and ◇ represent any three counting numbers, is this sentence always true?

$$\frac{\triangle}{\diamond} + \frac{\square}{\diamond} = \frac{\square}{\diamond} + \frac{\triangle}{\diamond}$$

Does the commutative property of addition apply to fractional numbers?

9. What do the parentheses mean in the number sentence shown below?

$$\left(\frac{1}{7} + \frac{3}{7}\right) + \frac{2}{7} = n$$

What is the sum of $\frac{1}{7}$ and $\frac{3}{7}$? Add $\frac{2}{7}$ to your answer. What fraction is the correct replacement for n?

10. In the following sentence, what two numbers should you add first? Why?

$$\frac{1}{7} + \left(\frac{3}{7} + \frac{2}{7}\right) = n$$

What is $\frac{3}{7} + \frac{2}{7}$? What is the sum of this number and $\frac{1}{7}$? What fraction is the correct replacement for n?

11. In Ex. 9 and 10, did the same number make both sentences true? Is the following sentence true?

$$\left(\frac{1}{7} + \frac{3}{7}\right) + \frac{2}{7} = \frac{1}{7} + \left(\frac{3}{7} + \frac{2}{7}\right)$$

— Solve these pairs of sentences.

12. $\left(\frac{3}{9} + \frac{1}{9}\right) + \frac{4}{9} = n$ $\quad\frac{3}{9} + \left(\frac{1}{9} + \frac{4}{9}\right) = n$

13. $\left(\frac{1}{5} + \frac{2}{5}\right) + \frac{4}{5} = n$ $\quad\frac{1}{5} + \left(\frac{2}{5} + \frac{4}{5}\right) = n$

14. $\left(\frac{3}{8} + \frac{4}{8}\right) + \frac{6}{8} = n$ $\quad\frac{3}{8} + \left(\frac{4}{8} + \frac{6}{8}\right) = n$

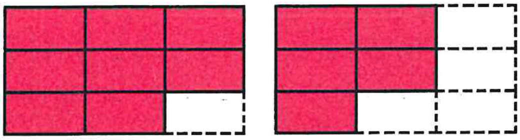
15. Does the associative property of addition apply to fractional numbers?

— Solve each of these sentences. Check by adding in a different order.

	a	b
16.	$\frac{3}{7} + \frac{1}{7} + \frac{2}{7} = n$	$\frac{1}{8} + \frac{3}{8} + \frac{7}{8} = n$
17.	$\frac{4}{5} + \frac{9}{5} + \frac{3}{5} = n$	$\frac{3}{11} + \frac{5}{11} + \frac{2}{11} = n$
18.	$\frac{4}{9} + \frac{7}{9} + \frac{1}{9} = n$	$\frac{7}{15} + \frac{3}{15} + \frac{2}{15} = n$

Subtracting fractional numbers named by like fractions

1. Amy's mother baked a cake and cut it into ninths. She ate one piece.

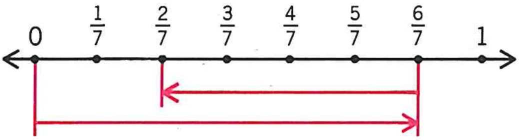


- The red part of the drawing at the left above represents the part of the cake remaining before Amy and her friends returned from school. There are ? ninths left.

- The red part of the drawing at the right above represents the part of the cake remaining after Amy and her friends ate some of it. Amy and her friends ate ? ninths of the cake. There are ? ninths left.

8 ninths − 3 ninths = ? ninths

2. The two arrows on this number line show the subtraction of $\frac{4}{7}$ from $\frac{6}{7}$.



What fractional number is shown at the tip of the top arrow? Is this sentence true? $\frac{6}{7} - \frac{4}{7} = \frac{2}{7}$

— Use the drawings above to help you solve these number sentences.

	a	b
3.	$\frac{5}{9} - \frac{2}{9} = n$	$\frac{8}{9} - \frac{4}{9} = n$
4.	$\frac{4}{7} - \frac{2}{7} = n$	$\frac{6}{7} - \frac{2}{7} = n$

5. Three ways to show the subtraction of $\frac{5}{9}$ from $\frac{7}{9}$ are given below.

A		B	
	7 ninths		$\frac{7}{9}$
	− 5 ninths		$-\frac{5}{9}$
	2 ninths		$\frac{2}{9}$

C $\frac{7}{9} - \frac{5}{9} = \frac{7-5}{9} = \frac{2}{9}$

- Study example C. How is the numerator of the answer related to the numerators shown by the original fractions? How is the denominator of the answer related to the denominators shown by the original fractions?

- Write a number sentence similar to example C to show the subtraction of $\frac{3}{5}$ from $\frac{4}{5}$; of $\frac{6}{11}$ from $\frac{10}{11}$.

- Tell how you subtract fractional numbers named by like fractions.

6. Suppose □, △, and ◇ represent any three counting numbers. Explain how the following sentence shows how to subtract fractional numbers named by like fractions.

$$\frac{\square}{\diamond} - \frac{\triangle}{\diamond} = \frac{\square - \triangle}{\diamond}$$

— Copy and solve these sentences.

	a	b
7.	$\frac{5}{6} - \frac{4}{6} = n$	$\frac{11}{15} - \frac{9}{15} = n$
8.	$\frac{3}{7} - \frac{2}{7} = n$	$\frac{10}{17} - \frac{4}{17} = n$
9.	$\frac{7}{5} - \frac{3}{5} = n$	$\frac{8}{3} - \frac{7}{3} = n$

Practice with fractional numbers

— Solve the following number sentences. Name each answer in lowest terms.

	a	b
1.	$\frac{2}{7}+\frac{4}{7}=n$	$\frac{5}{8}-\frac{2}{8}=n$
2.	$\frac{6}{5}-\frac{4}{5}=n$	$\frac{7}{3}+\frac{6}{3}=n$
3.	$\frac{6}{4}+\frac{5}{4}=n$	$\frac{11}{12}-\frac{5}{12}=n$
4.	$\frac{9}{2}-\frac{3}{2}=n$	$\frac{8}{9}+\frac{1}{9}=n$
5.	$\frac{3}{8}+\frac{9}{8}=n$	$\frac{12}{7}-\frac{12}{7}=n$

— Copy and compute. Name each answer in lowest terms.

	a	b	c	d
6.	$\begin{array}{r}\frac{6}{8}\\ -\frac{4}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{6}{7}\\ -\frac{2}{7}\\ \hline\end{array}$	$\begin{array}{r}\frac{5}{8}\\ +\frac{1}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\ +\frac{1}{4}\\ \hline\end{array}$
7.	$\begin{array}{r}\frac{3}{2}\\ -\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}\frac{9}{10}\\ -\frac{5}{10}\\ \hline\end{array}$	$\begin{array}{r}\frac{5}{6}\\ +\frac{1}{6}\\ \hline\end{array}$	$\begin{array}{r}\frac{4}{5}\\ +\frac{2}{5}\\ \hline\end{array}$
8.	$\begin{array}{r}\frac{7}{8}\\ -\frac{1}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{5}{3}\\ -\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}\frac{9}{2}\\ -\frac{3}{2}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{5}\\ -\frac{3}{5}\\ \hline\end{array}$
9.	$\begin{array}{r}\frac{1}{5}\\ \frac{2}{5}\\ +\frac{1}{5}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{8}\\ \frac{1}{8}\\ +\frac{3}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{2}{15}\\ \frac{7}{15}\\ +\frac{1}{15}\\ \hline\end{array}$	$\begin{array}{r}\frac{7}{16}\\ \frac{5}{16}\\ +\frac{2}{16}\\ \hline\end{array}$
10.	$\begin{array}{r}\frac{3}{7}\\ \frac{1}{7}\\ \frac{5}{7}\\ +\frac{2}{7}\\ \hline\end{array}$	$\begin{array}{r}\frac{2}{3}\\ \frac{4}{3}\\ \frac{1}{3}\\ +\frac{10}{3}\\ \hline\end{array}$	$\begin{array}{r}\frac{1}{8}\\ \frac{3}{8}\\ \frac{2}{8}\\ +\frac{1}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{5}\\ \frac{4}{5}\\ \frac{6}{5}\\ +\frac{2}{5}\\ \hline\end{array}$

11. Explain how you can use sentence **b** below to help you find the solution set for sentence **a**.

a. $3+n=8$ **b.** $8-3=n$

What is the solution set for each?

12. Will the solution set for sentence **b** below also be the solution set for sentence **a**? Why?

a. $\frac{2}{9}+n=\frac{7}{9}$ **b.** $\frac{7}{9}-\frac{2}{9}=n$

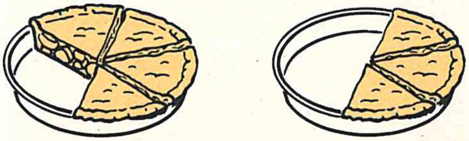
What is the solution set for each?

— Solve each of these sentences. Name all answers in lowest terms.

	a	b
13.	$\frac{3}{5}+n=\frac{4}{5}$	$n+\frac{4}{7}=\frac{6}{7}$
14.	$\frac{5}{9}-n=\frac{1}{9}$	$\frac{7}{8}-n=\frac{2}{8}$
15.	$n-\frac{3}{11}=\frac{2}{11}$	$n-\frac{4}{15}=\frac{7}{15}$
16.	$\frac{4}{9}+n=\frac{7}{9}$	$n+\frac{2}{3}=\frac{8}{3}$

17. Doris baked two cherry pies of the same size. She cut each pie into sixths and served her friends.

The picture below shows how many pieces of each pie she has left. Will all the pie fit on one pie pan? Explain.



If you need more practice, do Practice Set 6, page 325, and Practice Set 19, page 330.

Common multiples

1. Look at the set of numbers shown below.

{0, 3, 6, 9, 12, 15, 18, 21, 24, . . .}

Are the members of this set the multiples of 3? What is the next multiple of 3 after 24?

2. The set of numbers shown below is the set of multiples of what number?

{0, 4, 8, 12, 16, 20, 24, 28, . . .}

What is the next multiple of 4 after 28?

3. Shown below again are the set of multiples of 3 and the set of multiples of 4.

{0, 3, 6, 9, 12, 15, 18, 21, 24, . . .}
{0, 4, 8, 12, 16, 20, 24, 28, . . .}

- Is 12 a member of both of these sets? Then is 12 a multiple of both 3 and 4?
- What other numbers shown above are members of both sets? Then what other numbers are multiples of both 3 and 4?
- Extend the set of multiples of 3 to 39 and the set of multiples of 4 to 40. Study the results to find another number that is a multiple of both 3 and 4.
- Extend the two sets of multiples until you find two more numbers that are multiples of both 3 and 4.

4. The first set shown below is the set of multiples of 4. The second set is the set of multiples of ?.

{0, 4, 8, 12, 16, 20, 24, 28, . . .}
{0, 6, 12, 18, 24, 30, 36, 42, . . .}

- What numbers shown above are members of both sets? Then 0, 12, and ? are multiples of both 4 and 6.
- Extend the two sets of multiples until you find two more numbers that are multiples of both 4 and 6.

Suppose △ and □ stand for any two whole numbers. Any whole number that is a multiple of the two numbers represented by △ and □ is a common multiple of these numbers.

When we find the common multiples of two whole numbers, we shall think only of multiples other than 0.

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5. What is the product of 0 and any whole number? Then is 0 a multiple of every whole number?

6. Find three common multiples of 3 and 5 by listing the set of multiples of 3 and the set of multiples of 5. Explain why it is not necessary to list 0 in either set.

7. Find two common multiples of each pair of numbers shown below.

2 and 3
5 and 10
4 and 8
3 and 6
2 and 7
6 and 9

Least common multiples

{2, 4, 6, 8, 10, 12, 14, 16, 18, . . .}
{3, 6, 9, 12, 15, 18, 21, 24, . . .}

1. Use the sets shown above to find three common multiples of 2 and 3. Can you find another common multiple of 2 and 3 that is greater than 18? greater than 24? greater than 30?

Is there any common multiple that is less than 6? Then what is the **least common multiple** of 2 and 3?

2. Suppose you wish to find only the least common multiple of 3 and 5. Can you find it by listing in order the counting-number multiples of 3 until you find one that is also a multiple of 5?

{3, 6, 9, 12, 15}

Can you find the least common multiple of 3 and 5 by listing in order the counting-number multiples of 5 until you find one that is also a multiple of 3?

{5, 10, 15}

Which listing gives you the least common multiple more quickly?

3. To find the least common multiple of 4 and 6, list the counting-number multiples of 4 in order until you find one that is also a multiple of 6.

Now list the counting-number multiples of 6 in order until you find one that is also a multiple of 4.

Which listing gives you the least common multiple more quickly?

4. In Ex. 2 and 3, which gave you the least common multiple more quickly: listing the multiples of the smaller number or listing the multiples of the greater number?

5. Which is the greater number, 4 or 7? List in order the counting-number multiples of 7 until you find one that is also a multiple of 4. What is the least common multiple of 4 and 7?

6. Find the least common multiple of each pair of numbers shown below.

2 and 7 4 and 10 3 and 8

7. Which is the greater number, 3 or 9? Is 9 a multiple of 3? Then the least common multiple of 3 and 9 is ___?___.

8. Find the least common multiple of each pair of numbers shown below.

2 and 4 5 and 15 3 and 6

9. Tell how you would find the least common multiple of any two counting numbers.

— Find the least common multiple of each pair of numbers shown below.

	a	b	c
10.	2 and 5	4 and 8	8 and 12
11.	3 and 10	2 and 12	10 and 15
12.	9 and 6	16 and 8	15 and 6
13.	3 and 7	12 and 3	9 and 27
14.	5 and 4	6 and 24	16 and 12

Naming fractional numbers with like fractions

1. Suppose you wish to find the sum of $\frac{1}{6}$ and $\frac{5}{12}$. Are $\frac{1}{6}$ and $\frac{5}{12}$ like fractions? Does the pattern you use for adding fractional numbers work only with like fractions? Then to add $\frac{1}{6}$ and $\frac{5}{12}$, you must find a way to name $\frac{1}{6}$ and $\frac{5}{12}$ with like fractions.

- Look at the denominators shown in $\frac{1}{6}$ and $\frac{5}{12}$. Is either denominator a multiple of the other? Explain.
- Can you name $\frac{1}{6}$ with a fraction showing a denominator of 12?

$$\frac{1}{6} = \frac{2 \times 1}{2 \times 6} = \frac{?}{12}$$

- Are $\frac{1}{6}$ and $\frac{2}{12}$ equivalent? Then is the following sentence true?

$$\frac{1}{6} + \frac{5}{12} = \frac{2}{12} + \frac{5}{12}$$

2. To find the difference between $\frac{3}{8}$ and $\frac{1}{4}$, you can name these fractional numbers with like fractions.

- Look at the denominators shown in $\frac{3}{8}$ and $\frac{1}{4}$. Is one denominator a multiple of the other? Explain.
- Write a fraction showing a denominator of 8 that is equivalent to $\frac{1}{4}$. $\frac{1}{4} = \frac{?}{8}$
- Copy and solve this sentence.

$$\frac{3}{8} - \frac{1}{4} = \frac{3}{8} - \frac{n}{8}$$

— Copy and solve these sentences.

3. $\frac{2}{3} + \frac{1}{6} = \frac{n}{6} + \frac{1}{6}$

4. $\frac{3}{4} - \frac{1}{8} = \frac{n}{8} - \frac{1}{8}$

5. $\frac{3}{10} - \frac{1}{5} = \frac{3}{10} - \frac{n}{10}$

6. To find an equivalent fraction in higher terms, you ? both terms shown by a fraction by the same number.

— For each sentence below, tell the numerator that makes the sentence true.

	a	b	c
7.	$\frac{1}{4} = \frac{n}{12}$	$\frac{3}{4} = \frac{n}{8}$	$\frac{2}{5} = \frac{n}{10}$
8.	$\frac{1}{3} = \frac{n}{9}$	$\frac{5}{6} = \frac{n}{12}$	$\frac{7}{8} = \frac{n}{24}$

— To rename unlike fractions as like fractions, you must first find a common denominator. Tell what common denominator you would use to name each pair of fractional numbers shown below using like fractions.

9.	$\frac{5}{8}$ and $\frac{7}{16}$	$\frac{2}{3}$ and $\frac{5}{9}$	$\frac{1}{4}$ and $\frac{3}{8}$
10.	$\frac{1}{2}$ and $\frac{9}{10}$	$\frac{4}{5}$ and $\frac{7}{15}$	$\frac{3}{4}$ and $\frac{5}{12}$

11. Name each pair of fractional numbers shown in Ex. 9 and 10 using like fractions.

— Copy each sentence below. Replace n with a fraction so that both fractions at the right of the equals sign are like fractions and the sentence is true.

12. $\frac{1}{2} + \frac{3}{4} = n + \frac{3}{4}$

13. $\frac{2}{3} - \frac{1}{6} = n - \frac{1}{6}$

14. $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + n$

15. $\frac{5}{9} - \frac{1}{3} = \frac{5}{9} - n$

16. $\frac{1}{4} + \frac{5}{12} = n + \frac{5}{12}$

Finding the least common denominator

1. Suppose you wish to find the sum of $\frac{1}{3}$ and $\frac{2}{5}$. Will it be easier if you first name these numbers using like fractions?

- Look at the denominators shown in $\frac{1}{3}$ and $\frac{2}{5}$. Is either denominator a multiple of the other?
- What is the product of the two denominators? Is 15 a common multiple of 3 and 5? Then you can use 15 as a common denominator to name $\frac{1}{3}$ and $\frac{2}{5}$ with like fractions.

$$\frac{1}{3} = \frac{?}{15} \qquad \frac{2}{5} = \frac{?}{15}$$

- Is this number sentence true?

$$\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15}$$

2. To find the difference between $\frac{1}{4}$ and $\frac{1}{6}$, it is easier if you name both numbers using $\underline{\ ?\ }$ fractions.

- Look at the denominators shown in $\frac{1}{4}$ and $\frac{1}{6}$. Is either denominator a multiple of the other?
- Is the product of 4 and 6 a common multiple of 4 and 6? Then can you use 24 as a common denominator to name $\frac{1}{4}$ and $\frac{1}{6}$ using like fractions? Do it.
- Is 24 the *least* common multiple of 4 and 6? What number is? Then what number is the **least common denominator** for $\frac{1}{4}$ and $\frac{1}{6}$?

$$\frac{1}{4} = \frac{?}{12} \qquad \frac{1}{6} = \frac{?}{12}$$

- Is this number sentence true?

$$\frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12}$$

3. Explain how finding the least common denominator for $\frac{1}{4}$ and $\frac{1}{6}$ is the same as finding the least common multiple of 4 and 6.

4. What is the least common multiple of 6 and 9? Then what is the least common denominator for $\frac{5}{6}$ and $\frac{4}{9}$?

5. Use the answer you found for Ex. 4 to help you name $\frac{5}{6}$ and $\frac{4}{9}$ using like fractions.

— Name each pair of fractional numbers shown below using like fractions.

	a	b	c
6.	$\frac{1}{2}$ and $\frac{2}{3}$	$\frac{3}{4}$ and $\frac{1}{6}$	$\frac{5}{8}$ and $\frac{1}{12}$
7.	$\frac{1}{5}$ and $\frac{3}{4}$	$\frac{3}{10}$ and $\frac{1}{15}$	$\frac{2}{3}$ and $\frac{5}{6}$
8.	$\frac{3}{8}$ and $\frac{1}{6}$	$\frac{5}{6}$ and $\frac{1}{10}$	$\frac{3}{8}$ and $\frac{7}{16}$

9. Which fraction in the following set shows the greatest denominator? $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{8}\}$ Is 8 a multiple of 2 and of 3? Double 8. Is 16 a multiple of 2 and of 3? Triple 8. Is 24 a multiple of 2 and of 3? Then $\underline{\ ?\ }$ is the least common denominator of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{8}$.

10. Find the least common denominator for the following sets of fractional numbers.

$\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}\}$ $\qquad$ $\{\frac{1}{2}, \frac{1}{5}, \frac{1}{10}\}$ $\qquad$ $\{\frac{2}{3}, \frac{1}{6}, \frac{5}{9}\}$

Another look at adding fractional numbers

1. Below is one way to show the addition of $\frac{1}{6}$ and $\frac{1}{4}$.

$$\boxed{\frac{1}{6} + \frac{1}{4} = \frac{2}{12} + \frac{3}{12} = \frac{2+3}{12} = \frac{5}{12}}$$

- What common denominator was used to name $\frac{1}{6}$ and $\frac{1}{4}$ using like fractions?
- What is the sum of $\frac{1}{6}$ and $\frac{1}{4}$?

2. At the right is another way to show the addition of $\frac{1}{6}$ and $\frac{1}{4}$.

$$\begin{array}{rcr} \frac{1}{6} & = & \frac{2}{12} \\ +\frac{1}{4} & = & \frac{3}{12} \\ \hline & & \frac{5}{12} \end{array}$$

- How are the numerators of $\frac{2}{12}$ and $\frac{3}{12}$ related to the numerator of the sum?
- How are the denominators of $\frac{2}{12}$ and $\frac{3}{12}$ related to the denominator of the sum?

3. To find the sum of $\frac{3}{8}$ and $\frac{1}{4}$, it is easier if you first express $\frac{3}{8}$ and $\frac{1}{4}$ as $\underline{\ ?\ }$ fractions.

$$\begin{array}{rcr} \frac{3}{8} & = & \frac{?}{8} \\ +\frac{1}{4} & = & \frac{?}{8} \\ \hline & & \frac{?}{8} \end{array}$$

- What is the least common denominator of $\frac{3}{8}$ and $\frac{1}{4}$? Was this number used in the incomplete example shown above? What fraction shows a denominator of 8 and is equivalent to $\frac{1}{4}$?
- Is the sum of $\frac{3}{8}$ and $\frac{2}{8}$ equal to the sum of $\frac{3}{8}$ and $\frac{1}{4}$? Why?
- Copy and complete the example. What is the sum of $\frac{3}{8}$ and $\frac{1}{4}$?

4. Suppose you wish to find the sum of $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{4}{9}$. Can you add these numbers in any order that you wish? Why?

- What is the greatest denominator in $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{4}{9}$? Is 9 a multiple of 6 and of 3? Double 9. Is 18 a multiple of 6 and of 3? Then what is the least common denominator for $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{4}{9}$?

$$\begin{array}{rcr} \frac{1}{3} & = & \frac{?}{18} \\ \frac{1}{6} & = & \frac{?}{18} \\ +\frac{4}{9} & = & \frac{?}{18} \\ \hline & & \frac{?}{18} \end{array}$$

- Copy and complete the example above to find the sum of $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{4}{9}$.

— Copy the following examples and find the sums. Name all sums in lowest terms.

	a	b	c	d
5.	$\begin{array}{r}\frac{3}{8}\\+\frac{1}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{4}{7}\\+\frac{2}{7}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{4}\\+\frac{1}{2}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{3}\\+\frac{2}{5}\\\hline\end{array}$
6.	$\begin{array}{r}\frac{1}{2}\\\frac{1}{4}\\+\frac{1}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{3}\\\frac{1}{2}\\+\frac{1}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{5}\\\frac{3}{8}\\+\frac{1}{4}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{8}\\\frac{3}{4}\\+\frac{1}{12}\\\hline\end{array}$

— Copy and solve the following number sentences.

40

	a	b
7.	$\frac{1}{2} + \frac{1}{4} = n$	$\frac{3}{4} + \frac{1}{8} = n$
8.	$\frac{2}{3} + \frac{1}{9} = n$	$\frac{1}{6} + \frac{4}{9} = n$
9.	$\frac{3}{8} + \frac{1}{6} = n$	$\frac{5}{16} + \frac{3}{8} = n$

Using the least common denominator

1. To find the sum of $\frac{1}{6}$ and $\frac{1}{4}$, Bill and Betty wrote the examples shown at the right.

- Look at Bill's example. To find the sum of $\frac{1}{6}$ and $\frac{1}{4}$, Bill used _?_ as a common denominator. His first answer was $\frac{10}{24}$. Is $\frac{10}{24}$ in lowest terms? Explain how Bill got $\frac{5}{12}$ as his final answer.
- What number did Betty use as a common denominator? When she added, what was the first answer she got? Is $\frac{5}{12}$ named in lowest terms?
- Which is the least common denominator for $\frac{1}{6}$ and $\frac{1}{4}$, 24 or 12? Who used this number as a common denominator in his work, Bill or Betty? Whose example has fewer steps?

BILL

$$\begin{array}{rcl} \frac{1}{6} & = & \frac{4}{24} \\ +\frac{1}{4} & = & \frac{6}{24} \\ \hline & & \frac{10}{24} = \frac{5}{12} \end{array}$$

BETTY

$$\begin{array}{rcl} \frac{1}{6} & = & \frac{2}{12} \\ +\frac{1}{4} & = & \frac{3}{12} \\ \hline & & \frac{5}{12} \end{array}$$

It is often easier to find the sum of fractional numbers if you use the *least* common denominator to name the addends using like fractions. Explain why.

— Copy and find the sums. Remember to name all answers in lowest terms.

	a	b	c	d	e	f	g	h	i
2.	$\frac{1}{2} + \frac{1}{5}$	$\frac{1}{2} + \frac{1}{3}$	$\frac{1}{3} + \frac{1}{4}$	$\frac{2}{3} + \frac{1}{6}$	$\frac{1}{2} + \frac{3}{8}$	$\frac{3}{4} + \frac{1}{8}$	$\frac{2}{3} + \frac{1}{2}$	$\frac{3}{5} + \frac{1}{2}$	$\frac{1}{8} + \frac{3}{5}$
3.	$\frac{3}{4} + \frac{2}{3}$ ✓	$\frac{1}{7} + \frac{5}{14}$ ✓	$\frac{3}{8} + \frac{5}{6}$ ✓	$\frac{2}{5} + \frac{3}{4}$ ✓	$\frac{1}{7} + \frac{2}{3}$ ✓	$\frac{5}{8} + \frac{3}{16}$ ✓	$\frac{3}{4} + \frac{4}{5}$ ✓	$\frac{1}{7} + \frac{1}{8}$ ✓	$\frac{1}{6} + \frac{2}{5}$ ✓

— Solve the following sentences.

	a	b	c	d	e
4.	$\frac{3}{4} + \frac{1}{3} = n$	$\frac{5}{12} + \frac{1}{6} = n$	$\frac{1}{6} + \frac{1}{3} = n$	$\frac{1}{2} + \frac{1}{5} = n$	$\frac{2}{3} + \frac{3}{4} = n$
5.	$\frac{2}{3} + \frac{5}{12} = n$	$\frac{3}{8} + \frac{1}{6} = n$	$\frac{1}{4} + \frac{3}{8} = n$	$\frac{3}{7} + \frac{1}{3} = n$	$\frac{2}{15} + \frac{3}{10} = n$
6.	$\frac{3}{8} + \frac{1}{3} = n$	$\frac{2}{3} + \frac{5}{8} = n$	$\frac{1}{6} + \frac{1}{4} = n$	$\frac{2}{5} + \frac{1}{5} = n$	$\frac{1}{12} + \frac{5}{9} = n$
7.	$\frac{2}{7} + \frac{1}{4} = n$	$\frac{3}{4} + \frac{5}{12} = n$	$\frac{1}{2} + \frac{4}{5} = n$	$\frac{5}{6} + \frac{1}{8} = n$	$\frac{7}{10} + \frac{2}{5} = n$

Another look at subtracting fractional numbers

1. If □ stands for a whole number greater than the whole number △ stands for, is this number sentence true?

$$\frac{\square}{9} - \frac{\triangle}{9} = \frac{\square - \triangle}{9}$$

2. Explain how you subtract fractional numbers when they are named using like fractions.

3. Suppose you wish to solve this number sentence. $\frac{7}{9} - \frac{2}{3} = n$

- Are $\frac{7}{9}$ and $\frac{2}{3}$ like fractions? Is the denominator of $\frac{7}{9}$ a multiple of the denominator of $\frac{2}{3}$? Write a fraction showing a denominator of 9 that is equivalent to $\frac{2}{3}$.
- Is this number sentence true?

$$\frac{7}{9} - \frac{2}{3} = \frac{7}{9} - \frac{6}{9}$$

- Solve this sentence. $\frac{7}{9} - \frac{6}{9} = n$ Now what is the solution set for this sentence? $\frac{7}{9} - \frac{2}{3} = n$

4. To solve sentence **a** below, is it easier if you first name $\frac{3}{4}$ and $\frac{1}{3}$ using like fractions? Has this been done in sentence **b**?

a. $\frac{3}{4} - \frac{1}{3} = n$ **b.** $\frac{9}{12} - \frac{4}{12} = n$

What is the solution set for sentence **b**? Then what is the solution set for sentence **a**?

— Solve these number sentences.

	a	b
5.	$\frac{3}{5} - \frac{1}{5} = n$	$\frac{5}{8} - \frac{1}{4} = n$
6.	$\frac{2}{3} - \frac{1}{2} = n$	$\frac{5}{6} - \frac{3}{4} = n$

7. At the right is another way to show the subtraction of fractional numbers.

$$\begin{array}{rcr} \frac{3}{4} & = & \frac{9}{12} \\ -\frac{1}{6} & = & \frac{2}{12} \\ \hline & & \frac{7}{12} \end{array}$$

- What is the least common denominator for $\frac{3}{4}$ and $\frac{1}{6}$? Explain how the example shows $\frac{3}{4}$ and $\frac{1}{6}$ named using like fractions.
- Is the answer shown in the example named in lowest terms? What is the difference between $\frac{3}{4}$ and $\frac{1}{6}$?

8. What is the least common denominator for $\frac{7}{8}$ and $\frac{3}{4}$?

To find what $\frac{7}{8} - \frac{3}{4}$ is, copy and complete the example.

$$\begin{array}{rcr} \frac{7}{8} & = & \frac{?}{8} \\ -\frac{3}{4} & = & \frac{?}{8} \\ \hline & & \frac{?}{8} \end{array}$$

— Copy and find the answers.

	a	b	c	d
9.	$\begin{array}{r}\frac{7}{8}\\ -\frac{1}{4}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\ -\frac{5}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{11}{12}\\ -\frac{1}{6}\\ \hline\end{array}$	$\begin{array}{r}\frac{7}{8}\\ -\frac{2}{3}\\ \hline\end{array}$
10.	$\begin{array}{r}\frac{11}{15}\\ -\frac{3}{10}\\ \hline\end{array}$	$\begin{array}{r}\frac{5}{8}\\ -\frac{1}{6}\\ \hline\end{array}$	$\begin{array}{r}\frac{2}{3}\\ -\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}\frac{5}{6}\\ -\frac{1}{9}\\ \hline\end{array}$

11. Find the difference between the numbers named in each pair below.

$\frac{3}{5}, \frac{1}{7}$ $\frac{3}{4}, \frac{7}{8}$ $\frac{2}{5}, \frac{3}{10}$ $\frac{1}{12}, \frac{1}{6}$

— Solve the following sentences.

	a	b
12.	$\frac{11}{12} - \frac{3}{4} = n$	$\frac{3}{5} - \frac{3}{10} = n$
13.	$\frac{5}{9} - \frac{1}{6} = n$	$\frac{2}{3} - \frac{1}{4} = n$

Adding and subtracting fractional numbers

1. When a symbol for a number, such as $\square$, is used twice in the same sentence as shown below, what does it mean?

$\frac{1}{2}$ $\frac{1}{4}$

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{\square} + \frac{1}{\square}$$

What number does $\square$ stand for in the sentence above to make it a true sentence?

— Copy each number sentence below and replace the frames with numerals for the numbers that make the sentence true.

2. $\frac{2}{3} + \frac{1}{5} = \frac{\square}{15} + \frac{\triangle}{15}$

3. $\frac{4}{5} - \frac{1}{2} = \frac{8}{\square} - \frac{5}{\square}$

4. $\frac{3}{4} + \frac{2}{3} = \frac{\triangle}{\square} + \frac{8}{\square}$

5. $\frac{2}{3} - \frac{1}{4} = \frac{8}{\square} - \frac{3}{\square}$

6. $\frac{3}{4} - \frac{1}{5} = \frac{15}{\triangle} - \frac{\square}{\triangle}$

— Copy and find the sums. Be sure to name all answers in lowest terms.

	a	b	c	d
7.	$\frac{1}{3}$ $\frac{3}{4}$ $+\frac{1}{2}$	$\frac{1}{6}$ $\frac{1}{2}$ $+\frac{3}{4}$	$\frac{5}{8}$ $\frac{2}{3}$ $+\frac{1}{4}$	$\frac{1}{2}$ $\frac{2}{3}$ $+\frac{3}{4}$
8.	$\frac{1}{5}$ $\frac{2}{3}$ $+\frac{2}{3}$	$\frac{2}{3}$ $\frac{3}{4}$ $+\frac{1}{6}$	$\frac{1}{3}$ $\frac{3}{4}$ $+\frac{1}{3}$	$\frac{1}{4}$ $\frac{1}{2}$ $+\frac{1}{5}$

— Copy the examples below and compute.

	a	b	c	d
9.	$\frac{3}{4}$ $-\frac{2}{3}$	$\frac{3}{5}$ $-\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{3}$	$\frac{7}{8}$ $-\frac{2}{3}$
10.	$\frac{1}{2}$ $-\frac{2}{9}$	$\frac{5}{6}$ $-\frac{1}{4}$	$\frac{4}{5}$ $-\frac{1}{3}$	$\frac{5}{7}$ $-\frac{1}{2}$

— Solve the following number sentences.

	a	b
11.	$\frac{2}{5} + \frac{1}{2} = n$	$\frac{5}{7} - \frac{1}{2} = n$
12.	$\frac{7}{8} - \frac{1}{3} = n$	$\frac{3}{4} + \frac{1}{5} = n$
13.	$\frac{3}{4} - \frac{1}{3} = n$	$\frac{4}{5} + \frac{2}{3} = n$
14.	$\frac{1}{3} + \frac{5}{8} = n$	$\frac{8}{9} - \frac{1}{2} = n$
15.	$\frac{1}{4} + n = \frac{2}{3}$	$n + \frac{3}{5} = \frac{3}{4}$
16.	$\frac{1}{2} + n = \frac{7}{8}$	$n + \frac{5}{6} = \frac{11}{12}$
17.	$\frac{2}{7} + n = \frac{3}{4}$	$n + \frac{1}{10} = \frac{2}{3}$

Think twice!

18. Mary cut an apple pie into four equal parts and gave one piece to her father. She cut each of the remaining pieces in half and gave one of these smaller pieces to her brother. What part of the whole pie did Mary give to her father and brother?

If you need more practice, do Practice Set 7, page 326, and Practice Set 20, page 330.

Keeping up in mathematics

Oral

1. Round 806,608 to the nearest thousand.

2. What is the greatest whole number of 8's in 39?

3. What is the greatest number named by a two-digit numeral?

4. What is the greatest remainder you can have with a divisor of 37?

5. From the set of whole numbers, tell the solution set for this sentence.

$$\triangle \times 40 \leq 169$$

6. A good estimate for the sum of 298 and 105 is $\underline{?}$.

7. Name using a mixed form.

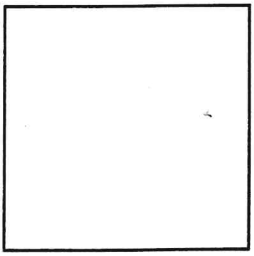
$\frac{17}{5}$ $\frac{13}{4}$ $\frac{13}{3}$ $\frac{25}{8}$ $\frac{27}{4}$

8. Which of the following fractions are names for whole numbers?

$\frac{3}{5}$ $\frac{7}{14}$ $\frac{12}{3}$ $\frac{5}{9}$ $\frac{8}{8}$ $\frac{6}{5}$ $\frac{0}{4}$

9. If you owe 31¢ and give the clerk a half dollar, how much change should you receive?

10. What is the perimeter of this square?



Written

— Copy and compute.

	a	b	c
1.	86 54 + 90	685 708 + 94	\$36.52 47.08 + 1.95
2.	862 − 348	\$19.41 − 8.07	29,736 − 18,659
3.	63 × 27	506 × 78	787 × 90
4.	1,000 + 780	2,000 − 608	1,090 × 305
5.	\$12.00 − 5.50	407 × 256	\$5.98 + 3.00
6.	$8\overline{)5,959}$	$9\overline{)6,820}$	$7\overline{)31,842}$
7.	$62\overline{)198}$	$54\overline{)380}$	$21\overline{)1,302}$
8.	$81\overline{)2,387}$	$56\overline{)841}$	$51\overline{)3,714}$

9. An airplane due to arrive at 2:47 P.M. arrived at 3:25 P.M. How late was it?

10. At 84¢ a pound, find the cost of $\frac{3}{4}$ pound of lunch meat.

— Solve these sentences.

	a	b
11.	$n + 19 = 105$	$n - 65 = 98$
12.	$12 \times n = 156$	$n \div 8 = 72$

Using letters in open sentences

You have often used a symbol such as □, △, or n in open sentences to represent any number from a set of numbers. As you continue your study of mathematics, you will find that most open sentences show a letter of the alphabet to represent a number.

— In the sentences shown below, each letter may be replaced by a name for any whole number. Find the solution set for each sentence.

	a	b
1.	$13 + 7 = b$	$27 - 3 = s$
2.	$14 = c + c$	$2 \times 6 > f$
3.	$36 \div 9 < x$	$6 \times y \leq 42$
4.	$18 \div w = 7$	$18 + m < 12$

5. Suppose a and b represent any two whole numbers. Is the expression shown in the box above a number sentence? Explain.

$$\boxed{a + b}$$

What number is represented by $a + b$ if a is replaced by 3 and b is replaced by 5? if a is replaced by 24 and b by 38?

— If m is replaced by 24 and n is replaced by 192, what number is represented by each expression below?

	a	b	c
6.	$m + n$	$m \times n$	$n - m$
7.	$m + 65$	$n \div m$	$6 \times m$
8.	$n \times 12$	$m - 8$	$n - 12$

— If a is replaced by 6 and b is replaced by 8, what number is represented by each of the following expressions?

	a	b	c
9.	$\frac{a}{b}$	$\frac{b}{a}$	$\frac{4 \times a}{4 \times b}$
10.	$\frac{6 \times b}{a}$	$\frac{a + b}{2}$	$\frac{b - a}{5}$
11.	$\frac{a \times b}{b \times a}$	$\frac{a + b}{b + a}$	$\frac{a + b}{b - a}$

— If p is replaced by 7 and q is replaced by 4, which of the following number sentences will be true?

	a	b
12.	$p - q = 3$	$(p + q) + 6 < 15$
13.	$p + q > 25$	$(3 \times q) - p = 6$
14.	$2 \times q < p$	$4 \times p = 7 \times q$

— If x is replaced by 12 in each of these sentences, find the solution set for y from the set of whole numbers.

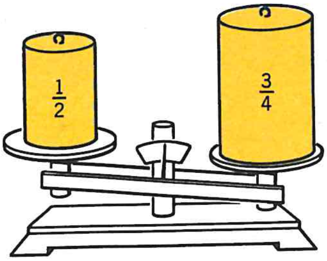
15.	$x + y = 24$	$y - x = 17$
16.	$x \div y = 4$	$y \times x > 30$
17.	$x \div 2 \leq y$	$y \times 2 = x$
18.	$x \div y < 1$	$x \div y > 1$
19.	$x + x = y$	$y + y = x$

20. In Ex. 15–19, if x is replaced by 5, what is the solution set for y from the set of whole numbers?

If x is replaced by 2, what is the solution set for y?

Scales and number sentences

The pupils in Miss Hunter's class are experimenting with a set of scales. They are using the results of their experiments to write number sentences.



1. Look at the scales shown in the picture above. What number of pounds is shown on each dish? Do the scales balance? On which dish should you put another weight to make the scales balance?

2. To find what weight to put on the left-hand dish, Daniel thought: "$\frac{1}{2}$ pound with some other weight will make $\frac{3}{4}$ pound and will balance the scales." He wrote this sentence.

$$\frac{1}{2} + w = \frac{3}{4}$$

Solve the sentence.

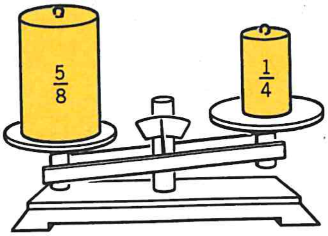
3. Susan thought, "If I take away $\frac{1}{2}$ pound from $\frac{3}{4}$ pound, the result will be the weight that is needed to balance the scales." She wrote this sentence.

$$\frac{3}{4} - \frac{1}{2} = w$$

Solve the sentence.

4. Did you find that the sentences in Ex. 2 and 3 have the same solution set? Are the sentences related? Explain how.

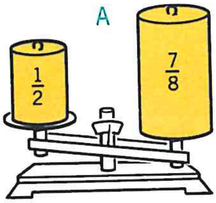
5. Look at the scales below.



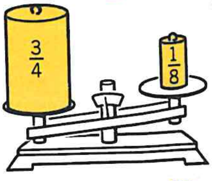
- Why don't the scales balance?
- On which dish should you place another weight to make the scales balance?
- Write a number sentence that can be used to find the weight needed to balance the scales. Solve.

— For each set of scales shown below, write a number sentence that can be used to find the weight needed to balance the scales. Then solve.

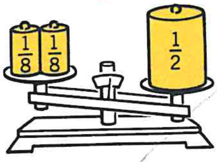
A

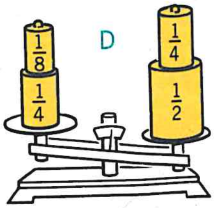


B



C

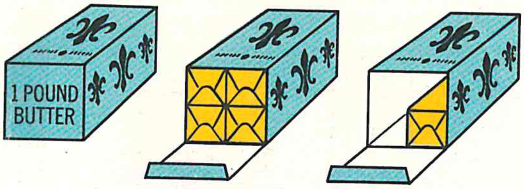




Subtracting numbers less than 1 from counting numbers

1. How much of 1 pound of butter is left after $\frac{3}{4}$ pound is used?

Use the illustration below to help you subtract $\frac{3}{4}$ from 1.



How many fourths equal one whole? Then, 1 pound is the same as $\frac{?}{4}$ pound. $\frac{4}{4} - \frac{3}{4} = \underline{\ ?\ }$

2. How much cake is left from 2 cakes after $\frac{1}{3}$ of a cake is eaten?

- Two cakes is the same as $\frac{?}{3}$ cakes.
- What is $\frac{6}{3} - \frac{1}{3}$? Name $\frac{5}{3}$ using a mixed form.
- Are both sentences below true?

$\frac{6}{3} - \frac{1}{3} = \frac{5}{3}$ $\qquad$ $\frac{6}{3} - \frac{1}{3} = 1\frac{2}{3}$

3. Suppose you wish to subtract $\frac{7}{8}$ from 5.

- Name 5 using a fraction with a denominator of 8. $5 = \frac{?}{8}$
- Use the pattern for subtracting fractional numbers named by like fractions. $\frac{40}{8} - \frac{7}{8} = \frac{40 - 7}{8} = \underline{\ ?\ }$

4. Another way to show the subtraction of $\frac{7}{8}$ from 5 is given at the right. Explain it.

$$\boxed{\begin{array}{rcl} 5 & = & \frac{40}{8} \\ -\frac{7}{8} & = & \frac{7}{8} \\ \hline & & \frac{33}{8} \end{array}}$$

Name $\frac{33}{8}$ using a mixed form. $5 - \frac{7}{8} = \frac{33}{8}$, or $\underline{\ ?\ }$.

5. Subtract each of the fractional numbers named below from 1; from 2; from 3; from 5.

$\frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{5} \quad \frac{7}{8} \quad \frac{5}{6} \quad \frac{7}{10}$

— Use a mixed form to name your answer to each of the following problems.

6. If you cut $\frac{3}{4}$ yard of rope from a 4-yard piece, how many yards of rope will you have left?

7. Sally bought a 5-pound bag of sugar. She used $\frac{2}{3}$ pound of it. How much sugar was left?

8. Fred is in school for 3 hours each morning. A quarter hour of that time is recess time. How many hours of work time are there?

— Solve these sentences. Use a mixed form to name each answer.

	a	b
9.	$1 - \frac{3}{7} = n$	$2 - \frac{1}{2} = n$
10.	$3 - \frac{7}{8} = n$	$6 - \frac{1}{6} = n$
11.	$8 - \frac{2}{3} = n$	$7 - \frac{4}{5} = n$

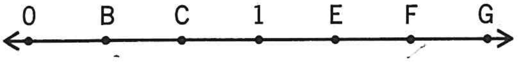
Think twice!

12. The boys in a scout troop bought 6 quarts of milk for a campfire supper. If they use $2\frac{3}{4}$ quarts of milk to make soup, how much milk will they have left?

If you need more practice, do Practice Set 21, page 330.

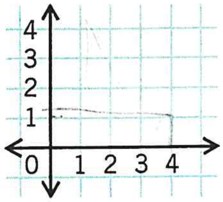
SELF-HELP TEST 7

1. On the number line shown below, what number is matched with point *B*? point *F*? *G*? **[119–21]**



2. Draw a graph similar to the one below. Then locate and label the following points.

A (1, 2) *B* (1, 4) *C* (3, 2) *D* (3, 4)



Show a line segment to connect point *A* to point *B*, *B* to *C*, and *C* to *D*. What letter of the alphabet is formed by these line segments? **[174–75]**

3. Find the average of the numbers in each set shown below. **[151]**

{3, 7, 11, 15} {47, 19, 12}

4. If *P* stands for the perimeter, *l* for the length, and *w* for the width of any rectangle, what is the formula you can use to find the perimeter of the rectangle? **[157]**

5. Which members of the set shown below name the quotient for 15 ÷ 4?

$\{\frac{4}{15}, \frac{15}{4}, 3\frac{3}{4}, 4\frac{1}{4}\}$ **[131]**

6. Is the quotient for 2,465 ÷ 66 30-some, 40-some, or 50-some? **[143]**

7. Is the quotient for 1,294 ÷ 97 10-some, 100-some, or 1,000-some? **[140]**

— Copy and complete. **[160–61]**

8. 153 cm make 1 m ? dm 3 cm

9. ? cm make 1 m

10. 20 cm make ? dm

SELF-HELP TEST 8

— Copy, find the answers, and check.

1. 7,263 + 9,158 **[25]**

2. 2,512 − 1,914 **[33]**

3. 857 × 472 **[55]**

4. $7\overline{)1,527}$ **[104]**

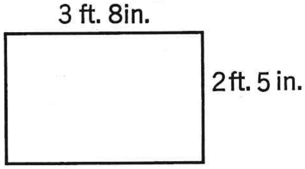
5. $27\overline{)347}$ **[140]**

6. $15\overline{)795}$ **[143]**

7. $66\overline{)1,284}$ **[143]**

8. $73\overline{)5,029}$ **[138]**

9. Find the perimeter of the rectangle shown below. **[157]**



10. One side of a square is 12 yards long. Find the area of the square region. **[168]**

17
32

IT'S CHECK-UP TIME

1. Which numbers named below are prime numbers?

3 8 12 19 21
11 9 2 20 5

2. List the set of whole-number factors for each number named below.

4 7 24 18 13

3. Name the greatest common factor of 12 and 16.

4. Name the least common multiple of 6 and 9.

5. Which fractions below are like fractions?

$\frac{3}{4}$ $\frac{6}{8}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{12}{18}$

— Copy and find the answers. Name each answer in lowest terms.

6. $\frac{1}{8} + \frac{5}{8} = \underline{?}$

7. $\frac{9}{11} - \frac{3}{22} = \underline{?}$

8. $5 - \frac{7}{8} = \underline{?}$

	a	b
9.	$\frac{5}{6} + \frac{1}{12}$	$\frac{4}{7} + \frac{2}{3}$
10.	$\frac{1}{2} + \frac{3}{4} + \frac{1}{6}$	$\frac{2}{5} + \frac{1}{3} + \frac{3}{10}$
11.	$\frac{6}{7} - \frac{1}{2}$	$\frac{3}{4} - \frac{2}{5}$

12
12
12
12
48

— Copy and solve each of the following number sentences.

12. $\frac{2}{3} = \frac{a}{12}$

13. $\frac{1}{2} + \frac{3}{4} = \frac{3}{4} + x$

14. $\frac{1}{6} + n = \frac{7}{8}$

15. $(\frac{1}{4} + \frac{3}{5}) + \frac{6}{7} = \frac{1}{4} + (p + \frac{6}{7})$

— If m is replaced by 4 and n is replaced by 10, what number is represented by each of the following expressions?

16. $\frac{m+n}{2}$

17. $\frac{n}{10 \times m}$

18. $\frac{m}{n-m}$

— If s is replaced by 6 in each number sentence below, list the solution set for t from the set of counting numbers.

	a	b
19.	$\frac{s}{t} > 1$	$\frac{s}{t} \leq 1$
20.	$\frac{t}{s} > 1$	$\frac{t}{s} \leq 1$

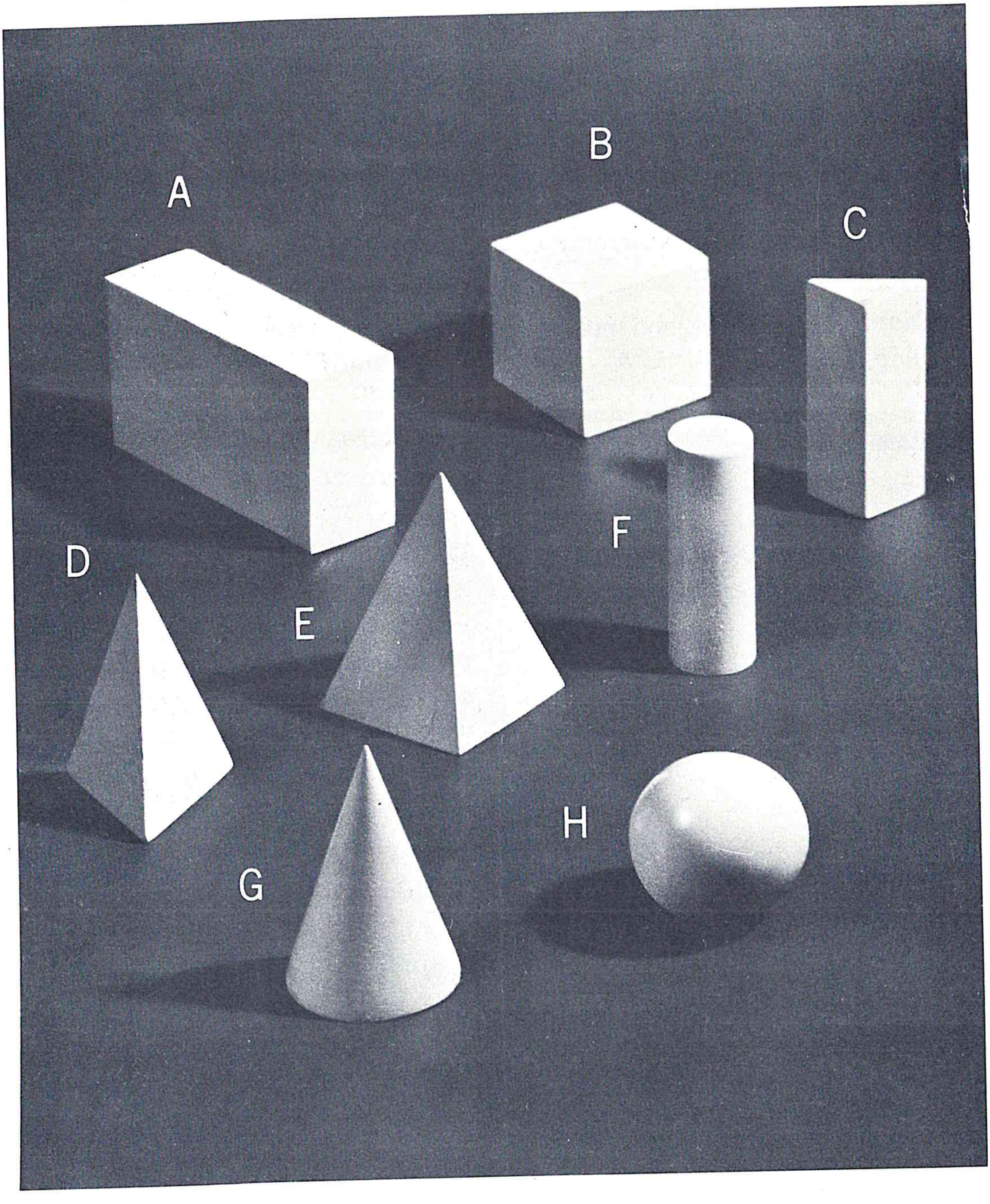
If you have time, try these —

— Copy and complete each of the following sentences.

21. $8 - \frac{1}{3} = \underline{?}$, so $8 - 3\frac{1}{3} = \underline{?}$

22. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \underline{?}$, so $3 \times \frac{1}{4} = \underline{?}$

Closed surfaces



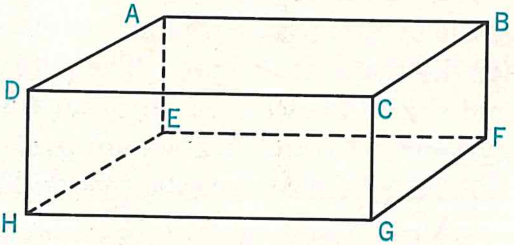
The rectangular prism and the cube

Look at the photograph on the opposite page. Does the skin, or surface, of each object remind you of a set of points?

A set of points in the shape of the surface of any object such as one of these is called a closed surface.

---

1. Shown below is a drawing of the first figure in the photograph. Is any part of the closed surface flat? How many flat parts are there? Each of these flat parts is called a **face**.



2. A closed surface such as that shown above is called a **rectangular prism**. How many faces does a rectangular prism have? What kind of region is each of these faces?

3. In the drawing above, the top and the front faces intersect in a line segment named $\overline{DC}$. Do the front and bottom faces intersect in a line segment? Name it.

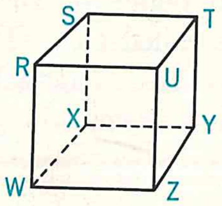
Name each line segment of the rectangular prism shown above that is the intersection of two faces.

4. When two faces of a closed surface intersect in a line segment, the line segment is called an **edge**. How many edges does a rectangular prism have?

5. How many line segments intersect at point A on the closed surface shown below Ex. 1? How many segments intersect at B? at C?

When three or more line segments of a closed surface intersect in a point, the point is called a **vertex**. How many vertices does a rectangular prism have?

6. Name all the edges of the rectangular prism shown below. Name all the vertices.

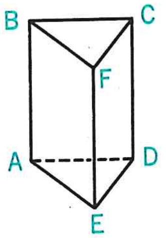


7. Is each face of this rectangular prism a rectangular region? Suppose all twelve edges of this rectangular prism are equal in length. Then what kind of region is each face?

If all six faces of a rectangular prism are square regions, then the rectangular prism is called a **cube**.

8. Is every cube also a rectangular prism? Is every rectangular prism also a cube? Explain.

The triangular prism and the pyramid

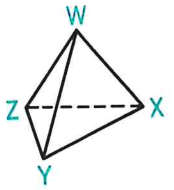


1. How many faces does the closed surface shown above have? Is each face a rectangular region? How many faces are rectangular regions? What kind of region is each of the other two faces?

2. A closed surface such as that shown above is called a **triangular prism**. How does a triangular prism differ from a rectangular prism?

3. In the prism shown above, region *ABCD* and region *CDEF* intersect in an edge called $\overline{CD}$. How many edges does this triangular prism have? Name them.

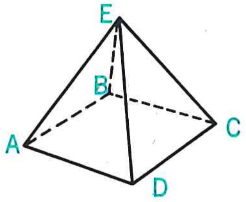
4. What kind of region is each of the faces of this closed surface?



A closed surface that is made up entirely of triangular regions is called a **triangular pyramid**. How does a triangular pyramid differ from a triangular prism?

5. How many faces does a triangular pyramid have? how many edges? how many vertices?

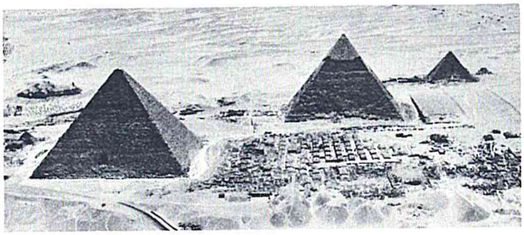
6. One of the faces of the pyramid shown below is not a triangular region. If $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are equal in length, what kind of region is *ABCD*?



Region *ABCD* is called the **base** of the pyramid. A pyramid is named for the shape of its base. The pyramid shown above is called a **square pyramid**. How does a square pyramid differ from a triangular pyramid?

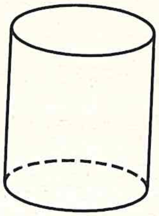
7. What shape is the base of a pentagonal pyramid? a hexagonal pyramid?

8. The picture below shows the great pyramids of Egypt. What kind of pyramid is each of these? Use an encyclopedia to find other interesting facts about these pyramids.



TWA

The cylinder and the cone

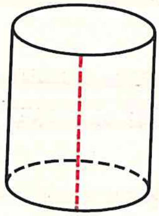


1. Does the drawing above remind you of a soup can? Name some other objects it suggests to you.

This drawing represents a closed surface called a **cylinder.**

2. How many different surfaces does a cylinder have? How many of these surfaces are flat? Are the two flat surfaces circular regions? Are they equal in size?

3. The curved surface of a cylinder is called the **lateral surface.** Think of the lateral surface of a cylinder cut in the way shown by the red marks on this drawing.

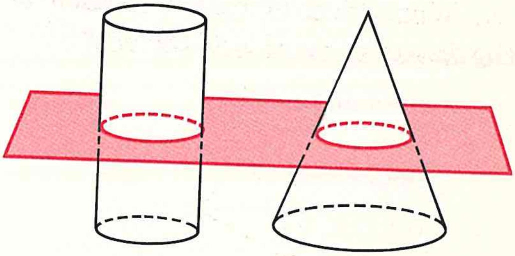


Now imagine the lateral surface laid out flat. What do you think the shape of the lateral surface will be after it is laid out flat? (Hint: What shape would the label on a soup can be if it were cut and laid out flat?) What kind of region is this?

4. The picture above shows a plastic container filled with candy. This container is a model of another closed surface called simply a **cone.**

- How many flat surfaces does a cone have? This flat surface is shaped like a $\underline{?}$ region. It is called the **base** of the cone.
- How does a cone differ from a cylinder?

5. Imagine a plane passing through or "cutting" a cylinder and a cone as shown in the drawing below. We say the plane *intersects* the cylinder and the cone.



- What closed path is formed by the intersection of the plane and the cylinder?
- What closed path is formed by the intersection of the plane and the cone?

The sphere

1. Look at the picture of a baseball.

If you ran your hand over the surface of a baseball, would any part of it be flat? The skin or surface of a baseball is a model of a closed surface called a **sphere**. Can you name some other models of a sphere?

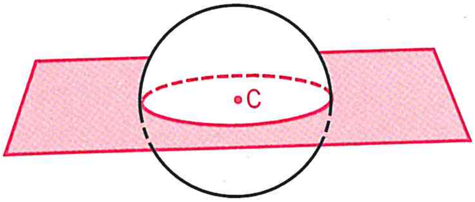
2. Like all closed surfaces, it is the shape of the baseball or its skin that reminds us of a sphere and not the set of points inside the skin.

Is a hollow rubber ball a model of a sphere? Is an egg a model of a sphere? Is a hat box? a bowling ball? Name some other models of a sphere.

Name the closed surface that is suggested by each of these objects.

- a 50-cent piece
- a globe
- a shoe box
- a tin can
- a marble

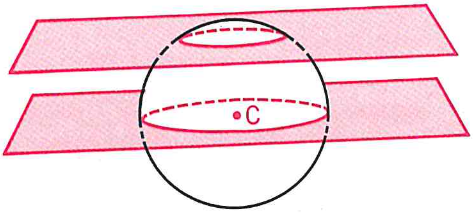
3. Every point of a sphere is the same distance from a point inside the sphere called its **center**. How is the center of a sphere like the center of a circle? How does a sphere differ from a circle?



4. The drawing above shows a sphere, its center C, and a plane intersecting the sphere through its center.

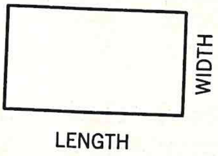
- What closed path is formed by the intersection of the plane and the sphere?
- What can you say about the distance from point C to any point on this closed path?
- What can you say about the distance from point C to any point on the sphere?

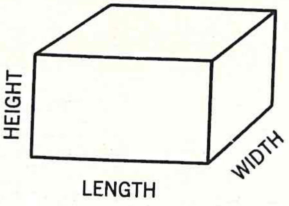
5. The drawing below shows a plane intersecting a sphere and passing through the center and another plane intersecting the sphere but not passing through the center.



- What closed path is formed by the intersection of the sphere and each of the two planes shown above?
- How do you think these two closed paths compare in size?

The meaning of volume

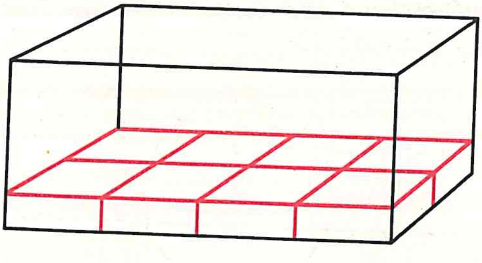




1. Shown above are three geometric figures: a line segment, a rectangle, and a rectangular prism.

- A line segment has only one dimension, called its ?.
- A rectangle has two dimensions, called its ? and ?.
- A rectangular prism has three dimensions, called its length, width, and ?.

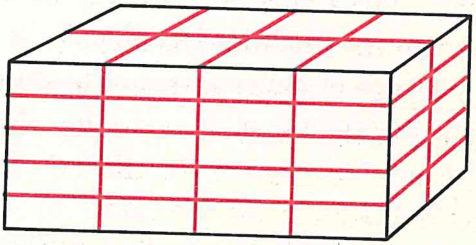
2. Fred is storing packages of paper in a box for his teacher. How many dimensions does the box have?



The picture above shows how Fred overed the entire bottom of the box vith packages of paper. All the ackages of paper have the same size nd shape. Are the packages shaped ke a rectangular prism? What hape does the box have?

3. Each row across the bottom of the box contains ? packages of paper. How many rows of packages does it take to cover the bottom? Explain why 4 × 2 names the number of packages it takes to cover the bottom of the box.

4. This drawing shows how the space inside the box looked after Fred filled it with packages of paper.



- Does each layer of packages contain 4 × 2 packages? Explain.
- How many layers are there? Then there are (4 × 2) × 5, or ?, packages of paper inside the box.

5. A closed surface and the space it encloses are called a **solid region.**

The solid region of Fred's box measures (4 × 2) × 5, or ?, packages of paper.

The measure of a solid region is called the volume of the solid region.

6. Using Fred's package of paper as a unit of measure, what is the volume of the box shown above?

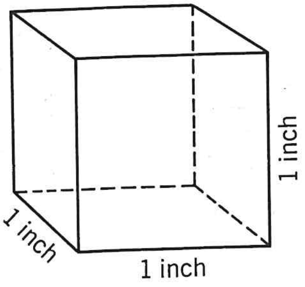
Standard units of volume

1. When you measure something, you usually use a standard unit of measure.

- To measure a line segment, you use another line segment as a standard unit. What are some names we give to standard units of length?
- To measure a flat surface such as a rectangular region, you use another region as a standard unit. Name some standard units of area.

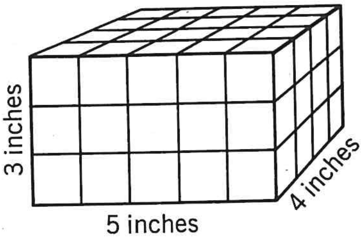
2. On the preceding page, you used a package of paper as a unit to measure the volume of Fred's box. Are the box and its interior a model of a solid region? Is the package of paper a model of a solid region? Then to measure a solid region, what should be used as a unit of measure?

3. The drawing below shows a cube that measures ? inch on each edge. We say this is a 1-inch cube.



The volume of a 1-inch cube is called 1 **cubic inch** (**cu. in.**). A cubic inch is a standard unit of measure used to find volume.

4. Shown below is a rectangular prism. How many dimensions does a rectangular prism have?

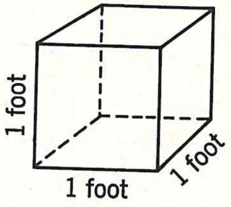


The length of the prism shown above is 5 inches. What is its width? What is its height? Are all three dimensions of this prism expressed in the same unit of measure?

5. Some 1-inch cubes have been placed inside the prism shown above.

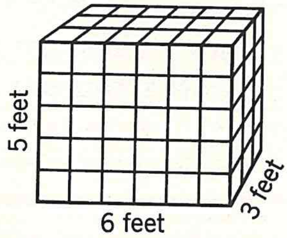
- How many 1-inch cubes are there in one row across the bottom of the prism? How is this number related to the length of the prism?
- How many rows of 5 one-inch cubes are there in each layer? How is this number related to the width of the prism?
- How many layers of 1-inch cubes are there inside the prism? How is this number related to the height of the prism?
- There are ? 1-inch cubes in each row, ? rows in each layer, and ? layers inside the prism. Thus there are (5 × 4) × 3, or ?, 1-inch cubes inside the prism. We say the volume of the prism is 60 cubic inches.

6. The drawing below shows a cube that measures 12 inches or ? foot on each edge. We say this is a 1-foot cube.



The volume of a 1-foot cube is called 1 **cubic foot** (**cu. ft.**). A cubic foot is another standard unit of measure used to find volume.

7. The dimensions of a box in which pianos are shipped are shown in the drawing below.



- How many 1-foot cubes are there in one row across the bottom? What is the length of the box?
- How many rows of 1-foot cubes are there in one layer of cubes? What is the width of the box?
- How many layers of 1-foot cubes fill the box? What is the height of the box?
- Explain why (6 × 3) × 5 names the number of cubic feet inside the box. What is the volume of the box?

Your discovery

Tell your discovery

If all three dimensions of a rectangular prism are expressed in the same unit of measure, what is a quick way to find the volume of the rectangular prism?

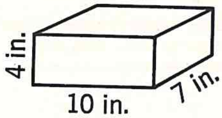
Use your discovery

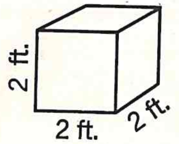
What is the volume of a rectangular prism that is 6 feet long, 4 feet wide, and 12 feet high?

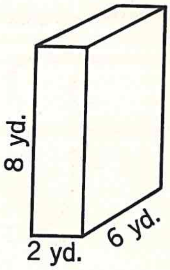
8. Another standard unit of measure used to find volume is a cube that measures 3 feet or 1 yard on each edge. The volume of a 1-yard cube is called 1 **cubic yard** (**cu. yd.**).

In cubic yards, what is the volume of a rectangular prism that is 3 yards long, 5 yards wide, and 2 yards high?

9. Find the volume of each rectangular prism shown below.







Think twice!

10. ? cu. in. make 1 cu. ft.

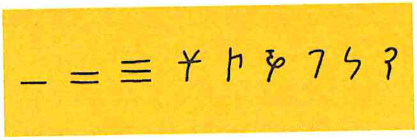
11. ? cu. ft. make 1 cu. yd.

12. ? cu. in. make 1 cu. yd.

Inquiry into history

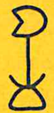
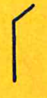
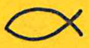
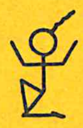
Ancient numeration systems

A numeration system is a way of naming numbers by writing symbols called numerals. The nine symbols shown in the picture at the right are early forms of our present-day numerals for 1 through 9. Our earliest knowledge of these symbols goes back about 2,200 years to the Hindu civilization of India.



Did the early Hindus use a different symbol for each whole number from 1 through 9? Did the Roman numeration system use a different symbol for each whole number from 1 through 9? Which of these systems do you think eventually used the idea of place value?

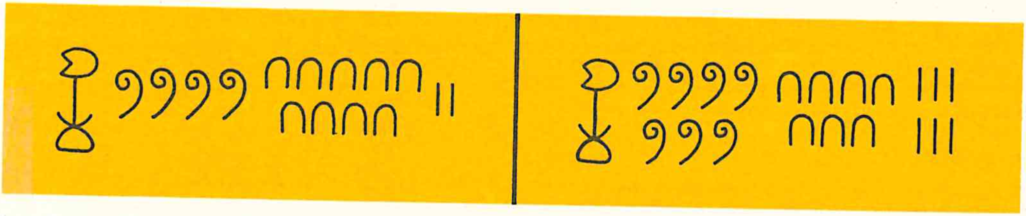
A numeration system more like the Roman system than like ours was used by the ancient Egyptians. They used a system of picture writing called hieroglyphics. Shown below are seven different picture symbols used by the ancient Egyptians, with the Hindu-Arabic numerals for the numbers they named.

EGYPTIAN SYMBOLS	|	∩	9				
HINDU-ARABIC SYMBOLS	1	10	100	1,000	10,000	100,000	1,000,000

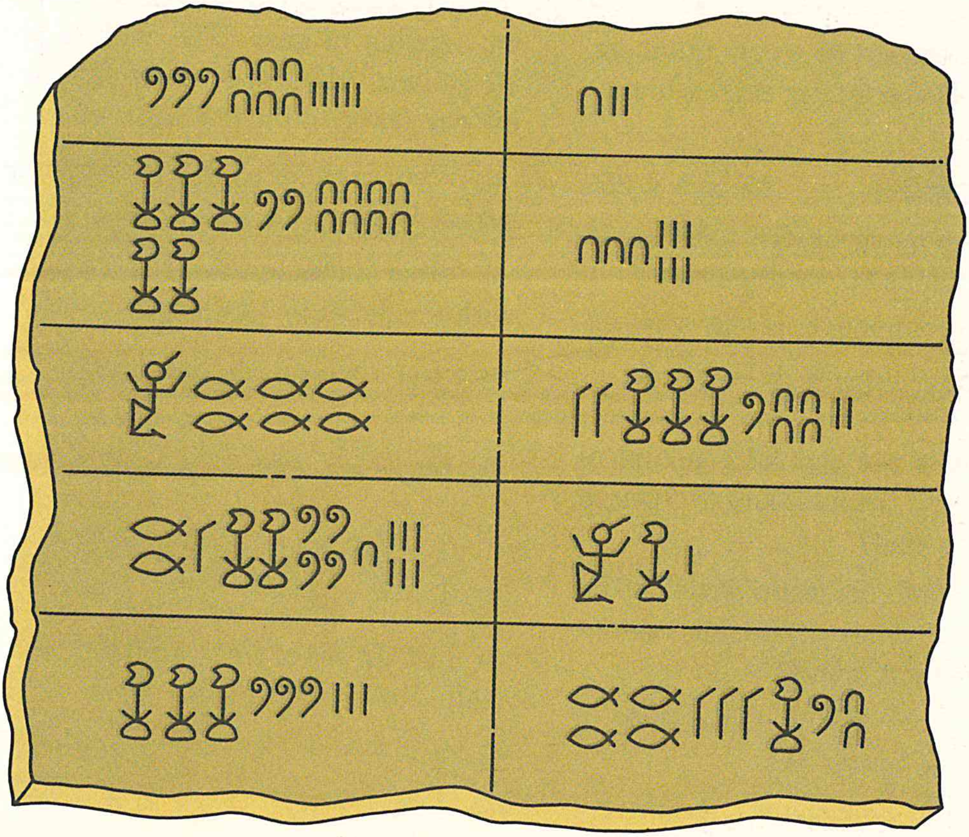
The Egyptians repeated a picture symbol as many times as necessary up to nine times. Instead of repeating the symbol ten times, they used another symbol for that number. For example, in place of ten |'s, they wrote one ∩. What would they write in place of ten ∩'s? ten 9's? ten 's?

When more than one kind of picture symbol was used to name a number, the Egyptians wrote the symbols next to one another. To find the number named, they added the values of the symbols. For example, since 𓍢 names 100 and 𓎆 names 10, then 𓍢𓍢𓎆𓎆𓎆 names 100 + 100 + 10 + 10 + 10, or __?__.

What two numbers are named in the chart below?



On the clay tablet pictured below, ten different numbers are shown using Egyptian picture symbols. See how many of these numbers you can identify.



Keeping up in mathematics

Oral

1. Round each number named below to the nearest thousand.

769,542 843,499

2. Tell the numerator shown by each fraction below. Tell the denominator.

$\frac{3}{7}$ $\frac{18}{5}$ $\frac{6}{9}$ $\frac{10}{4}$ $\frac{5}{12}$

3. Which fractions in Ex. 2 are not in lowest terms?

4. Which of the numbers named below are prime numbers? Which are composite numbers?

15 6 23 21 17 3

5. From the set of whole numbers, tell the solution set for each sentence below.

$50 \times a \leq 250$ $55 \times b \leq 250$

6. Is the quotient for $2{,}892 \div 36$ about 8, about 80, or about 800?

7. A bus traveling at an average rate of 60 miles per hour will go _?_ miles in 5 hours.

8. What is the cost of $\frac{3}{4}$ pound of hamburger if the hamburger is priced at 80¢ a pound?

9. Sarah and her sister saved $21 in 7 months. They saved an average of _?_ dollars a month.

10. Tell how many sides each of these plane figures has.

pentagon hexagon triangle

Written

— Copy and find the answers.

	a	b	c
1.	$\begin{array}{r}48\\29\\+\ 64\\\hline\end{array}$	$\begin{array}{r}382\\517\\+\ 436\\\hline\end{array}$	$\begin{array}{r}\$4.52\\7.28\\+\ 3.04\\\hline\end{array}$
2.	$\begin{array}{r}674\\-\ 521\\\hline\end{array}$	$\begin{array}{r}3{,}840\\-\ 2{,}165\\\hline\end{array}$	$\begin{array}{r}\$5.98\\-\ 3.49\\\hline\end{array}$
3.	$\begin{array}{r}52\\\times\ 37\\\hline\end{array}$	$\begin{array}{r}625\\\times\ 49\\\hline\end{array}$	$\begin{array}{r}4{,}018\\\times\ 26\\\hline\end{array}$
4.	$62\overline{)312}$	$7\overline{)163}$	$8\overline{)7{,}296}$

5. Mr. Foley's car travels 14 miles on 1 gallon of gas. The tank holds 16 gallons when full; how far can his car travel on a full tank of gas?

6. What is the area of a rectangular region 16 feet long and 12 feet wide?

7. What is the perimeter of a square with a side 39 inches long?

— Solve each number sentence.

	a	b
8.	$15 \times m = 900$	$n \times 47 = 376$
9.	$\frac{3}{4} + \frac{5}{8} = y$	$\frac{3}{5} - \frac{1}{7} = c$
10.	$\frac{1}{3} + d = \frac{5}{6}$	$t + \frac{5}{8} = \frac{2}{3}$

— List the set of whole numbers that make the sentence true.

11. $3 \times p < 20$ $5 + q < 27$

12. $4 \times 3 > w$ $8 + 7 > x$

Understanding division

1. The town of Bayville is planning a parade to celebrate its two-hundredth birthday.

Bayville's 350 school children are to be in the parade. They will march in sets of 42. How many sets of 42 children will there be?

- Which number named in the example at the right tells how many children will be in the parade?

$$\begin{array}{r} 8\text{ r}14 \\ 42\overline{)350} \\ \underline{336} \\ 14 \end{array}$$

- Which number tells how many children will march in each set?
- Which number tells how many sets there will be?
- Does the number 14 shown as the remainder in the example tell how many sets are left over or how many children are left over?
- The children left over will ride on a special float. In the parade there will be ? sets of children with ? children in each set marching and ? children riding on a float. In all, there will be ? children in the parade.

2. How many sets of children could march in the parade if Bayville had 378 school children?

3. How many sets of children could march in the parade if Bayville had 400 school children? How many children would ride on the float?

4. If 32 pupils want to share equally the job of moving 750 chairs to the auditorium, how many chairs will be in each set carried by a pupil?

- Which number named in the example at the right tells the total number of chairs?
- Which number tells into how many sets the 750 chairs are to be separated?

$$\begin{array}{r} 23\text{ r}14 \\ 32\overline{)750} \\ \underline{640} \\ 110 \\ \underline{96} \\ 14 \end{array}$$

- Can 750 chairs be separated into 32 sets with the same number in each set with no chairs left over? How many chairs are left over?
- For 32 pupils to share equally the job of carrying 750 chairs, each pupil will carry ? chairs and there will be ? chairs still to be carried.

5. If 30 pupils want to share equally the job of moving 750 chairs to the auditorium, how many chairs will be in each set carried by a pupil? Can 30 pupils share the job equally?

6. Twenty-five girls want to share equally 106 balloons. How does the division example at the right show that the best the girls can do is for each one to take 4 balloons? How many balloons will be left over?

$$\begin{array}{r} 4\text{ r}6 \\ 25\overline{)106} \\ \underline{100} \\ 6 \end{array}$$

Estimating quotients

1. Before dividing 96 by 32, Sally estimated the quotient.

Is the quotient for 96 ÷ 32 less than 10? Then how many digits will the name for the quotient for 96 ÷ 32 have?

$32\overline{)96}$

Sally thought: "There are 3 tens in 32 and 9 tens in 96. So the number of 3's in 9 will give me a hint of the number of 32's in 96."

How many 3's are in 9? Is the following number sentence true?

$$3 \times 32 \leq 96$$

Then did Sally's method of estimating work for this example?

2. The next example Sally worked looked like the one shown at the right.

To estimate the quotient, Sally thought: "There are 4 tens in 43 and 8 tens in 82. So the number of 4's in 8 will give me a hint of the number of 43's in 82."

$43\overline{)82}$

How many 4's are in 8? Is this number sentence true?

$$2 \times 43 \leq 82$$

Then did Sally's method of estimating work for this example?

What is the answer when you divide 82 by 43?

Sally's method does not always give the quotient, but it does give a close estimate of the quotient.

3. Suppose you wish to divide 139 by 25.

- How many tens are in 25? How many tens are in 139? Now how many 2's are in 13?
- Does 6 give you a hint as to the quotient? Does this mean that 6 is the quotient? Could it be 5?
- Which of the following sentences is true?

$$6 \times 25 \leq 139 \qquad 5 \times 25 \leq 139$$

Then which is the quotient, 6 or 5?

- What is the answer when you divide 139 by 25?

4. For the example at the right, will the quotient be less than 10? Explain. Then is 9 a good estimate for the quotient? Why? Copy and divide.

$46\overline{)414}$

5. How many 3's are in 19? Then would 6 be one estimate for the quotient? Copy the example and try 6 as the quotient.

$31\overline{)196}$

6. For this example, might you use 6 as an estimate for the quotient? Why?

$34\overline{)196}$

Does 6 work as the quotient? Then what number should you try next?

What is the answer when you divide 196 by 34?

Estimating and finding quotients

1. One of the examples on a division test Bob and Joan are taking is $29\overline{)89}$.

- Bob thought: "8 tens divided by 3 tens is 2. So 2 is my estimate of the quotient." Is 2 the greatest whole number that makes the following sentence true?

$$n \times 29 \leq 89$$

Then what number should Bob use as his next estimate?

- Joan thought: "29 is about 3 tens and 89 is about 9 tens. 9 tens divided by 3 tens is 3. My estimate is 3." Is 3 the greatest whole number that makes this sentence true?

$$n \times 29 \leq 89$$

- What is the answer when you divide 89 by 29?

— For each example below, first estimate the quotient. Then copy and find the answer.

	a	b	c
2.	$58\overline{)185}$	$75\overline{)369}$	$42\overline{)340}$
3.	$82\overline{)325}$	$66\overline{)558}$	$56\overline{)395}$
4.	$36\overline{)180}$	$39\overline{)234}$	$41\overline{)211}$
5.	$49\overline{)276}$	$45\overline{)280}$	$47\overline{)312}$
6.	$33\overline{)82}$	$55\overline{)445}$	$14\overline{)39}$
7.	$51\overline{)426}$	$43\overline{)81}$	$26\overline{)115}$
8.	$74\overline{)321}$	$82\overline{)724}$	$28\overline{)214}$
9.	$65\overline{)512}$	$98\overline{)818}$	$71\overline{)144}$

10. Another example on the division test is $35\overline{)2{,}135}$.

- Bob thought: "2,135 is greater than 10 × 35 but less than 100 × 35. My answer will have 2 digits. Since 21 ÷ 3 equals 7, the quotient will be about 70." Is the quotient 70-some? What number should Bob try next?

- Joan thought: "2,135 is about 21 hundreds. 35 is between 3 tens and 4 tens. 21 hundreds ÷ 3 tens is 7 tens. 21 hundreds ÷ 4 tens is about 5 tens. The quotient will be 50-some, 60-some, or 70-some. Since 35 is between 30 and 40, I will try 60 as my estimate." Is the quotient 60-some?

- What is the answer when you divide 2,135 by 35?

— For each example below, first estimate the quotient. Then copy and find the answer.

	a	b	c
11.	$32\overline{)1{,}730}$	$47\overline{)800}$	$65\overline{)455}$
12.	$26\overline{)1{,}352}$	$68\overline{)550}$	$56\overline{)4{,}196}$
13.	$53\overline{)2{,}328}$	$38\overline{)269}$	$47\overline{)2{,}872}$
14.	$46\overline{)2{,}668}$	$55\overline{)343}$	$69\overline{)5{,}589}$
15.	$38\overline{)3{,}649}$	$43\overline{)275}$	$24\overline{)1{,}399}$
16.	$57\overline{)4{,}801}$	$75\overline{)482}$	$36\overline{)2{,}324}$
17.	$66\overline{)5{,}710}$	$15\overline{)128}$	$94\overline{)9{,}118}$

Practice in division

SALE!

AMERICAN RED PINE

AGE	HEIGHT	COST OF 10	COST OF 25	COST OF 50
3-year seedlings	6″-10″	$.90	$2.00	$3.50
4-year transplants	8″-12″	2.00	4.50	8.00
5-year transplants	12″-18″	2.70	6.25	11.50

MOSS TREE FARM—Deer Creek, Pennsylvania

— Use the advertisement shown above to help find the answers for Ex. 1–5.

1. What will be the cost of each 3-year seedling if you buy 10? if you buy 25? if you buy 50?

2. What will be the cost of each 4-year transplant if you buy 10? if you buy 25? if you buy 50?

3. What will be the cost of each 5-year transplant if you buy 10? if you buy 25? if you buy 50?

4. How much do 25 four-year transplants cost? 50 four-year transplants? Then what is the cost of 75 four-year transplants?

5. The pupils of a fifth-grade class plan to buy 75 four-year transplants to plant around the school yard.

There are 25 pupils in the class. If they share the cost equally, what will be each pupil's share?

6. Find the greatest multiple of 10 that makes each sentence true.

$a \times 48 \leq 2{,}782$ $\qquad$ $b \times 24 \leq 1{,}869$

7. Find the greatest multiple of 100 that makes each sentence true.

$m \times 53 \leq 8{,}799$ $\qquad$ $n \times 13 \leq 6{,}248$

8. Find the greatest multiple of 1,000 that makes each sentence true.

$p \times 8 \leq 9{,}821$ $\qquad$ $q \times 3 \leq 8{,}125$

— For each example below, estimate the quotient. Then copy the example and find the answer.

	a	b	c
9.	$8\overline{)3{,}553}$	$6\overline{)2{,}592}$	$8\overline{)728}$
10.	$9\overline{)6{,}489}$	$5\overline{)2{,}735}$	$7\overline{)791}$
11.	$9\overline{)5{,}682}$	$9\overline{)8{,}562}$	$9\overline{)648}$
12.	$6\overline{)2{,}635}$	$7\overline{)2{,}878}$	$7\overline{)294}$
13.	$79\overline{)6{,}454}$	$57\overline{)4{,}737}$	$37\overline{)235}$
14.	$46\overline{)3{,}852}$	$68\overline{)5{,}802}$	$49\overline{)307}$
15.	$26\overline{)1{,}594}$	$37\overline{)2{,}999}$	$23\overline{)188}$
16.	$28\overline{)1{,}160}$	$48\overline{)2{,}990}$	$32\overline{)674}$
17.	$57\overline{)3{,}848}$	$45\overline{)3{,}465}$	$47\overline{)994}$
18.	$73\overline{)2{,}995}$	$67\overline{)5{,}762}$	$71\overline{)249}$
19.	$64\overline{)2{,}400}$	$83\overline{)7{,}636}$	$68\overline{)372}$
20.	$96\overline{)3{,}936}$	$55\overline{)2{,}555}$	$77\overline{)643}$

Finding quotients named by three-digit numerals

1. Suppose you wish to divide 9,072 by 21. Your first step should be to estimate the quotient.

- Are there as many as 100 twenty-ones in 9,072? Are there as many as 1,000 twenty-ones in 9,072? Then the quotient will be equal to or greater than ?, but less than ?. In what place of your answer should the first digit of the quotient be shown?

- Think: 9 thousands ÷ 2 tens is about 4 hundreds. Write 4 in the hundreds place in the answer. Multiply 21 by 400. What is the remainder when you subtract 400 twenty-ones from 9,072?

$$\begin{array}{r} 4?? \\ 21\overline{)9{,}072} \\ \underline{8{,}400} \\ 672 \end{array}$$

- Think: 672 is about 600; 6 hundreds ÷ 2 tens is ? tens. Write 3 in the tens place in the answer. Multiply 21 by 30. What is the remainder when you subtract 30 twenty-ones from 672?

$$\begin{array}{r} 43? \\ 21\overline{)9{,}072} \\ \underline{8{,}400} \\ 672 \\ \underline{630} \\ 42 \end{array}$$

- How many twenty-ones are in 42? What digit should be shown in the ones place of the answer?

- Copy and complete the example for 9,072 divided by 21.

- Check your work by multiplying 21 by your answer.

2. Study the example shown at the right. Is it correct as far as the work is shown? What digit should be written in the ones place of the answer? Copy and complete the example. Check your answer.

$$\begin{array}{r} 41? \\ 53\overline{)22{,}018} \\ \underline{21{,}200} \\ 818 \\ \underline{530} \\ 288 \end{array}$$

3. Look at this example. In what place of the answer is the digit 3 shown? What does the 3 in the answer mean?

$$\begin{array}{r} 3?? \\ 34\overline{)11{,}799} \\ \underline{10{,}200} \\ 1{,}599 \end{array}$$

Copy and complete the example. Check your answer.

— For each example below, first estimate the answer. Then copy, find the answer, and check.

	a	b	c
4.	33)9,652	81)10,203	65)7,789
5.	71)8,388	92)15,861	46)4,263
6.	28)7,984	49)34,917	37)8,650
7.	62)9,125	85)32,238	76)4,236
8.	56)4,368	24)21,479	38)5,667
9.	69)9,991	57)43,460	83)9,285
10.	35)9,438	44)16,324	75)8,403

Zero in the quotient

— Copy and find the quotients.

	a	b	c
1.	$35\overline{)35}$	$35\overline{)350}$	$35\overline{)3,500}$
2.	$83\overline{)83}$	$83\overline{)830}$	$83\overline{)8,300}$
3.	$13\overline{)26}$	$13\overline{)260}$	$13\overline{)2,600}$
4.	$42\overline{)126}$	$42\overline{)1,260}$	$42\overline{)12,600}$

5. Suppose you wish to find the quotient for 2,060 divided by 20.

- Is 2,060 greater than 10 × 20? greater than 100 × 20? as much as 1,000 × 20? Then will the quotient be named by a two-digit, three-digit, or four-digit numeral?

- To estimate, you think: 2 thousands ÷ 2 tens is 1 hundred. Write 1 in the hundreds place of the answer. What is the remainder when you subtract 100 twenties from 2,060?

$$\begin{array}{r} 1\,?\,? \\ 20\overline{)2,060} \\ \underline{2,000} \\ 60 \end{array}$$

- Look at the remainder, 60. Are there as many as 10 twenties in 60? Then the digit in the tens place of the answer should be ?.

- What is the greatest whole number that makes this sentence true?

$$n \times 20 \leq 60$$

Then the digit in the ones place of the answer should be ?.

$$\begin{array}{r} 10\,? \\ 20\overline{)2,060} \\ \underline{2,000} \\ 60 \end{array}$$

- Copy and complete the example.

6. Look at this example. To find the quotient, think: 6 thousands ÷ 3 tens is ? hundreds. The digit in the hundreds place should be ?. What is the remainder when you subtract 200 thirties from 6,090?

$$\begin{array}{r} 2\,?\,? \\ 30\overline{)6,090} \\ \underline{6,000} \\ 90 \end{array}$$

- Is 90 as great as 10 × 30? Then the digit in the tens place of the answer should be ?.

- How many 30's are in 90? Then the digit in the ones place of the answer should be ?.

- Copy and complete the example. What is the quotient for 6,090 divided by 30?

— Copy and complete each of the examples shown below.

	a	b
7.	$\begin{array}{r} 1\,?\,? \\ 80\overline{)8,160} \\ \underline{8,000} \\ 160 \end{array}$	$\begin{array}{r} 4\,?\,? \\ 56\overline{)22,849} \\ \underline{22,400} \\ 449 \end{array}$
8.	$\begin{array}{r} 1\,?\,? \\ 64\overline{)6,528} \end{array}$	$\begin{array}{r} 2\,?\,? \\ 14\overline{)2,870} \end{array}$
9.	$60\overline{)6,480}$	$40\overline{)8,360}$
10.	$52\overline{)5,570}$	$22\overline{)9,000}$
11.	$32\overline{)3,246}$	$24\overline{)4,872}$
12.	$20\overline{)8,160}$	$32\overline{)3,392}$

13. Suppose you wish to divide 375 by 36. Is 375 greater than 10 × 36? Is 375 as much as 100 × 36? Then the quotient will be greater than ? but less than ?.

- Think: 4 hundreds ÷ 4 tens is ? ten. Then the digit in the tens place of the answer probably is ?. What is 10 × 36? What is the remainder when you subtract 10 thirty-sixes from 375?

$$\begin{array}{r} 1? \\ 36\overline{)375} \\ \underline{360} \\ 15 \end{array}$$

- Look at the remainder, 15. What is the greatest whole number that makes this number sentence true? $n \times 36 \leq 15$ Then the digit in the ones place of the answer should be ?. The remainder is ?.

$$\begin{array}{r} 10 \\ 36\overline{)375} \\ \underline{360} \\ 15 \end{array}$$

14. To check your answer for Ex. 13, see if this sentence is true.

$$375 = (10 \times 36) + 15$$

Write a similar sentence to see if the answer shown for the example in the box at the right is correct.

$$\begin{array}{r} 50\text{ r}30 \\ 41\overline{)2{,}080} \\ \underline{2{,}050} \\ 30 \end{array}$$

15. Study the work shown for the example at the right. Is the work correct?

Is this number sentence true?

$$2{,}550 = (102 \times 25)$$

$$\begin{array}{r} 102 \\ 25\overline{)2{,}550} \\ \underline{2{,}500} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

16. Study the work shown in the example below. Is it correct as shown?

$$\begin{array}{r} 408\text{ r}1 \\ 56\overline{)22{,}849} \\ \underline{22{,}400} \\ 449 \\ \underline{448} \\ 1 \end{array}$$

Is this number sentence true?

$$22{,}849 = (408 \times 56) + 1$$

— Copy and complete each of the examples shown below. Then write a true number sentence to check.

	a	b
17.	$\begin{array}{r} 40\text{ r ??} \\ 23\overline{)935} \\ \underline{920} \\ 15 \end{array}$	$\begin{array}{r} 7? \\ 65\overline{)4{,}559} \\ \underline{4{,}550} \\ 9 \end{array}$
18.	$\begin{array}{r} 8? \\ 56\overline{)4{,}516} \\ \underline{4{,}480} \end{array}$	$\begin{array}{r} 6? \\ 32\overline{)1{,}929} \end{array}$
19.	$63\overline{)28{,}987}$	$61\overline{)30{,}999}$
20.	$54\overline{)11{,}056}$	$75\overline{)1{,}500}$
21.	$93\overline{)3{,}720}$	$63\overline{)12{,}743}$
22.	$78\overline{)51{,}032}$	$35\overline{)7{,}232}$
23.	$57\overline{)23{,}083}$	$35\overline{)1{,}418}$
24.	$21\overline{)12{,}187}$	$75\overline{)45{,}639}$
25.	$49\overline{)24{,}857}$	$65\overline{)6{,}007}$
26.	$34\overline{)10{,}383}$	$37\overline{)17{,}413}$

Division examples to find amounts of money

1. Eight classes of the Post Road Elementary School have voted to share equally the cost of magazine subscriptions for the library. All the subscriptions together cost $64.40. How much should each class pay?

```
8)$64.40
```

- Copy the example shown at the right.
- Think of $64.40 as 6,440 cents and divide as you would divide 6,440 by 8.
- Is the work correct as far as it is shown at the right? Does the 805 in the answer stand for 805 cents? Is $8.05 the same amount as 805 cents?

```
   8 05
8)$64.40
   64 00
      40
      40
       0
```

- Copy and complete the example by writing a dollar sign and cents point in the answer.

— Copy and find the answers.

	a	b
2.	3)$6.93	5)$15.65
3.	9)$10.80	4)$24.04
4.	6)$3.66	7)$22.47
5.	4)$15.28	6)$19.80
6.	2)$11.10	5)$65.10
7.	7)$1.27	8)$47.40
8.	8)$23.32	2)$4.07

9. There are 35 pupils in one class in the Post Road Elementary School. If they share equally the $8.05 cost of the magazine subscriptions, what will be each pupil's share?

- Shown below is the way Ben worked his example.

```
     $ .23
35)$8.05
     7 00
     1 05
     1 05
        0
```

- Ben said, "Each pupil should give 23 dollars." Is Ben's answer sensible? What mistake did he make? What should his answer be?

— Copy, find the answers, and check.

	a	b
10.	33)$17.49	42)$7.14
11.	41)$7.79	41)$9.02
12.	51)$115.26	71)$153.36
13.	35)$44.10	36)$72.36
14.	64)$79.36	75)$150.75
15.	37)$74.37	63)$252.63
16.	22)$4.62	20)$8.40
17.	52)$78.08	54)$108.72

Practice in finding the quotient

1. Anne wishes to find the answer for the example at the right.

36)210

- Anne copied the example. She thought: "36 is about 4 tens. 21 tens ÷ 4 tens is about 5. So an estimate for the quotient is 5."

Does 5 make this sentence true? $n \times 36 \leq 210$ Is 5 the greatest whole number that makes the sentence true? Try 6.

- Copy and complete the example. Write a number sentence to check your answer.

2. If each school bus can carry 28 pupils, how many buses will be needed to take 168 children to the zoo?

28)168

- To estimate the quotient, David thought: "168 is about 16 tens and 28 is about 3 tens. 16 tens ÷ 3 tens is about 5."

Does 5 make this sentence true? $n \times 28 \leq 168$ Is 5 the greatest whole number that makes the sentence true? Try 6.

- Copy and complete the example. Write a number sentence to check your answer.

If your estimate for the quotient is too small to be the quotient, try the next greater whole number. If that is still too small, try the next greater whole number, and so on, until you find the quotient.

— First estimate the quotient. Then find the answer and check.

	a	b	c	d	e	f
3.	78)613	29)187	53)468	28)140	54)487	27)172
4.	49)306	77)645	17)69	28)198	19)67	29)105
5.	27)118	96)725	86)629	59)413	28)107	34)268
6.	18)56	78)606	58)410	68)523	19)58	26)185
7.	15)645	48)3,607	15)7,734	27)1,429	36)564	56)4,368
8.	37)2,773	39)1,446	36)2,051	89)7,256	48)382	69)3,661

How well can you divide?

1. During the month of March, Larry gathered 2,170 eggs. That was an average of ? eggs a day.

- To find the average, you should divide 2,170 by ?.
- Will the answer be named by a one-digit or a two-digit numeral?
- What is the answer?

2. At 49¢ each, how many books can you buy for $10.00? How much money will you have left over?

3. Mrs. Hunter bought a set of 24 books for $29.76. What was the average price of each book?

4. Twenty-eight members of the speech club shared equally the cost of a tape recorder. The tape recorder cost $30.80. Each member's share was ?.

5. An express train averages 68 miles an hour. How many hours does it take this train to travel 1,836 miles?

6. Fifty-one boy scouts are going to share the cost of a trip to their state capital. If the trip costs a total of $209.10, what will each boy have to pay?

— Copy each example. Estimate the answer. Then find the answer. Write a true number sentence to check.

	a	b	c	d	e
7.	34)6,246	66)3,276	26)5,226	35)7,000	82)6,806
8.	41)12,382	57)2,009	62)1,242	57)2,465	67)13,420
9.	34)6,936	46)18,630	96)4,735	25)6,758	54)32,836
10.	66)1,259	63)19,225	92)7,728	62)12,424	92)4,734

— These examples will help you find out how well you can divide. Copy each example and find the quotient.

11.	75)2,800	37)15,133	50)1,971	75)4,513	28)1,425
12.	38)19,266	95)3,870	79)47,716	86)3,574	63)4,541
13.	27)8,109	36)3,610	24)9,602	23)9,629	47)9,436
14.	97)9,777	85)8,585	49)98,196	45)7,654	96)28,817

If you need more practice, do Practice Set 44, page 337.

3
45
6
270

40
×5
200

Tell what you think

1. Jerry paid 60¢ for a box of candy bars. There are 12 bars in the box. How can you find out how much each bar cost him?

- Jerry thought this way: "12 times the cost of one bar is 60¢, or $12 \times n$ equals 60. To find what number makes this sentence true, I'll divide 60 by 12.

"The cost of one bar is 60 ÷ 12, or __?__¢."

- Tom thought this way: "12 bars cost 60¢; so 1 bar costs $\frac{1}{12}$ of 60, or __?__¢."

Whose thinking do you understand better, Jerry's or Tom's?

25
3
75

— Tell what you think in solving each of these problems.

2. At 2 for 25¢, 6 racing cars will cost __?__¢.

13

3. At 3 for 10¢, a dozen oranges will cost __?__¢; at 3 for 10¢, $1\frac{1}{2}$ dozen oranges will cost __?__¢.

4. At 2 for 5¢, how many plums can you buy for 25¢?

80 3

40
×5
200

5. Jean saves 40¢ a week. At this rate, how many weeks will it take her to save 2 dollars?

6. Tom walked 8 blocks in 12 minutes. Did he walk more than a block a minute, exactly a block a minute, or less than a block a minute?

7. George has 15 problems to solve for homework. It has taken him 8 minutes to do 5 of them. At that rate, how many more minutes will it take George to finish solving the problems?

8. Patricia wants to buy 28 cupcakes. If they cost 72¢ a dozen, how much will she have to pay?

9. One week Peter worked 5 hours and 15 minutes in Mr. Ramon's garden. Mr. Ramon agreed to pay him 40¢ an hour. How much should he pay Peter for the week's work?

10. Whistles cost 96¢ a dozen. How much will the hiking club have to pay for 21 whistles?

12
12
24

12 60

11. Irene wants to buy 48 inches of material priced at 60¢ a yard. How much will she have to pay for the material?

12. Mrs. Bunch drove 14 miles in 15 minutes. At that rate, how far could she drive in $1\frac{1}{2}$ hours?

13. At 80¢ a pound, how much money will you need to buy 12 ounces of popcorn?

16 oz = 1 lb

14. Murray practices the trumpet $\frac{3}{4}$ hour each day. If he practices 6 days a week, how many hours of practice time does he have each week?

2 8
3 12
3 6
4 12
 14

17
9
13
39

PROBLEM TEST 5

1. A recipe calls for $\frac{2}{3}$ cup of brown sugar and $\frac{3}{4}$ cup of granulated sugar. How much sugar is needed all together?

2. Janet used $\frac{7}{8}$ yard of material to make an apron and $\frac{3}{8}$ yard of the same material for a potholder. How much of the material did she use?

3. One evening Fred spent 2 hours doing his homework. He spent $\frac{3}{4}$ hour on mathematics. How long did he spend on the rest of his work?

4. Ruth rides her bicycle $\frac{7}{10}$ mile to school each day. Her friend Jane rides $\frac{3}{5}$ mile to school. Which girl rides farther? How much farther?

5. If you traveled around the earth along the equator, you would travel a distance of about 24,900 miles. Traveling at the rate of 75 miles per hour, how many hours of travel time would be needed to make this trip around the earth?

6. An empty room measures 17 feet by 13 feet by 9 feet. How many cubic feet of air does the room contain?

7. Mr. Howden drove 1,764 miles in 36 hours. What was his average rate of travel?

8. A baseball stadium has 58,825 seats. The stadium is separated ~~into~~ 65 sections with the same number of seats in each section. How many seats are in each section?

9. A small swimming pool is shaped like a rectangular prism. It is 12 feet long, 7 feet wide, and 4 feet deep. What is the volume of the pool?

10. Marianne charges 75¢ an hour to baby-sit. How much money should she be paid if she baby-sits for $3\frac{2}{3}$ hours?

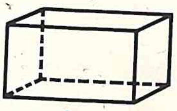
Write your score on your Problem Test Record.

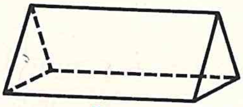
Mathamusements

- Fred's house number is a prime number and so are the house numbers of his neighbors on either side. What is Fred's house number?
- The word "prime" spelled backwards is "emirp." Let us say that an emirp number is one like 14, since if the digits in the numeral for 14 are reversed, they name 41, and 41 is a prime number. Are all prime numbers also emirp? Are all emirp numbers also prime? What numbers between 10 and 50 are both prime and emirp?

IT'S CHECK-UP TIME

1. Name each figure shown below.


A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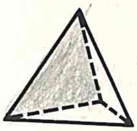
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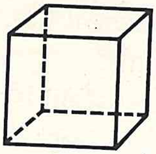
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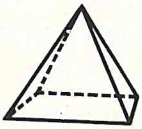
E

2. How many faces does each figure shown above have?

3. What kind of region is the base of a cone?

4. The closed path formed by the intersection of a sphere and a plane passing through the center of the sphere is called a _?_.

5. A rectangular prism with twelve edges that are equal in length is also called a _?_.

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6. The curved surface of a cylinder is called the _?_ surface.

7. To find the volume of the solid region of a rectangular prism, you multiply the length times the width times the _?_.

8. Find the greatest whole number that makes this sentence true.

$$w \times 45 \leq 214$$

9. Find the greatest multiple of 10 that makes this sentence true.

$$a \times 77 \leq 2{,}400$$

10. Find the greatest multiple of 100 that makes this sentence true.

$$l \times 36 \leq 27{,}472$$

— Copy and find the answers.

11. $9\overline{)5{,}214}$

12. $3\overline{)1{,}704}$

13. $48\overline{)3{,}884}$

14. $35\overline{)6{,}847}$

15. $87\overline{)4{,}345}$

16. $64\overline{)47{,}936}$

17. $26\overline{)2{,}759}$

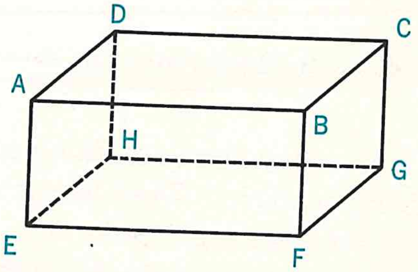
18. $91\overline{)\$719.05}$

19. What is the volume of a packing crate that is 7 feet long, 4 feet wide, and 8 feet high?

20. At 96¢ a dozen, how much will 16 cupcakes cost?

If you have time, try these —

21. Name all the segments contained in the solid region shown below that have *E* as one endpoint and any of the vertices as the other endpoint.

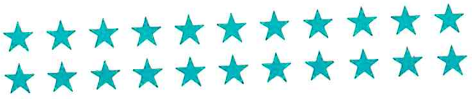


22. The volume of a box is 2,112 cubic inches. The dimensions of the box are 11 inches, 8 inches, and _?_ inches.

CHAPTER 11

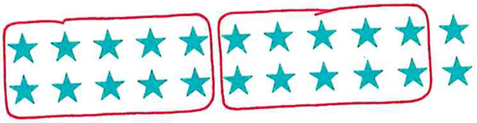
A different numeration system

In the thousands of years since he started to count, man has used many different ways to name and show numbers. The system we use today for naming numbers is called the decimal numeration system. The decimal system is based on sets of ten. This is why it is also called the **base-ten system.**



1. How many stars are shown in the set above? Write a decimal numeral to name this number.

2. To write the decimal numeral for the number of stars in the set, you thought of the stars as being grouped into sets of tens and ones.

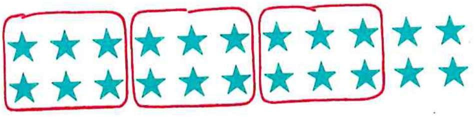


- How many stars are inside each ring drawn above? Then how many sets of ten stars are there?
- How many stars are left over?
- In the decimal numeral 22, the 2 on the left stands for ? sets of ten and the 2 on the right stands for ? sets of one.

3. Many people believe we use ten for the base because humans have ten fingers. What do you suppose might be our base if humans had only six fingers in all?

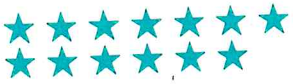
A **base-six system** is a numeration system based on sets of ?.

4. Look again at the set of stars we were considering. To write a base-six numeral for the number of stars, we must first group the stars into sets of sixes and ones.

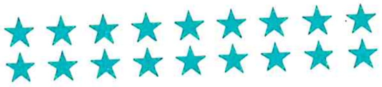


The rings show us there are ? sets of six and ? left over.

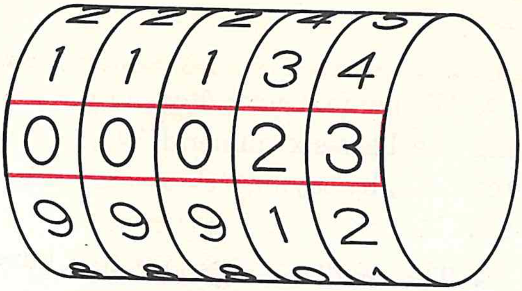
— Copy the two sets of stars shown below. Draw rings to separate them into as many sets of six as you can.



5. There are ? sets of six and ? left over.



6. There are ? sets of six and ? left over.



7. The odometer pictured above shows that the car in which it belongs has traveled 23 miles. Suppose the car travels 7 more miles.

- What happens to the wheel at the right on the odometer, where the 3 now shows? What new numeral will appear on this wheel?
- What happens to the wheel where the 2 now shows? What new numeral will appear on this wheel?

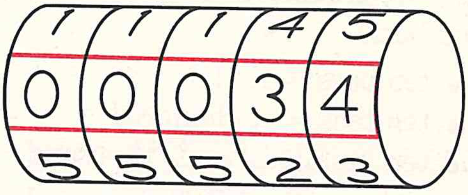
8. Think of a car with an odometer set as shown below.

$\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$

As the car starts from rest and travels its first nine miles, the wheel at the right will show each of the ten digits in turn. One mile after the first nine miles are traveled, what happens to the first two wheels at the right?

- Tell what happens one mile after the odometer shows 00019.
- Tell what happens one mile after the odometer shows 00099.
- Tell what happens one mile after the odometer shows 00999.

9. Shown below is a base-six odometer. Only six digits appear on each wheel. What are the digits?



As the car travels, the first wheel on the right will show 0, then 1, then 2, then 3, then 4, and then 5.

- What numeral will appear on the first wheel to show that the car has gone one mile more than five miles?
- What will the second wheel show to tell the driver that he has traveled one mile more than five miles?

10. On a base-six odometer, the first wheel shows ones. The second wheel shows the number of sets of six each. On the odometer pictured above, the 3 stands for 3 sixes and the 4 stands for 4 ones.

How would you read each of the following odometer settings?

$\boxed{0}\boxed{0}\boxed{0}\boxed{2}\boxed{5}$

- ? sixes ? ones

$\boxed{0}\boxed{0}\boxed{0}\boxed{4}\boxed{4}$

- ? sixes ? ones

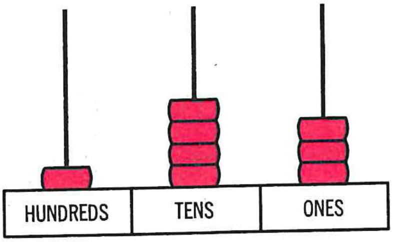
11. What will the base-six odometer show one mile after $\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{5}$? one mile after $\boxed{0}\boxed{0}\boxed{0}\boxed{1}\boxed{5}$? one mile after $\boxed{0}\boxed{0}\boxed{0}\boxed{5}\boxed{5}$?

Place value — base six

1. In the decimal or base-ten system, we group in sets of ten to name numbers.

- ten ones = $\underline{?}$ ten
- ten tens = $\underline{?}$ hundred
- ten hundreds = $\underline{?}$ thousand

2. Look at the abacus shown below.

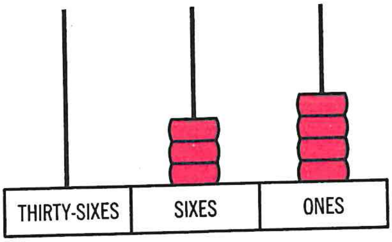


- How many beads are on the ones wire? on the tens wire? on the hundreds wire?
- Write a decimal numeral for the number represented on the abacus.
- Is this number sentence true?

143 = 1 hundred 4 tens 3 ones

3. In the base-six system, we group in sets of $\underline{?}$ to name numbers.

Look at the abacus shown below.



How many beads are on the ones wire? on the sixes wire? The beads show $\underline{?}$ sixes $\underline{?}$ ones.

4. We name 3 sixes 4 ones by writing the base-six numeral 34_{six}. We read 34_{six} by saying "three-four, base six."

- The small *six* written below and to the right of the digits is called a *subscript*. It shows that the numeral is based on sets of $\underline{?}$.
- Is this number sentence true?

34_{six} = 3 sixes 4 ones

- Why do you think we should not read 34_{six} by saying "thirty-four"?

5. Look at the base-six numeral shown at the right. In what place is the digit 2 written? In what place is the digit 4 written?

24_{six}

- The 2 stands for 2 $\underline{?}$.
- The 4 stands for 4 $\underline{?}$.
- Is this a true number sentence?

24_{six} = 2 sixes 4 ones

6. Copy the stars shown below and draw rings to show 24_{six}.

— Copy and complete each of the following sentences.

7. 12_{six} = $\underline{?}$ six $\underline{?}$ ones

8. 21_{six} = $\underline{?}$ sixes $\underline{?}$ one

9. 45_{six} = $\underline{?}$ sixes $\underline{?}$ ones

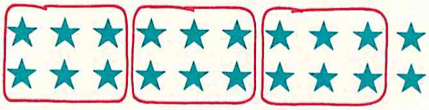
10. 11_{six} = $\underline{?}$ six $\underline{?}$ one

11. Look at the base-six numeral shown at the right. Read the numeral.

$$\boxed{10_{six}}$$

- What does the digit 1 stand for?
- The digit 0 stands for 0 $\underline{?}$.
- Does 10_{six} name the same number as the decimal numeral 10?
- Does 10_{six} name the same number as the decimal numeral 6?

12. Look at the set of stars below.



- How many sets of six are there?
- Are there any stars left over?
- Does the picture show 3 sixes 2 ones?
- Write the base-six numeral that names 3 sixes 2 ones.

— Copy and complete each of the following sentences.

13. $20_{six} = \underline{?}$ sixes $\underline{?}$ ones

14. $30_{six} = \underline{?}$ sixes $\underline{?}$ ones

15. $52_{six} = \underline{?}$ sixes $\underline{?}$ ones

16. 1 six 4 ones $= \underline{?}_{six}$

17. 5 sixes 2 ones $= \underline{?}_{six}$

18. 2 sixes 5 ones $= \underline{?}_{six}$

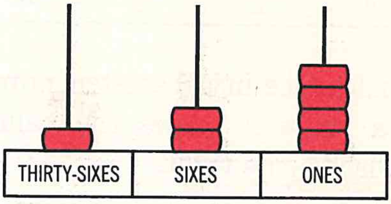
19. What base-six numeral names the next whole number after 5_{six}? after 15_{six}? 25_{six}? 35_{six}? 45_{six}?

20. Write base-six numerals for the whole numbers from 0_{six} to 55_{six}.

21. In the base-ten system, when we have ten sets of ten, we say we have one set of one hundred and we write the digit 1 in the $\underline{?}$ place.

When we have six sets of six, we say we have one set of thirty-six. In what place do you think we write the digit 1 to show six sets of six in the base-six system?

22. Look at the abacus shown below.



- On the base-six abacus, six beads on the ones wire represent the same number as 1 bead on the sixes wire. Six beads on the sixes wire represent the same number as 1 bead on the $\underline{?}$ wire.
- Write the base-six numeral for the number shown on the abacus above.
- Is this number sentence true?

$124_{six} = 1$ thirty-six 2 sixes 4 ones

— Copy and complete.

23. $213_{six} = \underline{?}$ thirty-sixes $\underline{?}$ six $\underline{?}$ ones

24. $142_{six} = \underline{?}$ thirty-six $\underline{?}$ sixes $\underline{?}$ ones

25. $\underline{?}_{six} =$ 3 thirty-sixes 1 six 5 ones

26. $\underline{?}_{six} =$ 4 thirty-sixes 2 sixes 0 ones

Base-ten and base-six numerals

1. Look at the place-value chart for base-ten numerals shown below.

THOUSANDS 10 X (10 X 10 X 1)	HUNDREDS 10 X (10 X 1)	TENS 10 X 1	ONES 1
3	2	5	4

Each place in a base-ten numeral has a value $\underline{?}$ times the value of the place at its right.

2. Now look at the place-value chart for base-six numerals shown below.

TWO HUNDRED SIXTEENS 6 X (6 X 6 X 1)	THIRTY-SIXES 6 X (6 X 1)	SIXES 6 X 1	ONES 1
3	2	5	4

Each place in a base-six numeral has a value $\underline{?}$ times the value of the place at its right.

3. In the base-ten numeral 3,254
the 4 stands for 4×1;
the 5 stands for $5 \times (\underline{?} \times 1)$;
the 2 stands for $2 \times (\underline{?} \times 10 \times 1)$;
the 3 stands for $3 \times (\underline{?} \times 10 \times 10 \times 1)$.

4. In the base-six numeral 3254_{six}
the 4 stands for 4×1;
the 5 stands for $5 \times (\underline{?} \times 1)$;
the 2 stands for $2 \times (\underline{?} \times 6 \times 1)$;
the 3 stands for $3 \times (\underline{?} \times 6 \times 6 \times 1)$.

5. Read the base-six numeral shown at the right.

$$\boxed{132_{six}}$$

- Is this number sentence true?

132_{six} = 1 thirty-six 3 sixes 2 ones

- Does 132_{six} name the same number as $(1 \times 36) + (3 \times 6) + 2$? the same number as $36 + 18 + 2$?
- Write a base-ten numeral for the number named by 132_{six}.
- Is this number sentence true?

$$132_{six} = 56$$

6. How should you read the base-six numeral shown here?

$$\boxed{203_{six}}$$

- Does 203_{six} name the same number as 2 thirty-sixes 3 ones? the same number as $(2 \times 36) + 3$?
- Write a base-ten numeral for the number named by 203_{six}.
- Is this number sentence true?

$$203_{six} = 75$$

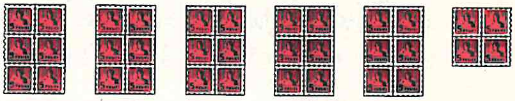
— Write a base-ten numeral for each number named below.

	a	b	c	d
7.	15_{six}	20_{six}	34_{six}	55_{six}
8.	100_{six}	135_{six}	140_{six}	213_{six}

Be your own teacher

— You now should know how to write a base-ten numeral for a number named by a base-six numeral. Now try the exercises on this page to see if you can discover how to write a base-six numeral for a number named by a base-ten numeral.

1. Tommy had thirty-four stamps. He said, "I know a way to find the base-six numeral that names this number."



He separated the thirty-four stamps into as many sets of six as he could. He found he could separate the stamps into 5 sets of six with 4 left over.

Is this sentence true? $34 = 54_{six}$

2. Robbie said, "I know another way to find the base-six numeral that names thirty-four. I will divide thirty-four by six to find the greatest number of sixes in thirty-four."

$$\begin{array}{r} 5\ \text{r}4 \\ 6\overline{)34} \\ \underline{30} \\ 4 \end{array}$$

In the division example at the right, does the 5 name the greatest number of sixes in 34? What is the remainder?

Robbie said, "Five sixes with four left over can be named by the numeral five-four, base six."

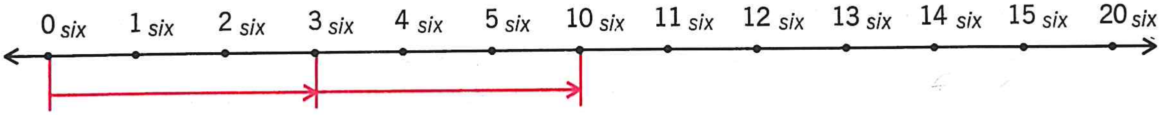
3. Tommy tried Robbie's way to find the base-six numeral that names the number nineteen. He said, "Nineteen divided by six is $\underline{\ ?\ }$ with a remainder of $\underline{\ ?\ }$. So nineteen is $\underline{\ ?\ }$ sixes $\underline{\ ?\ }$ one."

Write the base-six numeral that names nineteen.

4. Copy and complete this table, using Robbie's way.

NUMBER	SIXES	ONES	BASE-SIX NUMERAL
twenty-nine	4	5	?
thirty-one	5	?	?
fourteen	?	?	?
eight	?	?	?
thirty-five	?	?	?
sixteen	?	?	?
ten	?	?	?

Addition facts — base six



1. Does the number line above show the whole numbers in order beginning with the least whole number? If the number line were extended to the right, what base-six numeral would show the next whole number after 20_{six}?

2. The number line shows how to find the sum of 3_{six} and 3_{six}. Start at 0_{six} and move to 3_{six}. Then move three more spaces to the right. You stop at $\underline{?}$. $3_{six} + 3_{six} = \underline{?}$

3. Find the sum of 13_{six} and 3_{six} on the number line.

What do you think is the sum of 23_{six} and 3_{six}? of 33_{six} and 3_{six}? of 43_{six} and 3_{six}?

Which of these is the sum of 53_{six} and 3_{six}: 60_{six} or 100_{six}?

4. Use the number line to find the sum of 4_{six} and 3_{six}.

5. Shown at the right is a table of base-six addition facts. The arrows show how to find the sum of 4_{six} and 3_{six}. First look down the left column of the table until you find 4. Now find the column with 3 at the top. Look across the row containing 4 and down the column containing 3 until you see the box where they intersect. This box contains the numeral for the sum of 4_{six} and 3_{six}. $4_{six} + 3_{six} = \underline{?}$

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	10
2	2	3	4	5	10	11
3	3	4	5	10	11	12
4	4	5	10	11	12	13
5	5	10	11	12	13	14

6. Now use the table to find the sum of 2_{six} and 5_{six}. Find the row with 2 at the left and the column with 5 at the top. The numeral in the box formed by the intersection of this row and this column is $\underline{?}$. $2_{six} + 5_{six} = \underline{?}$

7. Use the table to find these sums.

a	b	c	d
$5_{six} + 5_{six} = \underline{?}$	$3_{six} + 4_{six} = \underline{?}$	$4_{six} + 5_{six} = \underline{?}$	$5_{six} + 3_{six} = \underline{?}$

8. Copy and solve. Use the table or the number line if you need help.

$13_{six} + 22_{six} = a$ $\quad$ $42_{six} + 52_{six} = n$ $\quad$ $45_{six} + 30_{six} = m$ $\quad$ $24_{six} + 41_{six} = y$

Be your own teacher

Try the work on this page to see if you can add using base-six numerals. Every numeral with digits on this page is a base-six numeral, so the subscript *six* has been omitted.

1. You know that a name for 45 is 4 sixes 5 ones. Is 3 sixes 15 ones another name for 45? Explain.

Tell two other names for 45.

2. Write a base-six numeral for 12 sixes 23 ones; for 55 sixes 12 ones.

3. The example below shows one way to find the sum of 34 and 14.

$$\begin{array}{rcl} 34 & = & \text{3 sixes} \quad \text{4 ones} \\ \underline{+\ 14} & = & \underline{\text{1 six} \quad\ \ \text{4 ones}} \\ & & \text{4 sixes 12 ones} = \\ & & \text{5 sixes 2 ones, or 52} \end{array}$$

How was 4 sixes 12 ones renamed above? Why was it renamed?

4. At the right is another way to show the addition of 34 and 14.

$$\boxed{\begin{array}{r} {}^{1} \\ 34 \\ +\ 14 \\ \hline 52 \end{array}}$$

- What is the greatest number that can be shown in the ones place of a base-six numeral? Is the sum of 4 and 4 greater than 5? Then is it necessary to rename the sum of the ones when you add 34 and 14?
- Add the ones. $4 + 4 = 12$ You write the 2 in the ones place. Where do you write the 1?
- What is the sum of the sixes?
- $34 + 14 = \underline{?}$

5. The example below is one way to show the addition of 43 and 45. Explain it.

$$\begin{array}{rcl} 43 & = & 40 + 3 \\ \underline{+\ 45} & = & \underline{40 + 5} \\ & & 120 + 12, \text{ or } 132 \end{array}$$

10. Explain this way to show the addition of 43 and 45.

$$\boxed{\begin{array}{r} {}^{1} \\ 43 \\ +\ 45 \\ \hline 132 \end{array}}$$

In the answer, the 2 stands for 2 ones. What does the 3 stand for? the 1?

11. To check your answer to the example in Ex. 10, express each addend as a base-ten numeral and add the numbers again.

What base-ten numeral names the same number as 4 sixes 3 ones? as 4 sixes 5 ones? Is 132 the base-six numeral for the sum of these numbers?

— Copy and find the sums. Use the number line or the addition-facts table on page 232 to help you.

	a	b	c	d
12.	$\begin{array}{r}24\\ +\ 23\\ \hline\end{array}$	$\begin{array}{r}21\\ +\ 35\\ \hline\end{array}$	$\begin{array}{r}14\\ +\ 13\\ \hline\end{array}$	$\begin{array}{r}15\\ +\ 32\\ \hline\end{array}$
13.	$\begin{array}{r}44\\ +\ 12\\ \hline\end{array}$	$\begin{array}{r}25\\ +\ 33\\ \hline\end{array}$	$\begin{array}{r}54\\ +\ 41\\ \hline\end{array}$	$\begin{array}{r}42\\ +\ 22\\ \hline\end{array}$

14. Check your answers to Ex. 12 and 13 by expressing the addends as base-ten numerals and adding again.

Keeping up in mathematics

Oral

1. Read this numeral. 8,040,369

2. What number is 500 greater than 4,598? 500 less than 4,598?

— Tell the products.

	a	b
3.	$10 \times 77 = \underline{?}$	$10 \times 606 = \underline{?}$
4.	$100 \times 56 = \underline{?}$	$100 \times 89 = \underline{?}$

— Tell the factors.

	a	b
5.	$50 \div 10 = \underline{?}$	$80 \div 20 = \underline{?}$
6.	$250 \div 5 = \underline{?}$	$600 \div 30 = \underline{?}$

7. The sum of the lengths of the sides of a rectangle is called the $\underline{?}$ of the rectangle.

8. The measure of a rectangular region is called the $\underline{?}$ of the region.

9. A rectangle is 4 feet long and 3 feet wide. What is the perimeter of the rectangle? What is the area of the rectangular region?

10. The dimensions of a rectangular box are 6 inches long, 3 inches wide, and 2 inches high. Find its volume.

11. What is the average of 5, 10, and 15?

12. A car traveled 84 miles in 2 hours, or an average of $\underline{?}$ mph.

13. How long will it take a plane to go 1,000 miles if it is flying at the average rate of 200 miles an hour?

14. Does $378 \div 36$ equal about 10 or about 100?

Written

— Copy, find the answers, and check.

	a	b	c
1.	$\begin{array}{r}1{,}686\\ +\ 279\\ \hline\end{array}$	$\begin{array}{r}2{,}804\\ +\ 376\\ \hline\end{array}$	$\begin{array}{r}\$40.09\\ +\ 9.85\\ \hline\end{array}$
2.	$\begin{array}{r}2{,}468\\ -\ 297\\ \hline\end{array}$	$\begin{array}{r}76{,}509\\ -\ 10{,}865\\ \hline\end{array}$	$\begin{array}{r}\$893.90\\ -\ 209.88\\ \hline\end{array}$
3.	$\begin{array}{r}7{,}548\\ \times\ 39\\ \hline\end{array}$	$\begin{array}{r}968\\ \times\ 407\\ \hline\end{array}$	$\begin{array}{r}718\\ \times\ 960\\ \hline\end{array}$
4.	$7\overline{)8{,}143}$	$8\overline{)9{,}040}$	$6\overline{)5{,}000}$

5. Divide 11,674 by 96. Check your answer.

6. Divide 24,400 by 48. Check your answer.

7. The dimensions of a rectangle are 16 feet by 9 feet. Find its perimeter. Find the area of the rectangular region.

$\begin{array}{r}16\\ 2\\ \hline 32\\ 18\\ \hline 50\end{array}$ $\begin{array}{r}9\\ \hline 18\end{array}$

8. On a 4-day trip, Dean's father traveled these distances: 456 miles, 387 miles, 279 miles, and 306 miles. What is the average number of miles he traveled each day?

9. How many yards are equivalent in length to 612 inches?

10. Write the numeral that names a number

- 21,000 greater than 8,746,250
- 21,000 less than 8,746,250.

A new notation in the decimal numeration system

1. One day Mr. Abrams took his son Jerry to work with him. Mr. Abrams test-drives new cars as they come off the assembly line at the factory.

When Jerry and his father got into the first car, the odometer looked like this.

0	0	0	0	0	0

Jerry said, "The mileage marker is at zero. How far is it around the test track?"

"About two and one-half miles," said his father.

After they had driven a short way, the odometer looked like this.

0	0	0	0	0	5

"The mileage marker isn't working," exclaimed Jerry. "I know we haven't driven five miles yet."

"The odometer is all right," said his father. "The blue numerals do not show miles but tenths of a mile. We have traveled five-tenths or one-half mile." $\frac{5}{10} = \frac{?}{2}$

2. Jerry watched the numerals change on the odometer. He read them, "6 tenths, 7 tenths, 8 tenths, 9 tenths." Then a 0 appeared in the tenths place and a 1 appeared in the place beside it. How far had they traveled? $\frac{10}{10} = \underline{?}$

3. Jerry said, "Ten tenths of a mile is equal to one mile."

0	0	0	0	1	0

Jerry continued to count as the numerals appeared on the odometer.

"1 and 1 tenth,	0	0	0	0	1	1
1 and 2 tenths,	0	0	0	0	1	2
1 and 3 tenths,	0	0	0	0	1	3
1 and _?_ tenths,	0	0	0	0	1	4
1 and _?_ tenths,	0	0	0	0	1	5
_?_ and _?_ tenths,	0	0	0	0	1	6
_?_ and _?_ tenths,	0	0	0	0	1	7
_?_ and _?_ tenths,	0	0	0	0	1	8
_?_ and _?_ tenths."	0	0	0	0	1	9

4. Draw a picture of what the odometer showed after they had driven a tenth mile farther than 1 and 9 tenths miles.

5. How should you read the mileage on each odometer shown below?

0	0	0	0	2	1

0	0	0	0	2	2

0	0	0	0	2	3

0	0	0	0	2	4

6. When they had gone completely around the test track, the odometer looked like this.

0	0	0	0	2	5

How many miles did Jerry's father say it was around the track? Was he correct? $2\frac{?}{10} = 2\frac{1}{2}$

Writing decimals

There are two ways of expressing the fractional numbers shown on the odometer pictured on page 235. One way is to write a fraction such as $\frac{7}{10}$; the other way is to write a **decimal** such as 0.7. Both ways are shown below.

FRACTION	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$	$1\frac{1}{10}$	$1\frac{2}{10}$
DECIMAL	0.7	0.8	0.9	1.0	1.1	1.2

The dot in the decimal 0.7 is called the **decimal point.**

Both $\frac{7}{10}$ and 0.7 name the same number. Both are read the same way, as "seven tenths."

One and four tenths may be named using the mixed form $1\frac{4}{10}$ or the decimal 1.4. They are read the same way, "one and four tenths."

The decimal point in a decimal separates the ones place and the tenths place. In 1.4, for example, the digit 1 is in the ones place and the digit 4 is in the tenths place.

— Copy and complete each set shown below.

1. {0.7, 0.8, 0.9, 1.0, ?, ?, ?}
2. {1.7, 1.8, 1.9, ?, 2.1, ?, ?}
3. {?, 3.1, 3.2, ?, ?, ?, 3.6}
4. {0.2, 0.4, 0.6, 0.8, ?, 1.2, ?}
5. {1.4, 1.3, ?, ?, 1.0, ?, ?}
6. {3.4, 3.2, ?, 2.8, ?, ?, ?}
7. {6.5, 7.0, ?, 8.0, ?, ?, 9.5}
8. {2.0, 2.5, ?, ?, 4.0, ?, ?}
9. {7.0, 6.5, ?, 5.5, ?, ?, ?}
10. {0.3, 0.6, ?, ?, 1.5, ?, ?}

— Write a decimal for each number.

	a	b	c	d
11.	$\frac{7}{10}$	$1\frac{2}{10}$	$2\frac{9}{10}$	$3\frac{0}{10}$
12.	$6\frac{7}{10}$	$10\frac{5}{10}$	$24\frac{8}{10}$	$146\frac{3}{10}$

13. What fraction names the same number as 0.8?

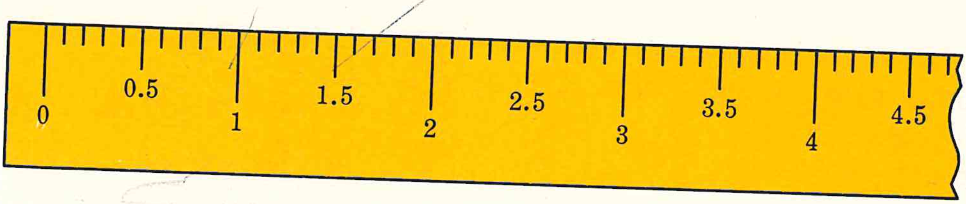
— Use mixed forms to name the numbers shown below.

14.	1.1	2.6	5.9	7.0
15.	8.3	15.1	112.3	125.5

— Write a fraction and a decimal for each number.

	a	b
16.	nine tenths	one and six tenths
17.	one tenth	ten and five tenths

Tenths as decimals



1. How would you use the ruler above to draw a line $\frac{1}{10}$ inch long?

2. Point to the mark on the ruler for 0.5 inch; for $\frac{5}{10}$ inch; for $\frac{1}{2}$ inch.
Is this number sentence true?

$$0.5 = \frac{5}{10} = \frac{1}{2}$$

— Use the ruler to show that the following number sentences are true.

	a	b
3.	$1.5 = 1\frac{1}{2}$	$2.5 = 2\frac{1}{2}$
4.	$3.5 = 3\frac{1}{2}$	$4.5 = 4\frac{1}{2}$

— Use the ruler to show that each sentence below is true.

5. 0.2 greater than 1.8 is 2.0

6. 1.3 greater than 0.7 is 2.0

7. 0.9 less than 2.1 is 1.2

8. 1.5 less than 3.0 is 1.5

9. Copy and complete the chart. Use the ruler if you need help.

ADDEND	ADDEND	SUM
0.7	0.4	?
1.6	0.8	?
2.6	?	3.2
?	0.2	4.0
2.7	1.6	?
0.5	?	4.1

10. A group in the fifth grade used a steel tape marked in tenths of feet to measure their heights. The measurements are given below. Read them.

Mary	4.2′	Joe	5.4′
John	5.1′	Tom	4.3′
Bill	4.8′	Nancy	4.7′
Bonnie	4.3′	Joan	5.1′

- Who is tallest?
- Which girl is tallest?
- How much taller is Joe than Bill? than John?
- How much taller is Joan than Nancy? than Bonnie?

11. The normal April rainfall for six cities is shown below.

Boston, Mass.	3.8 inches
Brownsville, Tex.	1.6 inches
New Orleans, La.	5.4 inches
Omaha, Neb.	2.6 inches
Phoenix, Ariz.	0.3 inch
San Diego, Calif.	0.8 inch

Arrange the names of the cities in order from the one with the least amount of rainfall to the one with the greatest amount.

Hundredths as decimals

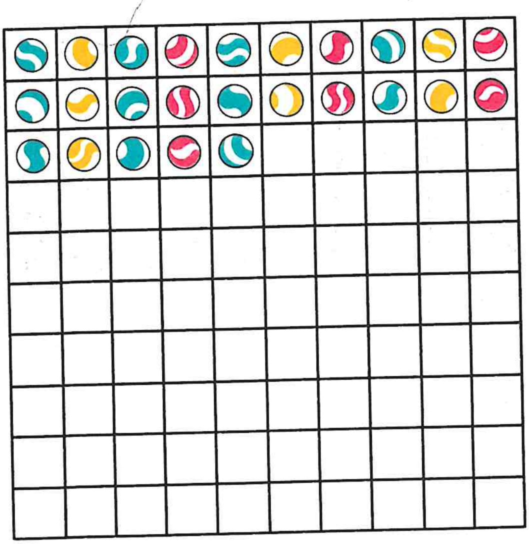
1. The picture at the right shows a square board separated into 100 small boxes.

The picture shows marbles in $\underline{?}$ of the small boxes.

We can name 25 out of 100 with a fraction, $\frac{25}{100}$, or with a decimal, 0.25.

How does the board show a numerator of 25? a denominator of 100?

On the board, 1 marble will fill $\frac{1}{100}$ of the boxes; 100 marbles will fill $\frac{?}{100}$ of the boxes. Is this number sentence true? $\frac{100}{100} = 1$



2. Copy and complete the table below.

FRACTION	$\frac{10}{100}$	?	$\frac{85}{100}$	$\frac{100}{100}$
DECIMAL	0.10	0.05	?	?

3. How many places to the right of the decimal point are needed to express hundredths?

4. Write a fraction and a decimal to show what part of the board above is full when 38 of the boxes contain marbles; when 65 of the boxes contain marbles; when 89 of the boxes contain marbles; when 14 of the boxes contain marbles.

5. Read these decimals.

0.05	0.07	0.09	0.16	0.75
0.82	0.60	0.06	0.99	0.24

— Copy and complete each of the following sets of decimals.

6. {0.03, 0.04, 0.05, $\underline{?}$, $\underline{?}$, $\underline{?}$}

7. {0.08, 0.09, $\underline{?}$, 0.11, $\underline{?}$, $\underline{?}$}

8. {0.50, 0.55, 0.60, $\underline{?}$, $\underline{?}$, $\underline{?}$}

9. Express as decimals.

$\frac{3}{100}$	$\frac{1}{100}$	$\frac{19}{100}$	$\frac{6}{100}$	$\frac{60}{100}$
$\frac{34}{100}$	$\frac{5}{100}$	$\frac{12}{100}$	$\frac{99}{100}$	$\frac{4}{100}$

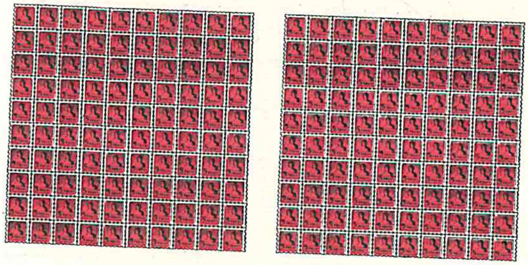
10. One row of marbles fills what part of the board shown above? Can this number be named by any of the following numerals?

$\frac{10}{100}$ $\frac{1}{10}$ 0.10 0.1

11. What part of the board above is filled by 2 rows of marbles? 3 rows? 5 rows?

Express each answer in four ways as was done in Ex. 10.

Understanding hundredths



1. Shown above are two sheets of stamps. Each sheet contains 100 stamps. Each stamp is $\frac{1}{100}$ or 0.01 of a sheet.

If 1 stamp is 0.01 of a sheet, then 20 stamps are what part of a sheet?

2. If 120 stamps equal 1.20 sheets, then 245 stamps equal ? sheets.

3. Each row of stamps on a sheet contains 10 stamps. Then does each row equal 0.10 of a sheet?

- If 1 row equals 0.10 of a sheet, then ? rows equal 0.60 of a sheet.
- If 14 rows equal 1.40 sheets, then ? rows equal 2.70 sheets.

— Write a fraction to name each number shown below.

	a	b	c	d	e
4.	0.15	0.08	0.40	0.04	0.4
5.	2.03	3.60	1.75	9.34	28.05

— Write a decimal to name each number shown below.

6.	$\frac{9}{10}$	$\frac{90}{100}$	$\frac{9}{100}$	$2\frac{7}{10}$	$3\frac{7}{100}$
7.	$5\frac{70}{100}$	$6\frac{8}{100}$	$8\frac{88}{100}$	$6\frac{80}{100}$	$7\frac{8}{10}$

8. Arrange in order beginning with the name for the least number.

5 50 0.05 0.5 1.0

9. Arrange in order beginning with the name for the least number.

0.70 0.07 1.00 7 70

10. Name a number that is

- 0.05 greater than 5.04
- 0.05 greater than 2.98
- 0.05 less than 4.00

— Count by hundredths following the pattern shown by the first three members of each set below. Then copy and complete the sets.

11. {0.38, 0.39, 0.40, ?, ?, ?, 0.44, ?, ?, ?}

12. {0.70, 0.69, 0.68, ?, ?, ?, ?, 0.63, ?, ?}

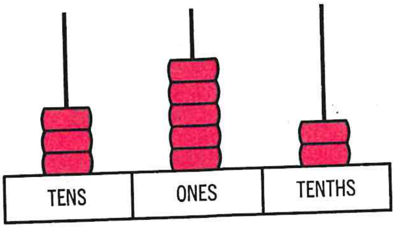
13. {2.00, 2.05, 2.10, ?, ?, ?, ?, ?, 2.40, ?}

14. {3.30, 3.20, 3.10, ?, ?, 2.80, ?, ?, ?, ?}

15. {2.50, 3.00, 3.50, ?, ?, ?, ?, ?, ?, 7.00}

16. {5.40, 5.20, 5.00, ?, ?, ?, ?, 4.00, ?, ?}

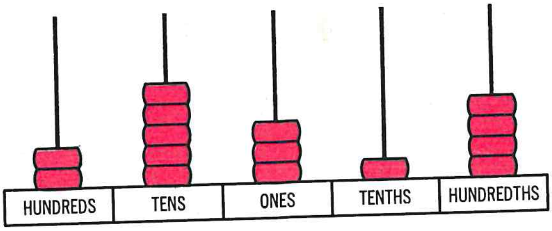
Tenths and hundredths



1. The abacus above shows the tens place, the ones place, and the tenths place.

- How many beads are on the tens wire? the ones wire? the tenths wire?
- The abacus shows ? tens ? ones ? tenths.
- Is this number sentence true?

  3 tens 5 ones 2 tenths = 35.2
- In the numeral 35.2, why is there a decimal point between the 5 and the 2?



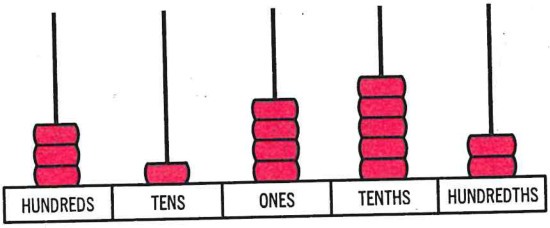
2. What places are shown by the wires on the abacus above? The beads on these wires show ? hundreds ? tens ? ones ? tenth ? hundredths.

Which of the following decimals names the number shown on the abacus?

2,531.4 253.14 25.314

3. Draw an abacus similar to the one in Ex. 2. Mark beads on the wires to show 415.27.

4. Name the number shown on the abacus below using a decimal.



- In the decimal you wrote, is the tenths place one place to the *right* of the ones place? The tens place is one place to the ? of the ones place.
- How many places to the right of the ones place is the hundredths place?
- The hundreds place is ? places to the ? of the ones place.

5. Tell what each digit stands for in the numeral 314.52.

If you forgot to mark the decimal point in 314.52, what would each digit stand for?

— Write a decimal to name each number shown below.

6. 3 tens 4 ones 5 tenths

7. 6 hundreds 2 tens 3 ones 8 tenths

8. 4 tens 6 ones 2 tenths 7 hundredths

9. Ten hundredths is another name for ? tenth. $\frac{10}{100} = \frac{?}{10}$ Ten tenths is another name for ? one. $\frac{10}{10} = ?$ Ten ones is another name for ? ten. Ten tens is another name for ? hundred.

10. Does the value of any digit, such as 5, depend upon its place in a decimal numeral? Look at the numeral 555.55. Does each 5 in this numeral have a different place value?

This 5 stands for 5 hundreds — 500.00
This 5 stands for 5 tens — 50.00
This 5 stands for 5 ones — 5.00
This 5 stands for 5 tenths — 0.50
This 5 stands for 5 hundredths — + 0.05
555.55 — 555.55

11. Tell what each digit stands for in 3.95; in 76.52; in 405.28; in 37.8; in 593.04.

12. Tell where to place the decimal point in 230 to have the 2 stand for ones; to have the 3 stand for ones; to have the 0 stand for ones.

13. Which decimal names the greater number, 0.02 or 0.20?

— For each set of decimals, arrange the numerals in order beginning with the name for the least number.

14. {0.05, 0.50, 50.00, 5.00}

15. {8.00, 0.08, 80.00, 0.80}

16. {3.40, 34.00, 0.34, 340.00}

17. {420.00, 0.42, 42.00, 4.20}

18. {100.00, 10.00, 1.00, 0.10, 0.01}

19. Write a numeral that stands for:

- 0.5 greater than 30.0
- 0.06 greater than 2.60
- 0.1 greater than 14.0
- 0.20 greater than 2.02
- 0.07 greater than 8.70
- 0.13 greater than 4.31
- 0.33 greater than 3.40
- 0.72 greater than 0.01

20. In the numeral 7.63, if you change the 6 to a 4, the new number named will be ? less than 7.63.

21. Write a numeral that stands for:

- 0.4 less than 7.4
- 0.06 less than 9.06
- 0.03 less than 2.83
- 0.1 less than 6.7
- 2.0 less than 2.4
- 0.12 less than 6.78
- 0.26 less than 4.66
- 0.04 less than 8.08

United States money and decimals

1. One dollar is the same amount of money as ? cents. What coin is worth $\frac{1}{100}$ of a dollar?

Two ways to express $\frac{1}{100}$ of a dollar are $.01 and 1¢.

2. What part of a dollar is $.02? $.03? $.04? $.05? $.10?

3. If $.17 means $\frac{17}{100}$ of a dollar, what part of a dollar is $.21? $.35? $.69? $.83? $.98? $.20?

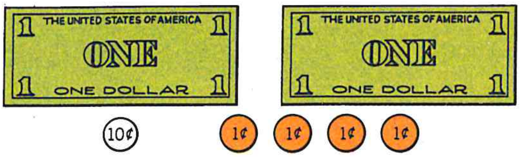
4. We read $2.34 as "two dollars and thirty-four cents," or as "two and thirty-four hundredths dollars."

Read each of these in two ways.

$3.49 $2.08 $12.10 $1.01 $6.66

5. One dollar is the same amount of money as ? dimes. What coin is worth $\frac{1}{10}$ of a dollar?

Is $\frac{1}{10}$ of a dollar the same amount as 10¢? as $.10?


THE UNITED STATES OF AMERICA
ONE
ONE DOLLAR
1
THE UNITED STATES OF AMERICA
ONE
ONE DOLLAR
1
10¢
1¢
1¢
1¢
1¢

6. Tom said, "In $2.14, the cents point can be thought of as a decimal point. The 2 stands for 2 dollars; the 1 stands for 1 dime or 1 tenth of a dollar; and the 4 stands for 4 cents or 4 hundredths of a dollar."

What would Tom say each digit stands for in each of the following?

$2.35 $1.48 $.68 $.04

7. Judy said, "When I think of $2.35 as 2 dollar bills and 35 pennies, I think of it as 2 and 35 hundredths dollars.

"But when I think of $2.35 as 2 dollar bills, 3 dimes, and 5 pennies, I think of it as 2 dollars, ? tenths of a dollar, and ? hundredths of a dollar."

8. In $1.11, the 1 at the left of the cents or decimal point stands for 1 whole dollar.

What does the first 1 at the right of the point stand for? the second 1 at the right of the point?

9. Sue said, "Dollars, dimes, and pennies make me think of ones, tenths, and ?."

— Using a dollar sign and cents point, write symbols for each of these amounts of money.

10. 3 dollars, 4 tenths of a dollar, and 7 hundredths of a dollar

11. 2 and 16 hundredths dollars

12. 1 and 10 hundredths dollars

13. 5 and 5 hundredths dollars

14. 5 and 5 tenths dollars

15. 5 and 55 hundredths dollars

16. 6 and 3 tenths dollars

17. 3 and 6 tenths dollars

18. 4 and 11 hundredths dollars

19. 7 and 9 hundredths dollars

20. 8 and 30 hundredths dollars

Changing notation

— Use a decimal to name each number shown below.

	a	b	c	d	e	f	g	h
1.	$\frac{7}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{4}{10}$	$\frac{6}{10}$	$\frac{5}{10}$
2.	$\frac{3}{100}$	$\frac{4}{100}$	$\frac{5}{100}$	$\frac{6}{100}$	$\frac{2}{100}$	$\frac{7}{100}$	$\frac{9}{100}$	$\frac{8}{100}$
3.	$\frac{13}{100}$	$\frac{25}{100}$	$\frac{18}{100}$	$\frac{32}{100}$	$\frac{45}{100}$	$\frac{83}{100}$	$\frac{50}{100}$	$\frac{75}{100}$
4.	$1\frac{1}{10}$	$3\frac{5}{10}$	$2\frac{7}{100}$	$6\frac{70}{100}$	$4\frac{23}{100}$	$9\frac{4}{100}$	$6\frac{4}{10}$	$8\frac{40}{100}$

— Use a fraction to name each number shown below.

5.	0.6	0.8	0.9	0.03	0.09	0.13	0.75	0.25
6.	1.2	3.7	4.6	8.25	2.07	2.70	3.15	2.75
7.	0.07	0.70	7.07	7.70	7.7	0.7	77.7	7.77

8. Bill has a sheet of 100 postage stamps. Write both a fraction and a decimal to show what part of the set of stamps he tears off when he tears off 5 stamps; when he tears off 10 stamps; when he tears off 25 stamps; 50 stamps; 45 stamps; 75 stamps; 95 stamps.

9. June says 10 stamps are 0.10 of the 100 stamps. Jerry says 10 stamps are 0.1 of the 100 stamps. Is $\frac{10}{100}$ equivalent to $\frac{1}{10}$?

Then do you think that 0.10 is equivalent to 0.1?

10. A metal plate is 0.2 inch thick. How many such plates placed one on top of the other would be an inch thick?

— Where should the decimal point be placed in the numerals in Ex. 11–16 to make sensible sentences?

11. Boots cost $475 a pair.

12. A boy's baseball mitt cost $650.

13. Mary weighs 895 pounds.

14. Eggs sell for $75 a dozen.

15. The classroom is 125 feet high.

16. John's shoes are 105 inches long.

— Copy and replace the ● with > or < to make a true sentence.

	a	b
17.	0.5 ● 0.6	0.21 ● 0.14
18.	0.40 ● 0.04	3.2 ● 3.9
19.	6.9 ● 8.4	6.1 ● 1.6
20.	0.80 ● 0.82	0.90 ● 0.09

Finding equivalent decimals

1. A decimal that is equivalent to $\frac{5}{10}$ is $\underline{?}$. Is $\frac{1}{2}$ also equivalent to $\frac{5}{10}$? Then is the following number sentence true?

$$0.5 = \frac{1}{2}$$

2. Mark wanted to find a decimal equivalent to $\frac{2}{5}$.

- He first found a fraction with a denominator of 10 or 100 that is equivalent to $\frac{2}{5}$.

$$\frac{2}{5} = \frac{2 \times 2}{2 \times 5} = \frac{?}{10}$$

- Then he renamed the number shown by $\frac{4}{10}$ as a decimal. Is the following number sentence true?

$$\frac{4}{10} = 0.4$$

3. Use Mark's way to find a decimal that is equivalent to $\frac{13}{20}$ by replacing n so that this number sentence is true.

$$\frac{13}{20} = \frac{n \times 13}{n \times 20} = \frac{65}{100}$$

What decimal is equivalent to $\frac{65}{100}$?

4. Equivalent decimals can be found for many fractions, such as $\frac{4}{5}$ or $\frac{3}{4}$, in this way: Rename the fractional number using a fraction with a denominator of 10 or 100. Then write the decimal that is equivalent to this fraction.

Copy and complete the following sentences.

$\frac{4}{5} = \frac{?}{10} = 0.8$ $\qquad$ $\frac{3}{4} = \frac{?}{100} = 0.??$

5. Copy and complete this chart.

FRACTION	EQUIVALENT FRACTION	EQUIVALENT DECIMAL
$\frac{1}{4}$	$\frac{25 \times 1}{25 \times 4} = \frac{25}{100}$	0.25
$\frac{1}{5}$	$\frac{2 \times 1}{2 \times 5} = \frac{2}{10}$	?
$\frac{18}{25}$	$\frac{4 \times 18}{4 \times 25} = \frac{?}{?}$	?
$\frac{9}{20}$	$\frac{? \times 9}{? \times 20} = \frac{?}{100}$	?

6. For each fraction shown below, find an equivalent decimal.

$\frac{3}{4}$ $\quad$ $\frac{4}{5}$ $\quad$ $\frac{19}{20}$ $\quad$ $\frac{27}{50}$

7. Count by halves two ways to complete this chart.

$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	?	?	?
0.5	1.0	1.5	2.0	?	3.0	?	?

8. Count by fourths two ways to complete this chart.

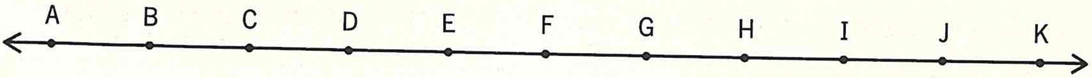
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	?	?
0.25	0.50	?	1.00	1.25	?	?

— Write an equivalent decimal for each number named below.

9. $5\frac{1}{2}$ $\quad$ $9\frac{3}{4}$ $\quad$ $3\frac{7}{10}$ $\quad$ $4\frac{3}{5}$ $\quad$ $5\frac{7}{20}$

10. $3\frac{2}{5}$ $\quad$ $7\frac{1}{4}$ $\quad$ $5\frac{3}{25}$ $\quad$ $6\frac{4}{50}$ $\quad$ $2\frac{17}{25}$

Numbers and the number line



Points on the number line above are named by capital letters.

1. Suppose the number 0 is matched with point *A* and 1 is matched with point *B*. Then what number would be matched with point *C*? point *D*? *E*? *F*? *G*?

2. Now suppose 50 is matched with point *B* and 100 is matched with point *C*. Then what number would be matched with point *D*?

What number would be matched with each of the other points named on the line?

— If you know what numbers are matched with any two of the points named on the line, can you tell the numbers that would be matched with all the other points named? Try it.

3. If 10 is matched with *B*, and 20 is matched with *C*, then $\underline{?}$ is matched with *G*.

4. If 2 is matched with *B* and 6 is matched with *D*, then $\underline{?}$ is matched with *C*.

5. If 200 is matched with *C* and 400 with *E*, then $\underline{?}$ is matched with *B* and $\underline{?}$ with *D*.

6. If $\frac{1}{2}$ is matched with *B* and $1\frac{1}{2}$ is matched with *D*, then $\underline{?}$ is matched with *C*.

7. If 0.5 is matched with *B* and 1.5 with *D*, then $\underline{?}$ is matched with *C* and $\underline{?}$ with *E*.

8. If 0.1 is matched with *B* and 0.4 with *E*, then $\underline{?}$ is matched with *G* and $\underline{?}$ with *K*.

9. If $\frac{2}{3}$ is matched with *C* and $1\frac{2}{3}$ with *F*, then $\underline{?}$ is matched with *B*, $\underline{?}$ with *H*, and $\underline{?}$ with *K*.

10. If *B* is matched with 20 and *C* is matched with 40, then point $\underline{?}$ is matched with 60.

11. If *C* is matched with $\frac{2}{5}$ and *E* with $\frac{4}{5}$, then $\underline{?}$ is matched with $\frac{1}{5}$, $\underline{?}$ with $1\frac{1}{5}$, and $\underline{?}$ with 1.

12. If *B* is matched with 0.25 and *C* with 0.50, then $\underline{?}$ is matched with 0.75, $\underline{?}$ with 1.00, $\underline{?}$ with 1.75, and $\underline{?}$ with 2.50.

13. If *B* is matched with 0.25 and *C* with 0.50, then *F* is matched with $\underline{?}$, *G* with $\underline{?}$, and *J* with $\underline{?}$.

14. If *B* is matched with 0.04 and *D* with 0.12, then *C* is matched with $\underline{?}$ and *E* with $\underline{?}$.

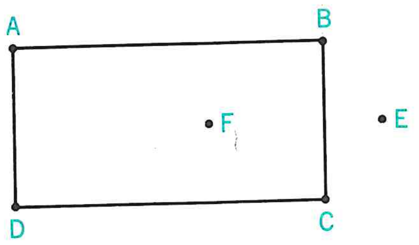
SELF-HELP TEST 9

1. The inverse of addition is ?. [30]

2. Solve this sentence. [49]

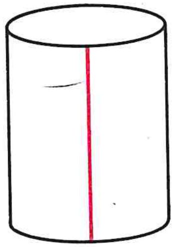
$3 \times 47 = (3 \times n) + (3 \times 7)$

3. Name the points shown below that are part of the rectangular region. [78]



4. What is the greatest common factor of 7 and 21? [181]

5. What is the least common multiple of 8 and 12? [189]



6. If the lateral surface of this cylinder could be cut along the red line segment and laid out flat, what kind of region would it form? [205]

7. Which pairs of fractions below are equivalent fractions? [114–15]

$\frac{3}{5}, \frac{4}{5}$ $\frac{2}{3}, \frac{4}{6}$ [✓ $\frac{1}{2}, \frac{2}{4}$] $\frac{3}{8}, \frac{1}{8}$

8. Which pairs of fractions above are like fractions? unlike fractions? [183]

9. List the set of whole-number factors for each number named below.

25 7 12 16 19 [180]

10. Which of the numbers named above are prime numbers? Which are composite numbers? [180]

SELF-HELP TEST 10

Copy and find the answers.

1. $\begin{array}{r} \frac{7}{8} \\ + \frac{3}{4} \\ \hline \end{array}$ [192]

2. $\begin{array}{r} \frac{1}{3} \\ - \frac{1}{4} \\ \hline \end{array}$ [194]

3. $\begin{array}{r} \frac{2}{5} \\ + \frac{5}{7} \\ \hline \end{array}$ [192]

4. $\begin{array}{r} \frac{1}{2} \\ - \frac{4}{9} \\ \hline \end{array}$ [194]

5. $\begin{array}{r} \frac{2}{3} \\ \frac{3}{4} \\ + \frac{1}{12} \\ \hline \end{array}$ [192]

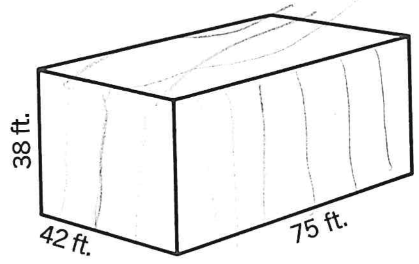
6. $\begin{array}{r} 7 \\ - \frac{3}{8} \\ \hline \end{array}$ [199]

7. $84\overline{)8{,}516}$ [218–19]

8. $32\overline{)\$27.54}$ [146]

9. How many triangles are formed by the edges of a triangular pyramid? a triangular prism? a square pyramid? [204]

10. Find the volume of the solid region shown below. [208–09]



IT'S CHECK-UP TIME

1. Write a decimal to name each number shown below.

$6\frac{7}{10}$ $4\frac{16}{100}$ $23\frac{87}{100}$ $5\frac{3}{10}$ $6\frac{3}{100}$

2. Write a fraction or a mixed form to name each number shown below.

0.7 4.3 8.29 6.09 60.9

— Study each set shown below for a pattern. Then copy and complete the set.

3. $\{3.5, 3.7, 3.9, \underline{\ ?\ }, \underline{\ ?\ }\}$

4. $\{7.75, 7.80, 7.85, \underline{\ ?\ }, \underline{\ ?\ }\}$

5. $\{1.6, 1.4, 1.2, \underline{\ ?\ }, \underline{\ ?\ }\}$

6. $\{4.24, 4.20, 4.16, \underline{\ ?\ }, \underline{\ ?\ }\}$

7. Write all the digits used in the base-six numeration system.

8. 2 sixes 4 ones = $\underline{\ ?\ }_{six}$

9. What base-six numeral names the next whole number after 35_{six}?

10. 332_{six} = $\underline{\ ?\ }$ thirty-sixes $\underline{\ ?\ }$ sixes $\underline{\ ?\ }$ ones

11. In 410_{six}, the digit 4 stands for $4 \times 6 \times \underline{\ ?\ }$.

12. Write a base-ten numeral for each number named below.

12_{six} 33_{six} 101_{six} 43_{six} 124_{six}

13. Write the decimal that names 6 hundreds 4 tens 2 ones 7 tenths.

14. Write the decimal that names two hundred and four hundredths.

15. What number is 0.6 greater than 7.0?

16. What number is 0.11 less than 12.97?

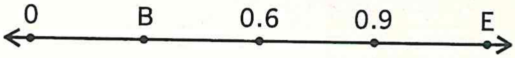
17. Write an equivalent decimal for each number named below.

$4\frac{1}{2}$ $8\frac{1}{4}$ $3\frac{3}{4}$ $\frac{7}{20}$ $\frac{3}{50}$

18. Write a symbol for each amount of money, using a dollar sign and cents point.

- 12 and 2 hundredths dollars
- 7 and 6 tenths dollars

19. What number should be matched with point *B* on the number line shown below? What number should be matched with point *E*?



20. Jack had a set of 100 stamps. He used 23 of them. Write a fraction and a decimal to name the part of the set of stamps that Jack used.

If you have time, try these —

21. Which number shown below names the sum of 0.50 and 0.25?

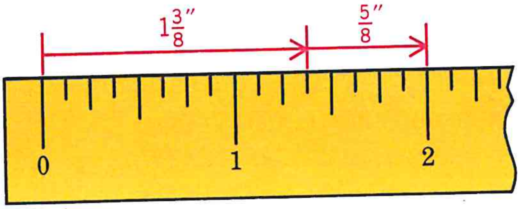
$\frac{1}{2}$ $\frac{3}{5}$ $\frac{1}{4}$ $\frac{3}{4}$

22. Which number shown below names the difference between 0.8 and 0.2?

$\frac{3}{4}$ $\frac{3}{5}$ $\frac{2}{3}$ $\frac{2}{5}$

CHAPTER 12

Addition of fractional numbers named with mixed forms



1. Use the ruler shown above to add $1\frac{3}{8}$ and $\frac{5}{8}$. Begin at 0 and move to the mark for $1\frac{3}{8}$ inches. Then count $\frac{5}{8}$ inch farther to the right. You stop at the mark for $\underline{?}$ inches.

$$1\tfrac{3}{8} + \tfrac{5}{8} = \underline{?}$$

2. Is $\frac{11}{8}$ another name for $1\frac{3}{8}$? Then is this sentence true?

$$1\tfrac{3}{8} + \tfrac{5}{8} = \tfrac{11}{8} + \tfrac{5}{8}$$

What whole number makes the following sentence true?

$$\frac{11}{8} + \frac{5}{8} = \frac{11+5}{8} = \frac{n}{8}$$

What whole number is named by $\frac{16}{8}$? Is this the same answer you found in Ex. 1?

3. Is $1\frac{3}{8}$ another name for $1 + \frac{3}{8}$? Then is this sentence true?

$$1\tfrac{3}{8} + \tfrac{5}{8} = (1 + \tfrac{3}{8}) + \tfrac{5}{8}$$

Explain each step shown below.

$$\begin{aligned} 1\tfrac{3}{8} + \tfrac{5}{8} &= (1 + \tfrac{3}{8}) + \tfrac{5}{8} \\ &= 1 + (\tfrac{3}{8} + \tfrac{5}{8}) \\ &= 1 + \tfrac{8}{8} \\ &= 1 + 1, \text{ or } \underline{?} \end{aligned}$$

Is this the same answer you found in Ex. 1 and 2?

4. Use any one of the ways shown in Ex. 1–3 to find the sum of $1\frac{1}{8}$ and $\frac{7}{8}$.

5. Another name for $2\frac{1}{8}$ is $2 + \underline{?}$. To find the sum of $2\frac{1}{8}$ and $\frac{6}{8}$, copy and complete the following sentences.

$$\begin{aligned} 2\tfrac{1}{8} + \tfrac{6}{8} &= (2 + \tfrac{1}{8}) + \tfrac{6}{8} \\ &= 2 + (\tfrac{1}{8} + \tfrac{6}{8}) \\ &= 2 + \underline{?}, \text{ or } \underline{?} \end{aligned}$$

6. Copy and complete the following to find the sum of $2\frac{1}{7}$ and $1\frac{3}{7}$.

$$\begin{aligned} 2\tfrac{1}{7} + 1\tfrac{3}{7} &= (2 + \tfrac{1}{7}) + (1 + \tfrac{3}{7}) \\ &= (2 + 1) + (\tfrac{1}{7} + \tfrac{3}{7}) \\ &= \underline{?} + \underline{?}, \text{ or } \underline{?} \end{aligned}$$

7. Another way to show the addition of $2\frac{1}{7}$ and $1\frac{3}{7}$ is to write an example like this.

$$\boxed{\begin{array}{r} 2\frac{1}{7} \\ +\,1\frac{3}{7} \\ \hline 3\frac{4}{7} \end{array}}$$

- Copy the example without the answer.
- Add $\frac{1}{7}$ and $\frac{3}{7}$. Write your answer under the $\frac{1}{7}$ and $\frac{3}{7}$.
- Add 2 and 1. Write your answer under the 2 and 1.

8. Copy and complete.

a	b	c	d	e
$3\frac{1}{5}$	$6\frac{5}{8}$	$4\frac{2}{7}$	$5\frac{1}{9}$	$6\frac{3}{5}$
$+\,1\frac{3}{5}$	$+\,4\frac{2}{8}$	$+\,2\frac{3}{7}$	$+\,2\frac{4}{9}$	$+\,8\frac{1}{5}$
$?\frac{4}{5}$	$10\frac{?}{?}$			

If you need more practice, do Practice Set 8, page 326.

Renaming in addition

1. Study the example shown below.

$$\begin{array}{rcl} 5\frac{1}{2} & = & 5\frac{2}{4} \\ +\,2\frac{1}{4} & = & 2\frac{1}{4} \\ \hline & & 7\frac{3}{4} \end{array}$$

To add fractional numbers named by unlike fractions, you rename the numbers as like fractions.

- Look at the denominators named in $5\frac{1}{2}$ and $2\frac{1}{4}$. Is either denominator a multiple of the other? Can you express $\frac{1}{2}$ as a fraction showing a denominator of 4?
- Are $5\frac{1}{2}$ and $5\frac{2}{4}$ equivalent?
- What is the sum of $\frac{2}{4}$ and $\frac{1}{4}$? of 5 and 2?

2. Now look at the example below.

$$\begin{array}{rcl} 4\frac{1}{4} & = & 4\frac{3}{12} \\ +\,2\frac{1}{3} & = & 2\frac{4}{12} \\ \hline & & 6\frac{7}{12} \end{array}$$

- Explain why $4\frac{1}{4}$ and $2\frac{1}{3}$ were renamed. Is 12 the least common denominator of $\frac{1}{4}$ and $\frac{1}{3}$?
- Is $6\frac{7}{12}$ the correct sum of $4\frac{1}{4}$ and $2\frac{1}{3}$?

3. Explain the example below.

$$\begin{array}{rcl} 13\frac{1}{3} & = & 13\frac{2}{6} \\ +\,5\frac{1}{2} & = & 5\frac{3}{6} \\ \hline & & 18\frac{5}{6} \end{array}$$

— Copy and find the answers.

	a	b	c	d
4.	$2\frac{3}{4} + 1\frac{1}{8}$	$5\frac{3}{8} + 2\frac{1}{4}$	$3\frac{1}{8} + 2\frac{1}{2}$	$2\frac{1}{3} + 4\frac{1}{2}$
5.	$7\frac{1}{5} + 2\frac{1}{10}$	$5\frac{2}{5} + 3\frac{1}{3}$	$7\frac{2}{3} + 3\frac{1}{6}$	$8\frac{1}{5} + 9\frac{7}{10}$
6.	$8\frac{1}{3} + 5\frac{1}{3}$	$5\frac{3}{4} + \frac{1}{5}$	$\frac{1}{3} + 2\frac{1}{2}$	$9 + 8\frac{7}{8}$

— Solve each sentence below.

	a	b
7.	$2\frac{2}{5} + 4\frac{1}{4} = n$	$3\frac{1}{7} + 4\frac{5}{7} = n$
8.	$5\frac{3}{8} + 4\frac{1}{2} = n$	$2\frac{3}{16} + 3\frac{3}{4} = n$
9.	$8\frac{1}{5} + 9\frac{2}{3} = n$	$2\frac{1}{3} + \frac{1}{2} = n$

Think twice!

— Can you use fractional numbers named with mixed forms to solve these problems?

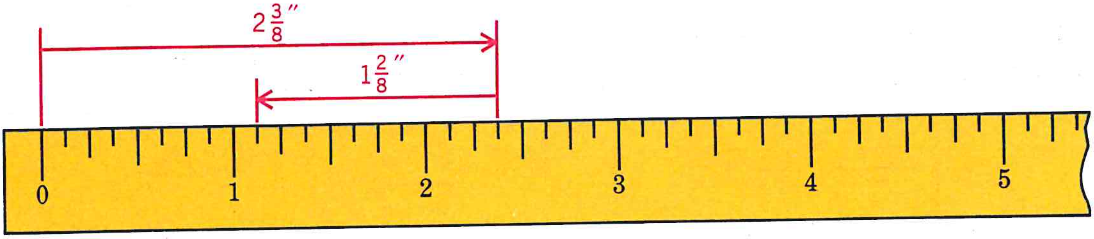
8

10. The trip from Tom's house to Dallas takes 1 hour 15 minutes on the bus, and 2 hours 30 minutes more on the train. How long does the entire trip take?

11. Nancy bought three chickens. One weighed 3 pounds 8 ounces; another weighed 2 pounds 4 ounces; the third weighed 2 pounds 3 ounces. How much did all three chickens weigh together?

If you need more practice, do Practice Set 9, page 326.

Subtraction of fractional numbers named with mixed forms



1. Use the ruler shown above to subtract $1\frac{2}{8}$ from $2\frac{3}{8}$.

- Begin at 0 and move to the right until you find the mark for $2\frac{3}{8}$ inches.
- Now count back $1\frac{2}{8}$ inches. You stop at the mark for _?_ inches.

$$2\frac{3}{8} - 1\frac{2}{8} = \underline{\ ?\ }$$

2. Is $\frac{19}{8}$ equivalent to $2\frac{3}{8}$? Is $\frac{10}{8}$ equivalent to $1\frac{2}{8}$? Then is this sentence true? $2\frac{3}{8} - 1\frac{2}{8} = \frac{19}{8} - \frac{10}{8}$

What whole number makes the following sentence true?

$$\frac{19}{8} - \frac{10}{8} = \frac{19 - 10}{8} = \frac{n}{8}$$

Express $\frac{9}{8}$ in mixed form. Is this the same answer you found in Ex. 1?

3. Look at this example.

$$\begin{array}{r} 2\frac{3}{8} \\ -\ 1\frac{2}{8} \\ \hline 1\frac{1}{8} \end{array}$$

- What is $\frac{3}{8} - \frac{2}{8}$? What is $2 - 1$?
- Is $1\frac{1}{8}$ the same answer you found in Ex. 1 and 2?

— Use any of the ways shown in Ex. 1–3 to solve each sentence below.

	a	b
4.	$1\frac{3}{4} - \frac{2}{4} = n$	$3\frac{6}{8} - 1\frac{3}{8} = n$
5.	$2\frac{1}{2} - 1\frac{1}{2} = n$	$4\frac{5}{8} - 3\frac{2}{8} = n$

— Use the ruler above to solve each of these sentences.

	a	b
6.	$3 - 1\frac{1}{4} = n$	$3 - 1\frac{3}{4} = n$
7.	$3 - \frac{3}{4} = n$	$3 - 2\frac{1}{4} = n$
8.	$3\frac{3}{8} - \frac{1}{2} = n$	$3\frac{3}{8} - \frac{5}{8} = n$

— Think of a longer ruler to help you solve these sentences.

	a	b
9.	$6\frac{7}{8} - 3\frac{2}{8} = n$	$5\frac{4}{8} - 1\frac{3}{8} = n$
10.	$8\frac{3}{4} - 2\frac{3}{4} = n$	$7\frac{2}{4} - 4\frac{1}{4} = n$
11.	$6\frac{7}{8} - 3 = n$	$9\frac{3}{8} - \frac{2}{8} = n$

12. For Ex. 9a, Harry wrote the example at the right.

$$\begin{array}{r} 6\frac{7}{8} \\ -\ 3\frac{2}{8} \\ \hline 3\frac{5}{8} \end{array}$$

- He subtracted $\frac{2}{8}$ from $\frac{7}{8}$ and wrote the answer under the $\frac{2}{8}$ and $\frac{7}{8}$.
- Then he subtracted 3 from 6 and wrote the answer under the 3 and 6.
- Is $3\frac{5}{8}$ the answer you got when you solved the sentence in Ex. 9?

13. Write an example like the one shown in Ex. 12 for each sentence in Ex. 9–11 and find the answer.

If you need more practice, do Practice Set 22, page 331.

Renaming in subtraction

1. To subtract fractional numbers, you first express them as like fractions.

$$\begin{array}{rcr} 4\frac{1}{2} & = & 4\frac{2}{4} \\ -\,1\frac{1}{4} & = & 1\frac{1}{4} \\ \hline & & 3\frac{1}{4} \end{array}$$

- In the example at the right, do $\frac{1}{2}$ and $\frac{1}{4}$ have the same denominator?
- Why was $4\frac{1}{2}$ renamed as $4\frac{2}{4}$?
- What is $\frac{2}{4} - \frac{1}{4}$? $4 - 1$? Then is $3\frac{1}{4}$ the answer for the example?

2. Copy the part printed in black in each example shown below. Without looking at the book, find the answer. Then check your work with the part of the example shown in red below.

$$\begin{array}{rcr} 3\frac{5}{6} & = & 3\frac{5}{6} \\ -\,2\frac{2}{3} & = & 2\frac{4}{6} \\ \hline & & 1\frac{1}{6} \end{array} \qquad \begin{array}{rcr} 5\frac{3}{5} & = & 5\frac{6}{10} \\ -\,2\frac{3}{10} & = & 2\frac{3}{10} \\ \hline & & 3\frac{3}{10} \end{array}$$

$$\begin{array}{rcr} 4\frac{1}{2} & = & 4\frac{3}{6} \\ -\,1\frac{1}{3} & = & 1\frac{2}{6} \\ \hline & & 3\frac{1}{6} \end{array} \qquad \begin{array}{rcr} 2\frac{5}{8} & = & 2\frac{5}{8} \\ -\,2\frac{1}{2} & = & 2\frac{4}{8} \\ \hline & & \frac{1}{8} \end{array}$$

— Copy and find the answers.

	a	b	c
3.	$\begin{array}{r} 8\frac{5}{6} \\ -\,4\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 2\frac{3}{5} \\ -\,1\frac{3}{10} \\ \hline \end{array}$	$\begin{array}{r} 3\frac{3}{4} \\ -\,1\frac{1}{3} \\ \hline \end{array}$
4.	$\begin{array}{r} 4\frac{5}{8} \\ -\,3\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 6\frac{1}{2} \\ -\,2\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{7}{9} \\ -\,3\frac{1}{6} \\ \hline \end{array}$

— Write an example such as the one in Ex. 1 to help you solve each of these sentences.

	a	b
5.	$3\frac{1}{2} - 2\frac{1}{4} = n$	$4\frac{1}{6} - \frac{1}{6} = n$
6.	$4\frac{5}{6} - 2\frac{2}{3} = n$	$5\frac{1}{2} - 3\frac{1}{3} = n$
7.	$6\frac{2}{3} - 2\frac{1}{3} = n$	$8\frac{1}{2} - 2\frac{1}{5} = n$

— Copy and find the answers. Watch the signs.

	a	b	c
8.	$\begin{array}{r} 3\frac{1}{2} \\ +\,2\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 4\frac{5}{8} \\ -\,3\frac{2}{8} \\ \hline \end{array}$	$\begin{array}{r} 2\frac{3}{4} \\ +\,1\frac{1}{8} \\ \hline \end{array}$
9.	$\begin{array}{r} 6\frac{7}{8} \\ -\,1\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 1\frac{3}{4} \\ +\,2\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} 4\frac{3}{4} \\ -\,2\frac{3}{8} \\ \hline \end{array}$
10.	$\begin{array}{r} 5\frac{3}{5} \\ +\,4\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} 3\frac{7}{8} \\ -\,2\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} 3\frac{3}{8} \\ +\,2\frac{1}{2} \\ \hline \end{array}$

— Solve each of the following number sentences.

	a	b
11.	$6\frac{3}{4} + 7\frac{1}{8} = n$	$3\frac{7}{8} - 1\frac{4}{8} = n$
12.	$7\frac{2}{3} - 4\frac{1}{2} = n$	$4\frac{1}{12} + 2\frac{1}{3} = n$
13.	$5\frac{4}{7} - 3\frac{2}{7} = n$	$3\frac{1}{3} + 2\frac{1}{3} = n$
14.	$3\frac{1}{2} + 2\frac{2}{5} = n$	$6\frac{7}{8} - 3\frac{1}{4} = n$
15.	$7\frac{5}{8} - 2\frac{1}{4} = n$	$3\frac{3}{5} - 2\frac{1}{2} = n$

If you need more practice, do Practice Set 23, page 331.

Renaming the answer

1. The example below shows a way to find the sum of $1\frac{3}{4}$ and $1\frac{1}{3}$.

$$\begin{array}{rcl} 1\frac{3}{4} & = & 1\frac{9}{12} \\ +\ 1\frac{1}{3} & = & 1\frac{4}{12} \\ \hline & & 2\frac{13}{12} = 3\frac{1}{12} \end{array}$$

- Why were $1\frac{3}{4}$ and $1\frac{1}{3}$ renamed?
- The answer, $2\frac{13}{12}$, was renamed as $3\frac{1}{12}$. $3\frac{1}{12}$ is the *more common* mixed form for the fractional number. Explain the sentences shown below.

$$\begin{aligned} 2\tfrac{13}{12} &= 2 + \tfrac{13}{12} \\ &= 2 + (\tfrac{12}{12} + \tfrac{1}{12}) \\ &= (2 + \tfrac{12}{12}) + \tfrac{1}{12} \\ &= (2 + 1) + \tfrac{1}{12} \\ &= 3 + \tfrac{1}{12} \\ &= 3\tfrac{1}{12} \end{aligned}$$

2. Copy and complete.

$$\begin{aligned} 5\tfrac{9}{8} &= 5 + \tfrac{9}{8} \\ &= 5 + (\tfrac{?}{8} + \tfrac{1}{8}) \\ &= (5 + \tfrac{?}{8}) + \tfrac{1}{8} \\ &= (5 + \underline{\ ?\ }) + \tfrac{1}{8} \\ &= \underline{\ ?\ } + \tfrac{1}{8}, \text{ or } \underline{\ ?\ } \end{aligned}$$

— Copy and solve these sentences.

	a	b
3.	$3\frac{4}{3} = 4\frac{\square}{3}$	$6\frac{7}{6} = \triangle\frac{1}{6}$
4.	$5\frac{11}{8} = 6\frac{\triangle}{8}$	$7\frac{3}{2} = \square\frac{1}{2}$

5. Study the example below.

$$\begin{array}{rcl} 2\frac{5}{6} & = & 2\frac{5}{6} \\ -\ 1\frac{1}{2} & = & 1\frac{3}{6} \\ \hline & & 1\frac{2}{6} = 1\frac{1}{3} \end{array}$$

Explain each step. Why was the first answer, $1\frac{2}{6}$, renamed?

— Estimate each answer for Ex. 6–8 to the nearest whole number. In Ex. 6a think: $3\frac{7}{8}$ is almost 4; $2\frac{1}{4}$ is a little greater than 2; so the answer is about 6. Then copy and find each answer. Express each answer in the more common mixed form and in lowest terms.

	a	b	c	d	e	f	g	h
6.	$3\frac{7}{8}$ $+\ 2\frac{1}{4}$	$2\frac{2}{3}$ $-\ 1\frac{1}{2}$	$4\frac{2}{3}$ $+\ 2\frac{3}{4}$	$2\frac{1}{2}$ $+\ 3\frac{3}{4}$	$\frac{2}{3}$ $+\ 2\frac{7}{8}$	$3\frac{7}{12}$ $-\ 2\frac{1}{2}$	$4\frac{7}{8}$ $-\ 2\frac{1}{4}$	$1\frac{5}{6}$ $+\ 3\frac{1}{2}$
7.	$6\frac{5}{8}$ $-\ 2\frac{1}{2}$	$3\frac{1}{2}$ $+\ 5\frac{2}{3}$	$3\frac{3}{4}$ $-\ 2\frac{2}{3}$	$6\frac{3}{4}$ $-\ 3\frac{1}{2}$	$3\frac{3}{4}$ $-\ \frac{1}{8}$	$7\frac{1}{2}$ $+\ 8\frac{5}{12}$	$3\frac{3}{4}$ $+\ 4\frac{7}{8}$	$2\frac{1}{3}$ $+\ 3\frac{5}{6}$
8.	$8\frac{7}{8}$ $-\ 5\frac{1}{12}$	$7\frac{1}{5}$ $+\ 5\frac{1}{4}$	$10\frac{5}{6}$ $-\ 6\frac{1}{4}$	$7\frac{1}{5}$ $+\ 4\frac{2}{3}$	$10\frac{3}{4}$ $+\ 3\frac{7}{8}$	$8\frac{1}{2}$ $+\ 7\frac{4}{5}$	$12\frac{4}{7}$ $-\ \frac{1}{2}$	$6\frac{9}{16}$ $-\ 1\frac{1}{2}$

9. Kate, Julia, and Pamela each found the sum of $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{3}{8}$.

- ▶ Kate expressed the sum as $\frac{14}{8}$.
- ▶ Julia expressed the sum as $\frac{7}{4}$.
- ▶ Pamela expressed the sum as $1\frac{3}{4}$. Are $\frac{14}{8}$, $\frac{7}{4}$, and $1\frac{3}{4}$ equivalent?
- ▶ Margaret said that $1\frac{6}{8}$ is another name for $\frac{14}{8}$. What is $1\frac{6}{8}$ expressed in lowest terms? Then is Margaret correct?

$$\begin{array}{rcl} \frac{5}{8} & = & \frac{5}{8} \\ \frac{3}{4} & = & \frac{6}{8} \\ +\,\frac{3}{8} & = & \frac{3}{8} \\ \hline & & \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4} \end{array}$$

When you express the answer as a fraction, you should always express it in lowest terms. For example, $\frac{14}{8}$ should be expressed as $\frac{7}{4}$.

When the answer is expressed in mixed form, it should be expressed in the more common mixed form as well as in lowest terms. For example, $2\frac{1}{2}$ should be used rather than $2\frac{2}{4}$, $1\frac{6}{4}$, or $1\frac{3}{2}$.

10. Explain each step in the example shown at the right. In how many ways is the answer shown? In which way is the answer expressed in the more common mixed form as well as in lowest terms?

$$\begin{array}{rcl} 4\frac{3}{4} & = & 4\frac{9}{12} \\ +\,2\frac{5}{12} & = & 2\frac{5}{12} \\ \hline & & 6\frac{14}{12} = 6\frac{7}{6} = 7\frac{1}{6} \end{array}$$

— Express each of the following fractional numbers in lowest terms two ways. For example, $\frac{9}{6}$ can be expressed as $\frac{3}{2}$ and as $1\frac{1}{2}$.

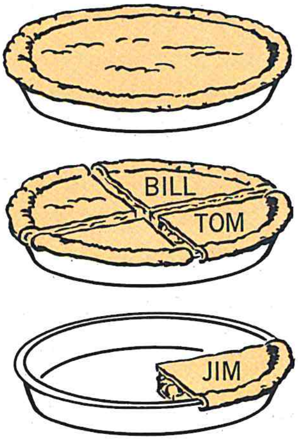
	a	b	c	d	e	f	g	h
11.	$\frac{9}{6}$	$\frac{12}{8}$	$1\frac{8}{6}$	$3\frac{9}{8}$	$\frac{16}{6}$	$2\frac{10}{6}$	$\frac{18}{4}$	$5\frac{15}{9}$
12.	$\frac{12}{3}$	$7\frac{8}{6}$	$4\frac{18}{7}$	$8\frac{24}{9}$	$\frac{15}{2}$	$\frac{19}{3}$	$\frac{28}{8}$	$\frac{27}{6}$

Copy and find the answers. Express all answers in lowest terms.

13.	$\begin{array}{r}\frac{1}{2}\\ \frac{5}{12}\\ +\frac{1}{4}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\ \frac{7}{12}\\ +\frac{5}{6}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{8}\\ \frac{3}{16}\\ +\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{10}\\ \frac{1}{2}\\ +\frac{3}{5}\\ \hline\end{array}$	$\begin{array}{r}\frac{1}{4}\\ \frac{2}{3}\\ +\frac{5}{12}\\ \hline\end{array}$	$\begin{array}{r}4\frac{5}{6}\\ 3\frac{2}{3}\\ +2\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}2\frac{3}{4}\\ 6\frac{7}{12}\\ +3\\ \hline\end{array}$	$\begin{array}{r}2\frac{1}{2}\\ 4\\ +1\frac{3}{5}\\ \hline\end{array}$
14.	$\begin{array}{r}3\frac{1}{12}\\ 2\frac{1}{3}\\ +5\frac{1}{6}\\ \hline\end{array}$	$\begin{array}{r}6\frac{3}{4}\\ \frac{1}{12}\\ +4\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}7\frac{11}{16}\\ 5\frac{1}{2}\\ +3\frac{5}{16}\\ \hline\end{array}$	$\begin{array}{r}4\frac{5}{6}\\ \frac{1}{12}\\ +5\frac{1}{12}\\ \hline\end{array}$	$\begin{array}{r}8\frac{1}{2}\\ \frac{1}{6}\\ +4\frac{1}{12}\\ \hline\end{array}$	$\begin{array}{r}2\frac{1}{3}\\ \frac{5}{12}\\ +1\frac{3}{4}\\ \hline\end{array}$	$\begin{array}{r}8\frac{3}{16}\\ 7\frac{3}{4}\\ +\frac{5}{8}\\ \hline\end{array}$	$\begin{array}{r}4\frac{1}{2}\\ 9\frac{5}{16}\\ +2\frac{3}{8}\\ \hline\end{array}$

If you need more practice, do Practice Set 10, page 327.

Another look at subtracting fractional numbers



1. Tom's mother had $2\frac{1}{4}$ pies. She gave Jim, Tom, and Bill each $\frac{1}{4}$ of a pie. Did she give away $\frac{3}{4}$ of a pie? Can you tell how much pie was left?

First she gave Jim the $\frac{1}{4}$ pie. Then she cut into one of her two whole pies to get pieces for Tom and Bill.

Find in the picture at the right the $\frac{1}{4}$ pie she gave to Jim. Find the $\frac{2}{4}$ pie she gave to Tom and Bill.

Did she have 1 whole pie and $\frac{2}{4}$ pie left? Are $1\frac{2}{4}$ and $1\frac{1}{2}$ equivalent? Then is this number sentence true? $2\frac{1}{4} - \frac{3}{4} = 1\frac{1}{2}$

2. Another way to show the subtraction of $\frac{3}{4}$ from $2\frac{1}{4}$ is given at the right.

$$\begin{array}{rcr} 2\frac{1}{4} & = & 1\frac{5}{4} \\ -\frac{3}{4} & = & \frac{3}{4} \\ \hline & & 1\frac{2}{4} = 1\frac{1}{2} \end{array}$$

- Can you subtract $\frac{3}{4}$ from $\frac{1}{4}$?
- Is $1\frac{5}{4}$ equivalent to $2\frac{1}{4}$? Copy and complete the following sentence.

$$2\frac{1}{4} = (1 + \frac{4}{4}) + \frac{1}{4} = 1 + (\frac{4}{4} + \frac{1}{4}) = 1 + \frac{?}{4}, \text{ or } 1\frac{?}{4}$$

- Can you subtract $\frac{3}{4}$ from $\frac{5}{4}$?
- Why was the answer, $1\frac{2}{4}$, renamed as $1\frac{1}{2}$?

3. Copy and complete the following number sentences.

- $3 = 2 + \frac{?}{8}$ $\qquad 2 = 1\frac{?}{6}$ $\qquad 4 = 3\frac{?}{9}$ $\qquad 5 = 4\frac{?}{8}$
- $2\frac{1}{6} = (1 + \frac{6}{6}) + \frac{1}{6} = 1 + (\frac{6}{6} + \frac{1}{6}) = 1 + \frac{7}{6} = \underline{\ ?\ }$
- $5\frac{3}{8} = (4 + \frac{?}{8}) + \frac{3}{8} = 4 + (\frac{?}{8} + \frac{3}{8}) = 4 + \underline{\ ?\ } = \underline{\ ?\ }$

4. Copy and solve each sentence.

$4\frac{1}{3} = 3\frac{n}{3}$ $\qquad 6\frac{1}{2} = 5\frac{b}{2}$ $\qquad 8\frac{3}{4} = 7\frac{e}{4}$ $\qquad 3\frac{2}{5} = 2\frac{f}{5}$ $\qquad 5\frac{3}{8} = 4\frac{m}{8}$

5. Explain each example below. Then copy the part in black. Find the answer. Check your work with the work in the book.

$2\frac{1}{6} = 1\frac{7}{6}$	$3 = 2\frac{8}{8}$	$4 = 3\frac{9}{9}$	$5\frac{3}{8} = 4\frac{11}{8}$
$-\frac{5}{6} = \frac{5}{6}$	$-\frac{5}{8} = \frac{5}{8}$	$-1\frac{5}{9} = 1\frac{5}{9}$	$-\frac{7}{8} = \frac{7}{8}$
$1\frac{2}{6} = 1\frac{1}{3}$	$2\frac{3}{8}$	$2\frac{4}{9}$	$4\frac{4}{8} = 4\frac{1}{2}$

6. Barbara wanted to subtract $1\frac{3}{4}$ from $5\frac{1}{2}$. When she wrote the example, she found she had to rename $5\frac{1}{2}$ twice.

$$\begin{array}{rcrcr} 5\frac{1}{2} & = & 5\frac{2}{4} & = & 4\frac{6}{4} \\ -\ 1\frac{3}{4} & = & 1\frac{3}{4} & = & 1\frac{3}{4} \\ \hline & & & & 3\frac{3}{4} \end{array}$$

- Why did she rename $5\frac{1}{2}$ as $5\frac{2}{4}$?
- Why was $5\frac{2}{4}$ renamed as $4\frac{6}{4}$?
- To check her answer, Barbara wrote this sentence. $3\frac{3}{4} + 1\frac{3}{4} = n$

What number makes the sentence true? Then is Barbara's example correct?

7. Explain each of these subtraction examples. Then copy the part in black. Find the answer. Check your answer two ways: by writing an addition number sentence, and by looking at the work in the book.

$$\begin{array}{rcrcr} 4\frac{1}{6} & = & 4\frac{1}{6} & = & 3\frac{7}{6} \\ -\ 2\frac{2}{3} & = & 2\frac{4}{6} & = & 2\frac{4}{6} \\ \hline & & & & 1\frac{3}{6} = 1\frac{1}{2} \end{array} \qquad \begin{array}{rcrcr} 2\frac{1}{3} & = & 2\frac{4}{12} & = & 1\frac{16}{12} \\ -\ \frac{3}{4} & = & \frac{9}{12} & = & \frac{9}{12} \\ \hline & & & & 1\frac{7}{12} \end{array} \qquad \begin{array}{rcrcr} 3\frac{1}{4} & = & 3\frac{2}{8} & = & 2\frac{10}{8} \\ -\ 1\frac{7}{8} & = & 1\frac{7}{8} & = & 1\frac{7}{8} \\ \hline & & & & 1\frac{3}{8} \end{array}$$

— Write an example to help you solve each of these sentences. Check.

	a	b	c	d	e
8.	$12\frac{1}{6} - 2\frac{5}{6} = n$	$5\frac{1}{4} - 2\frac{2}{3} = n$	$3\frac{1}{6} - \frac{2}{3} = n$	$7\frac{1}{4} - 1\frac{2}{3} = n$	$8\frac{1}{4} - 5\frac{3}{4} = n$
9.	$12\frac{1}{4} - 11\frac{1}{2} = n$	$6\frac{1}{4} - 2\frac{5}{8} = n$	$10 - \frac{3}{8} = n$	$4\frac{2}{3} - 1\frac{5}{6} = n$	$5\frac{1}{2} - \frac{2}{3} = n$
10.	$13\frac{3}{8} - 10\frac{5}{8} = n$	$7\frac{5}{10} - \frac{9}{10} = n$	$9\frac{1}{2} - 4 = n$	$7 - \frac{5}{10} = n$	$14\frac{1}{2} - \frac{7}{8} = n$
11.	$12\frac{1}{2} - 6\frac{3}{4} = n$	$3\frac{3}{4} - 1\frac{5}{8} = n$	$9 - 2\frac{7}{8} = n$	$2\frac{1}{8} - 1\frac{1}{4} = n$	$4 - 1\frac{5}{6} = n$

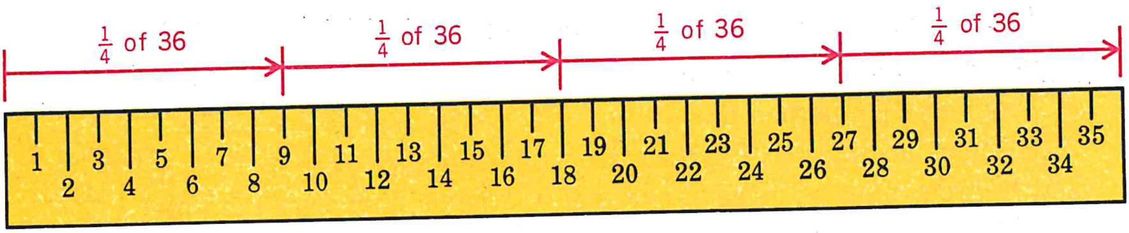
12. Henry weighed $90\frac{1}{2}$ pounds before a basketball game. He weighed $89\frac{7}{8}$ pounds after the game. How much weight did he lose during the game?

13. Frank spent $2\frac{1}{2}$ hours cutting and raking the lawn. It took him $\frac{3}{4}$ hour to do the raking. How long did he spend cutting?

14. On Monday a seedling was found to be $1\frac{3}{4}$ inches tall. The following Monday it was $2\frac{3}{16}$ inches tall. How much did it grow in a week?

If you need more practice, do Practice Set 24, page 331.

Fractional parts of measurements



1. How many inches make 1 yard? Use the yardstick shown above to find how many inches make $\frac{3}{4}$ yard.

- The first arrow shows $\frac{1}{4}$ yard. At what point does the first arrow stop? This shows that $\frac{1}{4}$ of 36 inches makes ? inches.
- The second arrow shows another $\frac{1}{4}$ yard. At what point does the second arrow stop? Then $\frac{2}{4}$ of 36 inches makes ? inches.
- The third arrow shows still another $\frac{1}{4}$ yard. The third arrow stops at ? inches. Since $\frac{3}{4}$ of 36 inches makes 27 inches, then $\frac{3}{4}$ yard makes ? inches.
- Since $\frac{1}{4}$ yard makes 9 inches, then $\frac{3}{4}$ yard makes 3 × 9, or ?, inches.

— Copy and complete these sentences.

	a	b	c
2.	$\frac{1}{2}$ lb. makes ? oz.	$\frac{1}{2}$ gal. makes ? qt.	$\frac{1}{2}$ pt. makes ? c.
3.	$\frac{1}{4}$ lb. makes ? oz.	$\frac{3}{4}$ lb. makes ? oz.	$\frac{1}{8}$ lb. makes ? oz.
4.	$\frac{1}{4}$ qt. makes ? c.	$\frac{3}{4}$ qt. makes ? c.	$\frac{1}{5}$ hr. makes ? min.
5.	$\frac{1}{3}$ ft. makes ? in.	$\frac{1}{3}$ yd. makes ? ft.	$\frac{1}{4}$ gal. makes ? qt.
6.	$\frac{1}{4}$ ft. makes ? in. $\frac{3}{4}$ ft. makes ? in.	$\frac{1}{4}$ hr. makes ? min. $\frac{3}{4}$ hr. makes ? min.	$\frac{1}{4}$ day makes ? hr. $\frac{3}{4}$ day makes ? hr.
7.	$\frac{1}{3}$ hr. makes ? min. $\frac{2}{3}$ hr. makes ? min.	$\frac{1}{4}$ doz. makes ? things $\frac{3}{4}$ doz. makes ? things	$\frac{1}{16}$ lb. makes ? oz. $\frac{5}{16}$ lb. makes ? oz.
8.	$\frac{1}{4}$ min. makes ? sec. $\frac{3}{4}$ min. makes ? sec.	$\frac{1}{3}$ doz. makes ? things $\frac{2}{3}$ doz. makes ? things	$\frac{1}{3}$ yr. makes ? mo. $\frac{2}{3}$ yr. makes ? mo.
9.	$\frac{5}{6}$ ft. makes ? in.	$\frac{3}{8}$ lb. makes ? oz.	$\frac{4}{5}$ hr. makes ? min.

Renaming measurements

1. The example at the right shows how to find the number of minutes in $1\frac{3}{4}$ hours.

- Is $1\frac{3}{4}$ equivalent to $1 + \frac{3}{4}$?
- How many minutes make 1 hour?
- How many minutes make $\frac{1}{4}$ hour? $\frac{3}{4}$ hour?
- Then $1\frac{3}{4}$ hours make $60 + 45$, or ?, minutes.

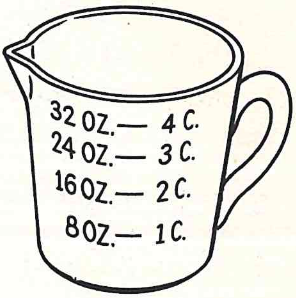
$$\begin{array}{rlr} 1 & \text{hr. makes} & 60 \text{ min.} \\ + \frac{3}{4} & \text{hr. makes} & 45 \text{ min.} \\ \hline 1\frac{3}{4} & \text{hr. make} & 105 \text{ min.} \end{array}$$

2. Find the number of minutes that make $1\frac{1}{2}$ hours; $2\frac{1}{4}$ hours; $3\frac{1}{3}$ hours.

— Copy and complete these sentences.

	a	b	c
3.	$1\frac{1}{2}$ ft. make ? in.	$1\frac{3}{4}$ gal. make ? qt.	$1\frac{1}{2}$ yd. make ? in.
4.	$1\frac{3}{8}$ lb. make ? oz.	$1\frac{2}{3}$ doz. make ? things	$1\frac{3}{4}$ day make ? hr.
5.	$1\frac{2}{3}$ min. make ? sec.	$2\frac{1}{4}$ dollars make ? ¢	$1\frac{2}{3}$ yd. make ? in.

6. The picture at the right shows a quart measuring pitcher. How many ounces make 1 quart? How many cups make 1 quart? Is the quart measure marked off both in ounces and in cups?

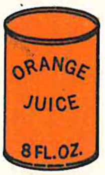

32 OZ.— 4 C.
24 OZ.— 3 C.
16 OZ.— 2 C.
8 OZ.— 1 C.

The ounces used to measure liquids are called *fluid* ounces (fl. oz.) so we can tell them from the ounces used as a unit of weight.

— Copy these sentences. Use the scales shown on the pitcher to help you complete them.

7.	1 qt. makes ? c.	1 qt. makes ? fl. oz.	1 c. makes ? fl. oz.
8.	1 pt. makes ? fl. oz.	$\frac{1}{2}$ pt. makes ? fl. oz.	$\frac{1}{4}$ pt. makes ? fl. oz.
9.	24 fl. oz. make ? c.	16 fl. oz. make ? c.	12 fl. oz. make ? c.
10.	$\frac{3}{4}$ qt. makes ? fl. oz.	$\frac{5}{8}$ qt. makes ? fl. oz.	18 fl. oz. make ? c.

11. How many fluid ounces of concentrated orange juice are contained in the can at the right? If you mix 3 of the same size cans of water with the concentrated orange juice, you will have ? fluid ounces of juice; ? cups of juice.

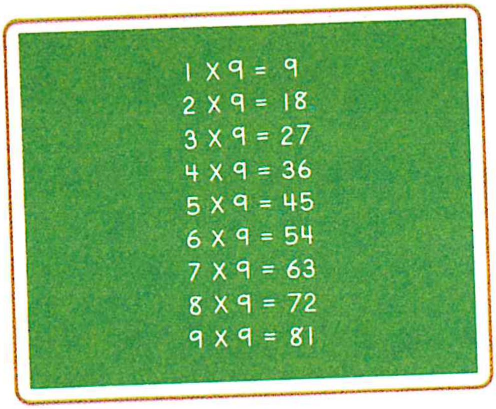


Inquiry into number

Patterns and the number 9

In your study of mathematics, you have been asked many times to examine a pattern to help you arrive at a conclusion. There are many interesting number patterns. Some of them are associated with the number 9. How many have you already discovered?

▶ You know that the product of two numbers is called a multiple of each factor. Shown at the right are a few multiples of 9.



$1 \times 9 = 9$
$2 \times 9 = 18$
$3 \times 9 = 27$
$4 \times 9 = 36$
$5 \times 9 = 45$
$6 \times 9 = 54$
$7 \times 9 = 63$
$8 \times 9 = 72$
$9 \times 9 = 81$

Look at the digits in the numeral for each multiple. Add the numbers named by these digits. For example, add the 1 and 8 of 18, the 2 and 7 of 27, the 3 and 6 of 36, and so on. What do you discover?

Is this number sentence true? $276 \times 9 = 2{,}484$ Then is 2,484 a multiple of 9? What is the sum of the numbers named by the digits in this multiple? $2 + 4 + 8 + 4 = \underline{?}$

What number makes this sentence true? $6{,}483 \times 9 = n$ Then is 58,347 a multiple of 9? What is the sum of the numbers named by the digits in this multiple? $5 + 8 + 3 + 4 + 7 = \underline{?}$

Multiply some other numbers by 9. Then find the sum of the numbers named by the digits in each multiple. What can you say about the sum of the numbers named by the digits in any multiple of 9? How can you use this discovery to check your answer when you find the product of any number and 9?

Does this pattern continue when you multiply a number by any multiple of 9? Try a few examples and see. When you multiply a number by any multiple of 9, what relation does the sum of the numbers named by the digits in your answer have to the number 9?

$$\begin{array}{r}18\ r0\\ 9\overline{)162}\end{array} \qquad 1+6+2=9$$

$$\begin{array}{r}59\ r0\\ 9\overline{)531}\end{array} \qquad 5+3+1=9$$

$$\begin{array}{r}488\ r0\\ 9\overline{)4{,}392}\end{array} \qquad 4+3+9+2=18;\quad 1+8=9$$

$$\begin{array}{r}73\ r1\\ 9\overline{)658}\end{array} \qquad 6+5+8=19;\quad 1+9=10;\quad 1+0=1$$

$$\begin{array}{r}36\ r2\\ 9\overline{)326}\end{array} \qquad 3+2+6=11;\quad 1+1=2$$

$$\begin{array}{r}813\ r4\\ 9\overline{)7{,}321}\end{array} \qquad 7+3+2+1=13;\quad 1+3=4$$

▶ Shown on the chalkboard above are six division examples in which the divisor is 9. At the right of each example are shown addition sentences using the digits of the dividend. Note the remainder for each example. Then note the sum obtained when the numbers named by the digits in the dividend are added. The arrows are drawn to help you. Do you see a pattern? Try a few more division examples with 9 to check your discovery.

How can you use this discovery to check your answers when you divide any number by 9?

▶ Study each set of sentences shown on the chalkboard below. Do you see a pattern? If so, use your discovery to copy and complete the last two sentences in each set without computing.

$(1 \times 9) + 2 = 11$

$(12 \times 9) + 3 = 111$

$(123 \times 9) + 4 = 1{,}111$

$(1{,}234 \times 9) + 5 = 11{,}111$

$(12{,}345 \times 9) + 6 = \underline{?}$

$(123{,}456 \times 9) + \underline{?} = 1{,}111{,}111$

$(9 \times 5) + 4 = 49$

$(9 \times 55) + 4 = 499$

$(9 \times 555) + 4 = 4{,}999$

$(9 \times 5{,}555) + 4 = \underline{?}$

$(9 \times 55{,}555) + 4 = \underline{?}$

Keeping up in mathematics

Oral

1. What number makes each of these sentences true?

$\frac{6}{8} + \frac{1}{8} = n$ $\qquad$ $\frac{3}{4} + \frac{1}{8} = n$

2. What number makes each of these sentences true?

$\frac{6}{9} - \frac{2}{9} = n$ $\qquad$ $\frac{2}{3} - \frac{2}{9} = n$

3. Tell three fractions that are equivalent to each of these.

$\frac{2}{8}$ $\quad$ $\frac{4}{5}$ $\quad$ $\frac{1}{8}$ $\quad$ $\frac{3}{7}$ $\quad$ $\frac{2}{9}$

4. Express each of the following in lowest terms.

$\frac{4}{8}$ $\quad$ $\frac{9}{12}$ $\quad$ $1\frac{2}{8}$ $\quad$ $4\frac{9}{15}$

5. Round to the nearest ten; to the nearest hundred; to the nearest thousand.

58,517 $\qquad$ 8,904,499

6. Is $34\overline{)7{,}050}$ about 2, about 20, or about 200?

7. How many faces does a rectangular prism have? How many faces does a triangular pyramid have?

8. Estimate to the nearest hundred the product of 41 and 59.

9. Which is greater, $\frac{1}{6}$ or $\frac{5}{6}$? $\frac{1}{8}$ or $\frac{1}{10}$? $\frac{3}{7}$ or $\frac{3}{5}$? $\frac{7}{3}$ or $\frac{5}{3}$?

10. Estimate to the nearest hundred the quotient for 5,080 ÷ 7.

Written

— Copy, find the answers, and check.

	a	b	c
1.	386 854 + 795	259 64 + 907	7,234 8,917 + 436
2.	618 − 275	815 − 491	6,030 − 3,594
3.	387 × 6	204 × 95	3,764 × 128
4.	$7\overline{)508}$	$45\overline{)572}$	$61\overline{)2{,}762}$

— Solve each sentence below.

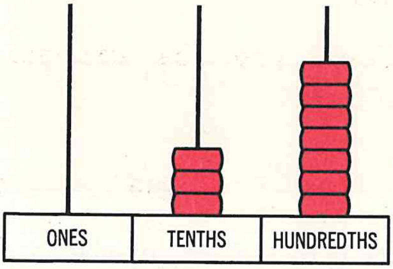
	a	b
5.	$362 + n = 5{,}046$	$\frac{3}{7} + \frac{1}{5} = n$
6.	$16 \times n = 5{,}136$	$n - \frac{4}{5} = \frac{2}{3}$
7.	$n - 541 = 3{,}800$	$\frac{5}{8} + n = \frac{11}{12}$
8.	$n \div 47 = 365$	$\frac{5}{9} - n = \frac{1}{3}$

9. Lee timed himself and found that he could do 32 exercises in 2 minutes. At this rate, could he do 80 exercises in 4 minutes?

10. Jack has saved 289 seeds from his prize-winning watermelons.

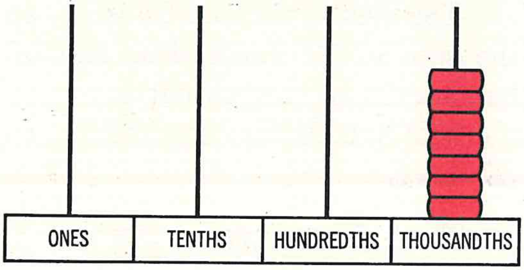
If he wishes to put 20 seeds in a package, how many packages can he fill?

Thousandths as decimals



1. On the abacus shown above, there are ? beads on the tenths wire and ? beads on the hundredths wire.

- Write a fraction that names the number shown on the abacus.
- Write a decimal that names the number shown on the abacus.
- Are $\frac{37}{100}$ and 0.37 equivalent?



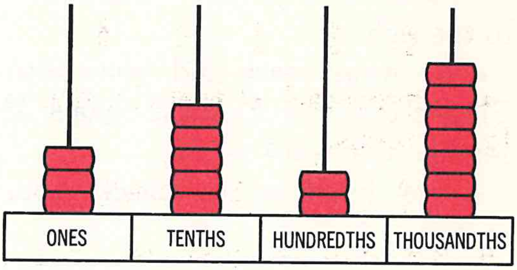
2. Look at the abacus shown above.

- The tenths place is ? place to the right of the ones place.
- The hundredths place is ? places to the right of the ones place, and ? place to the right of the tenths place.
- The *thousandths* place is ? places to the right of the ones place, ? places to the right of the tenths place, and ? place to the right of the hundredths place.

3. The number shown on the abacus above Ex. 2 is seven thousandths. Express this number as a fraction.

4. Another way to name the number shown on the abacus in Ex. 2 is to write the decimal 0.007.

We read 0.007 by saying "seven thousandths." The numeral 0.007 shows 0 ones 0 tenths ? hundredths ? thousandths.



5. The beads on the abacus above show ? ones ? tenths ? hundredths ? thousandths.

The decimal for this number is 3.527. What does each digit in 3.527 stand for?

6. Write a decimal for 6 tens 5 ones 7 tenths 3 hundredths 4 thousandths.

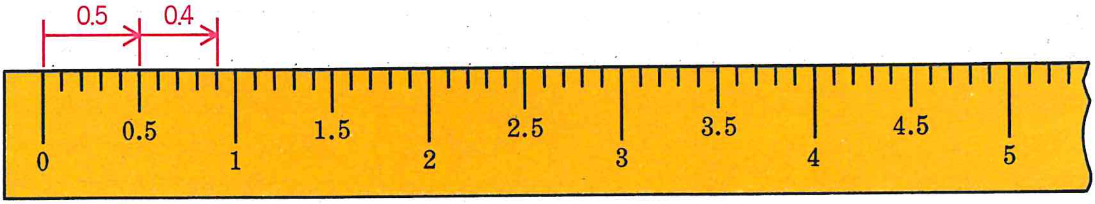
— Name each using a decimal.

	a	b	c
7.	$\frac{9}{1,000}$	$8\frac{409}{1,000}$	$\frac{346}{1,000}$
8.	$3\frac{281}{1,000}$	$6\frac{46}{1,000}$	$29\frac{7}{1,000}$

— Name each using a fraction.

9.	0.003	0.041	0.539
10.	6.179	8.093	7.007

Addition using decimals



1. The ruler shown above is marked off in tenths. Use this ruler to find the sum of 0.5 and 0.4.

- Begin at 0 and move 5 tenths to the right. Then move 4 tenths more to the right.
- At what mark did you stop? Then 5 tenths + 4 tenths equals $\underline{?}$ tenths. $0.5 + 0.4 = \underline{?}$
- Use fractions to check your answer. $\frac{5}{10} + \frac{4}{10} = \underline{?}$

2. Use the ruler to find the sum of 0.6 and 0.3. $0.6 + 0.3 = \underline{?}$

Check your answer using equivalent fractions. $\frac{6}{10} + \frac{3}{10} = \underline{?}$

3. Use the ruler to find the sum of 1.3 and 0.4. $1.3 + 0.4 = \underline{?}$

Now use equivalent fractions to check. $1\frac{3}{10} + \frac{4}{10} = \underline{?}$

— Use the ruler above to solve each of these sentences. Then check your answers using fractions.

	a	b
4.	$0.4 + 0.2 = n$	$0.5 + 0.3 = n$
5.	$0.6 + 0.7 = n$	$0.8 + 0.8 = n$
6.	$1.2 + 0.6 = n$	$3.1 + 0.8 = n$
7.	$2.4 + 0.7 = n$	$1.5 + 1.3 = n$

8. Is $0.14 + 0.25$ another way to express $\frac{14}{100} + \frac{25}{100}$?

- $\frac{14}{100} + \frac{25}{100} = \frac{14 + 25}{100} = \frac{?}{100}$
- If $\frac{14}{100} + \frac{25}{100}$ equals $\frac{39}{100}$, then what decimal can replace n to make this sentence true?

$$0.14 + 0.25 = n$$

9. Suppose you wish to find the sum of 0.32 and 0.56.

- The sum of 32 and 56 is $\underline{?}$, so the sum of 32 hundredths and 56 hundredths is $\underline{?}$ hundredths.
- $0.32 + 0.56$ is equivalent to $\frac{32}{100} + \frac{?}{100}$.
- $\frac{32}{100} + \frac{56}{100} = \frac{32 + 56}{100} = \frac{?}{100}$
- So, $0.32 + 0.56$ equals $\underline{?}$.

— Solve these pairs of sentences. Copy each and replace m with a fraction and n with a decimal.

10. $\frac{7}{10} + \frac{2}{10} = m$ $\quad 0.7 + 0.2 = n$

11. $\frac{3}{100} + \frac{9}{100} = m$ $\quad 0.03 + 0.09 = n$

12. $\frac{23}{100} + \frac{59}{100} = m$ $\quad 0.23 + 0.59 = n$

13. $1\frac{3}{10} + 2\frac{4}{10} = m$ $\quad 1.3 + 2.4 = n$

14. $7\frac{58}{100} + 5\frac{37}{100} = m$ $\quad 7.58 + 5.37 = n$

Writing addition examples using decimals

1. You can think of the whole number 34 as 3 tens $\underline{?}$ ones, and you can think of the whole number 15 as $\underline{?}$ ten $\underline{?}$ ones.

$$\begin{array}{rcl} 34 & = & 3 \text{ tens } 4 \text{ ones} \\ \underline{+\ 15} & = & \underline{1 \text{ ten } \ 5 \text{ ones}} \\ & & 4 \text{ tens } 9 \text{ ones} \end{array}$$

To add two whole numbers such as 34 and 15, you can write an example like the one in the box.

You add the ones. 4 ones + 5 ones = $\underline{?}$ ones
Then you add the tens. 3 tens + 1 ten = $\underline{?}$ tens

2. In the same way, you can think of the fractional number 0.53 as 5 tenths $\underline{?}$ hundredths and the fractional number 0.26 as $\underline{?}$ tenths $\underline{?}$ hundredths.

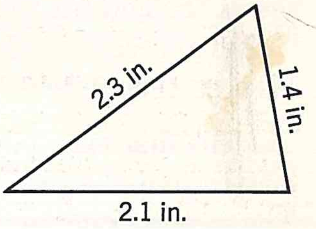
$$\begin{array}{rcl} 0.53 & = & 5 \text{ tenths } 3 \text{ hundredths} \\ \underline{+\ 0.26} & = & \underline{2 \text{ tenths } 6 \text{ hundredths}} \\ & & 7 \text{ tenths } 9 \text{ hundredths, or } 0.79 \end{array}$$

- Add the hundredths. 3 hundredths + 6 hundredths = $\underline{?}$ hundredths
- Then add the tenths. 5 tenths + 2 tenths = $\underline{?}$ tenths

3. Frank and Albert are finding the perimeter of the triangle shown at the right.

Did Frank use fractions in his example? Albert used $\underline{?}$ in his example. Explain the work of each. Did both get the same answer? Why?

FRANK	ALBERT
$2\frac{3}{10}$	2.3
$1\frac{4}{10}$	1.4
$+\ 2\frac{1}{10}$	+ 2.1
$5\frac{8}{10}$	5.8



4. Copy and find each sum. Then write another example naming the addends as decimals. Find the sum and compare the answers.

a	b	c	d	e	f	g
$\frac{72}{100}$	$1\frac{3}{10}$	$8\frac{32}{100}$	$3\frac{26}{100}$	$5\frac{16}{100}$	$12\frac{51}{100}$	$57\frac{62}{100}$
$+\ \frac{16}{100}$	$+\ 4\frac{6}{10}$	$+\ 3\frac{45}{100}$	$+\ \frac{43}{100}$	$+\ 9\frac{73}{100}$	$+\ 34\frac{28}{100}$	$+\ 28\frac{27}{100}$

5. Copy and find the sums. Check by using fractions.

0.75	0.37	0.53	1.82	5.46	2.55	6.73
+ 0.24	+ 0.31	+ 0.44	+ 3.17	+ 8.52	+ 1.32	+ 3.15

If you need more practice, do Practice Set 11, page 327.

Renaming in addition

1. To see how you can find the sum of 0.85 and 0.67, study the example at the right.

$$\begin{array}{rcl} 0.85 & = & \text{8 tenths } \ \text{5 hundredths} \\ \underline{+\,0.67} & = & \underline{\text{6 tenths } \ \text{7 hundredths}} \\ & & \text{14 tenths 12 hundredths, or 1.52} \end{array}$$

- Think of 0.85 as 8 tenths ? hundredths and 0.67 as ? tenths ? hundredths.
- Add the hundredths.

5 hundredths + 7 hundredths = ? hundredths

- Then add the tenths. 8 tenths + 6 tenths = ? tenths
- Is 12 hundredths equivalent to 1 tenth 2 hundredths? $\frac{12}{100} = 0.12$

Then rename 14 tenths 12 hundredths as 15 tenths ? hundredths.

- Is 15 tenths 2 hundredths equivalent to 1 one 5 tenths 2 hundredths? Write a decimal for 1 one 5 tenths 2 hundredths.

2. The first example at the right shows the addition of 38 and 97. The second example shows the addition of 0.38 and 0.97.

Explain each. How are they similar? How do they differ?

$$\begin{array}{r} 38 \\ \underline{+\,97} \\ 135 \end{array} \qquad \begin{array}{r} 0.38 \\ \underline{+\,0.97} \\ 1.35 \end{array}$$

3. The first example at the right shows how to find the sum of \$2.49 and \$3.68. The second example shows the addition of 2.49 and 3.68.

Explain each. How are they similar? How do they differ?

$$\begin{array}{r} \$2.49 \\ \underline{+\,3.68} \\ \$6.17 \end{array} \qquad \begin{array}{r} 2.49 \\ \underline{+\,3.68} \\ 6.17 \end{array}$$

— Copy and find the answers. Use fractions to check your answers.

	a	b	c	d	e	f	g
4.	0.39 + 0.48	0.62 + 0.59	0.87 + 0.64	3.23 + 2.98	4.86 + 6.37	6.86 + 5.38	7.36 + 8.92
5.	8.25 + 3.96	64.74 + 38.57	7.374 + 5.817	4.843 + 3.589	1.685 + 4.537	7.846 + 9.396	3.975 + 9.048

If you need more practice, do Practice Set 12, page 327.

Subtraction using decimals

1. Each box at the right shows a pair of subtraction examples.

In each box, does the example showing fractions help you to understand the example showing decimals? Explain.

Where do you write the decimal point in the answer when you subtract using decimals? Why?

$$\begin{array}{r} 6\frac{9}{10} \\ -\ 4\frac{5}{10} \\ \hline 2\frac{4}{10} \end{array} \qquad \begin{array}{r} 6.9 \\ -\ 4.5 \\ \hline 2.4 \end{array}$$

$$\begin{array}{r} 7\frac{46}{100} \\ -\ 3\frac{12}{100} \\ \hline 4\frac{34}{100} \end{array} \qquad \begin{array}{r} 7.46 \\ -\ 3.12 \\ \hline 4.34 \end{array}$$

2. Copy each example below and subtract. Then rename each sum and known addend by a decimal. Subtract again. Compare the answers.

a	b	c	d	e	f
$13\frac{9}{10}$	$12\frac{8}{10}$	$8\frac{68}{100}$	$15\frac{27}{100}$	$3\frac{637}{1,000}$	$35\frac{514}{1,000}$
$-\ 7\frac{4}{10}$	$-\ 9\frac{6}{10}$	$-\ 2\frac{36}{100}$	$-\ 8\frac{15}{100}$	$-\ 1\frac{323}{1,000}$	$-\ 12\frac{203}{1,000}$

3. You can think of the fractional number 0.76 as 7 tenths ? hundredths and 0.52 as ? tenths ? hundredths.

$$\begin{array}{rl} 0.76 & = 7 \text{ tenths } 6 \text{ hundredths} \\ -\ 0.52 & = 5 \text{ tenths } 2 \text{ hundredths} \\ \hline & \ 2 \text{ tenths } 4 \text{ hundredths, or } 0.24 \end{array}$$

- Subtract the hundredths.

6 hundredths − 2 hundredths = ? hundredths

- Subtract the tenths. 7 tenths − 5 tenths = ? tenths
- Write a decimal for 2 tenths 4 hundredths.

— Copy and find the answers. Check by subtracting using the equivalent fractions.

4.	4.5 − 1.3	8.95 − 2.74	5.79 − 4.30	0.864 − 0.243	13.95 − 8.21	27.49 − 12.18
5.	37.6 − 21.3	6.42 − 4.21	5.876 − 3.124	48.18 − 26.18	3.588 − 1.284	476.752 − 94.401

If you need more practice, do Practice Set 25, page 332.

Renaming in subtraction

When you subtract using decimals, you sometimes have to rename one number just as you do when you subtract with whole numbers.

1. To subtract 0.38 from 0.92, think of 0.92 as 9 tenths _?_ hundredths and 0.38 as _?_ tenths _?_ hundredths.

$$\begin{array}{r} 0.92 \\ -\ 0.38 \\ \hline \end{array}$$

- Do you know how to subtract 8 ones from 2 ones? Then can you subtract 8 hundredths from 2 hundredths?
- Study the example below to see how the numbers are renamed.

$$\begin{array}{rcccl} 0.92 & = & \text{9 tenths 2 hundredths} & = & \text{8 tenths 12 hundredths} \\ -\ 0.38 & = & \text{3 tenths 8 hundredths} & = & \text{3 tenths 8 hundredths} \\ \cline{1-1}\cline{3-3}\cline{5-5} & & & & \text{5 tenths 4 hundredths, or 0.54} \end{array}$$

- One tenth is equivalent to _?_ hundredths. Is 9 tenths equivalent to 8 tenths 10 hundredths? Is 9 tenths 2 hundredths equivalent to 8 tenths 12 hundredths?
- What is 12 hundredths − 8 hundredths? 8 tenths − 3 tenths? Write a decimal for 5 tenths 4 hundredths.

2. The first example at the right shows the subtraction of 148 from 214. The second example shows the subtraction of 1.48 from 2.14.

Explain each. How are they similar? different?

$$\begin{array}{r|r} 214 & 2.14 \\ -\ 148 & -\ 1.48 \\ \hline 66 & 0.66 \end{array}$$

3. The first example at the right shows how to find the difference between \$3.43 and \$1.86. The second example shows how to find the difference between 3.43 and 1.86.

Explain each. How are they similar? different?

$$\begin{array}{r|r} \$3.43 & 3.43 \\ -\ 1.86 & -\ 1.86 \\ \hline \$1.57 & 1.57 \end{array}$$

— Copy and find the answers. Check by adding.

	a	b	c	d	e	f
4.	0.74 − 0.28	5.85 − 3.77	0.946 − 0.653	12.35 − 8.17	3.493 − 1.284	6.807 − 5.907
5.	0.928 − 0.536	0.981 − 0.765	3.850 − 1.647	9.891 − 3.673	7.676 − 4.483	34.180 − 17.543

If you need more practice, do Practice Set 26, page 332.

Practice with decimals

— Tell the numbers that make these sentences true.

	a	b
1.	$0.1 = \frac{a}{10} = \frac{b}{100}$	$0.30 = \frac{x}{100} = \frac{y}{10}$
2.	$0.2 = \frac{m}{10} = \frac{n}{5}$	$1.2 = 1\frac{d}{10} = 1\frac{e}{100}$
3.	$2.5 = 2\frac{5}{10} = 2\frac{1}{r}$	$0.4 = \frac{w}{10} = \frac{x}{5}$
4.	$0.5 = \frac{c}{10} = \frac{p}{2}$	$0.6 = \frac{f}{10} = \frac{g}{100}$
5.	$0.25 = \frac{h}{100} = \frac{i}{4}$	$0.50 = \frac{50}{100} = \frac{1}{c}$

— In the following exercises, the decimal points have been omitted from the numerals. Tell where the points should be placed so that the numbers named are sensible.

6. Andy walks from his home to school in a half hour. He says the distance is 15 miles.

7. In baseball the pitcher's circle is 605 feet from home plate.

8. Serena says that the ceiling in her room is 825 feet high.

— If each set below is to be a set of numerals that name the same number, tell which numeral should not be shown as a member of the set.

9. $\{\frac{6}{10}, 0.60, \frac{3}{5}, 1.6\}$

10. $\{0.5, \frac{1}{5}, \frac{1}{2}, 0.50\}$

11. $\{0.75, \frac{75}{100}, 7.5, \frac{3}{4}\}$

12. $\{\frac{3}{2}, 1.5, \frac{15}{100}, \frac{15}{10}\}$

— Name as decimals.

13. 7 tenths

14. 15 hundredths

15. 10 and 8 tenths

16. 6 and 18 hundredths

17. 45 and 362 thousandths

18. Write an equivalent decimal for each fraction below.

$\frac{3}{10}$ $\frac{27}{100}$ $\frac{581}{1{,}000}$ $\frac{4}{5}$ $\frac{1}{2}$ $\frac{3}{4}$

19. Write an equivalent fraction in lowest terms for each decimal below.

0.70 0.83 0.421 0.6 0.45

— Copy, find the answers, and check using fractions.

	a	b	c	d
20.	$\begin{array}{r}0.32\\+0.64\\\hline\end{array}$	$\begin{array}{r}1.38\\+3.61\\\hline\end{array}$	$\begin{array}{r}0.462\\+0.849\\\hline\end{array}$	$\begin{array}{r}3.87\\+9.46\\\hline\end{array}$
21.	$\begin{array}{r}0.74\\-0.31\\\hline\end{array}$	$\begin{array}{r}8.96\\-3.44\\\hline\end{array}$	$\begin{array}{r}0.824\\-0.368\\\hline\end{array}$	$\begin{array}{r}5.42\\-2.87\\\hline\end{array}$

— Solve each of these sentences.

22. $6.18 + 12.05 = n$

23. $4.55 + 15.45 = n$

24. $12.05 - 7.18 = n$

25. $4.12 - 1.50 = n$

26. $3.765 + 2.483 = n$

27. $27.8 + n = 43.4$

28. $n + 6.38 = 9.46$

29. $82.2 - n = 17.3$

30. $n - 3.82 = 7.49$

Estimating answers

1. Howard wants a book that costs 22¢ and a knife that costs 69¢. Can he buy both with a dollar?

Think: 22¢ is about 20¢; 69¢ is about 70¢. So the total cost of the knife and the book will be about 20 + 70, or ? ¢.

Howard can buy both the book and the knife for less than a dollar.

2. To estimate the perimeter of a map 1 foot long and 10 inches wide, think: The two lengths total 2 feet and the two widths will be a little less than 2 feet. In all, the perimeter of the map is a little less than ? feet.

3. To estimate the cost of 14 feet of framing at $.24 a foot, think: $.24 is about a quarter. At a quarter a foot, I can get 4 feet for $1.00, or 12 feet for $3.00. The extra 2 feet will cost about ?.

The total cost of the framing will be about ?.

4. If you eat an egg each day for a year (365 days), do you estimate that you will have eaten about 20 dozen, 30 dozen, or 40 dozen eggs during that year?

5. There are 9 shelves in the science room. Each shelf is 4 feet long. Estimate how many 20-foot rolls of paper you would need to cover these shelves.

6. The pupils in a sewing class have saved $10.00 to buy 24 yards of curtain material. Estimate how much they can pay per yard.

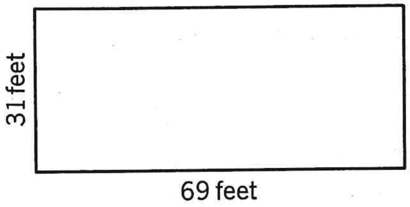
Can they pay as much as 10¢ a yard? 20¢? 40¢?

7. Estimate how much money David must save each week for a year (52 weeks) to be able to buy a 25-dollar chemistry set.

8. Fred and his father went to a father-son dinner. There were 27 tables with 6 chairs at each table.

- Fred thought that about 6 × 30 or ? people could be seated.
- His father thought that about 6 × 25 or ? people could be seated.
- Find the product of 6 and 27 to see whose estimate was nearer to the number of chairs at the tables.

9. Estimate the perimeter of the rectangle shown below. Then find the perimeter and use your estimate as a check on the answer.

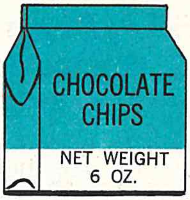


10. Estimate the area of the rectangular region shown above. Then find the area and use your estimate as a check on your answer.

Problem-solving practice

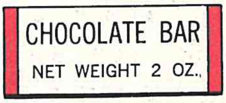
— Try to find the answers to the problems on this page without using a pencil and paper.

1. Mary is planning to make cookies. The recipe calls for $1\frac{1}{2}$ pounds of chocolate chips. A package of chocolate chips is shown in the picture at the right. Can Mary buy enough of these packages to make exactly $1\frac{1}{2}$ pounds? How many packages of chips should she buy?



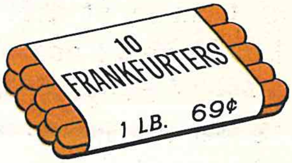
2. At 2 for 25¢, how much would a dozen strawberry tarts cost?

3. The picture at the right shows a chocolate bar that sells for 10¢. How much would a pound of these bars cost?



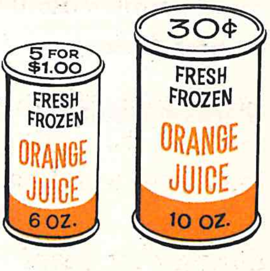
4. Elaine needs 18 feet of ribbon to sew around the edge of her bedspread. The ribbon she wants is sold by the yard. If it costs 60¢ a yard, how much will Elaine have to spend?

5. Look at the package of hot dogs shown at the right. How much would it cost to buy enough of these packages to have 2 hot dogs each for 2 dozen people?

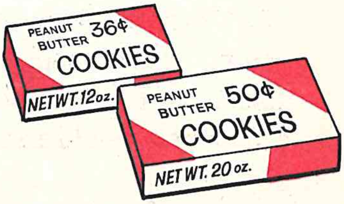


6. Look at the two cans of frozen orange juice shown in this picture.

- What is the difference in the contents of the two cans?
- How many ounces are contained in 5 cans of the smaller size?
- How many of the larger cans would you have to buy to have the same amount of orange juice as is contained in 5 of the smaller cans?
- Which is the better buy, 5 of the 6-ounce cans or 3 of the 10-ounce cans? Why?



7. Look at the picture at the right. Which box of cookies is the better buy? Why?



PROBLEM TEST 6

1. The three sides of a scarf measure $14\frac{1}{2}$ inches, $14\frac{1}{2}$ inches, and $18\frac{3}{4}$ inches. Tonia wants to sew a piece of rickrack around the scarf. How much rickrack does she need?

2. Jim is $53\frac{1}{4}$ inches tall. His brother is $48\frac{7}{8}$ inches tall. How much taller is Jim?

3. The hiking club can walk 10 miles in $2\frac{2}{3}$ hours. How many minutes is that?

4. Mr. Duffy drove 45.7 miles in one hour, 63.4 miles the second hour, and 58.9 miles the third hour. How far did he travel in these three hours?

5. The average yearly rainfall for Nevada is 8.60 inches; for Louisiana, it is 57.34 inches. How much greater is Louisiana's average yearly rainfall?

6. Sandra needs 30 inches of material. It costs 72¢ a yard. How much money will Sandra spend for the material?

7. Joe has 18.50 feet of fence. He wants to enclose a rectangular-shaped yard that is 12.25 feet long and 8.67 feet wide. How much more fence does he need?

8. Bakersville's snowfall was 8.6 inches in December, 6.3 inches in January, and 9.1 inches in February. What was Bakersville's average monthly snowfall for these three months?

9. Last year the Cuzak family spent an average of $130 a month for food and $24 a month for clothing. What was the total amount spent for food and clothing during the year?

10. In 1945, $45,924,000,000 was spent in the United States for food. In 1960, $86,823,000,000 was spent. Find the difference between the amounts of money spent for food during these two years.

Write your score on your Problem Test Record.

Mathamusements

One morning when John's mathematics class began, the clock read 15:00. The teacher wrote on the chalkboard that the class would have a test with 32 questions and that they would have 50 minutes to complete it, so they ought to try to answer each question in an average time of 230 seconds. After the test the teacher discussed the homework for a half hour. At the end of the discussion, the clock read 20:00, and it was time for lunch.

What was the class studying?

IT'S CHECK–UP TIME

1. Which numeral below is not equivalent to $3\frac{1}{4}$?

$\frac{26}{8}$ $3\frac{4}{16}$ $\frac{13}{4}$ $3\frac{4}{8}$ $2\frac{10}{8}$

2. Write a decimal for 5 tens 6 ones 8 tenths 3 hundredths 4 thousandths.

3. Name the numbers shown below using decimals.

$\frac{322}{1,000}$ $\frac{16}{1,000}$ $\frac{7}{1,000}$ $2\frac{99}{1,000}$ $7\frac{203}{1,000}$

4. Write a mixed form in lowest terms for each number named below.

2.4 13.25 10.077 9.005 5.05

— Copy and complete.

5. $\frac{1}{2}$ pt. makes ? fl. oz.

6. $1\frac{1}{3}$ ft. make ? in.

— Copy each example below and find the answer. Name each answer in lowest terms and in the more common mixed form.

	a	b
7.	$\begin{array}{r}6\frac{2}{5}\\+\ 7\frac{1}{5}\\\hline\end{array}$	$\begin{array}{r}13\frac{3}{5}\\-\ 6\frac{2}{5}\\\hline\end{array}$
8.	$\begin{array}{r}8\frac{1}{2}\\+\ 4\frac{1}{3}\\\hline\end{array}$	$\begin{array}{r}12\frac{5}{6}\\-\ 4\frac{1}{3}\\\hline\end{array}$
9.	$\begin{array}{r}37\frac{1}{8}\\-\ 18\frac{7}{8}\\\hline\end{array}$	$\begin{array}{r}18\frac{1}{4}\\+\ 18\frac{7}{8}\\\hline\end{array}$
10.	$\begin{array}{r}45\frac{1}{9}\\-\ 14\frac{1}{2}\\\hline\end{array}$	$\begin{array}{r}14\ \frac{1}{2}\\+\ 30\frac{11}{18}\\\hline\end{array}$

— Copy and find the answers.

	a	b
11.	$\begin{array}{r}54.87\\+\ 32.11\\\hline\end{array}$	$\begin{array}{r}86.98\\-\ 54.87\\\hline\end{array}$
12.	$\begin{array}{r}7.254\\-\ 6.877\\\hline\end{array}$	$\begin{array}{r}6.877\\+\ 0.377\\\hline\end{array}$

— Copy and solve each of the following number sentences.

13. $9\frac{7}{12} + 7\frac{3}{8} + 3\frac{2}{3} = n$

14. $7\frac{1}{4} - n = 4\frac{5}{6}$

15. $n - 5\frac{1}{10} = 6\frac{1}{5}$

16. $1.974 = 4.852 - n$

17. $32.49 = n - 27.67$

18. $12.536 = 7.638 + n$

19. At 3 for 15¢, how much will 2 dozen oranges cost?

20. Estimate the perimeter of a rectangle that is 26 inches long and 6 inches wide. Then find the perimeter and use your estimate to check your answer.

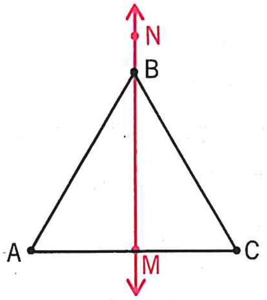
If you have time, try these —

21. Jessie wants 6 pieces of ribbon, each $1\frac{1}{2}$ feet long. The ribbon she wants is sold only by the yard. How many yards of ribbon should she buy?

22. Jack has saved \$12.75. His brother has saved $\frac{5}{3}$ as much money. How much money have the boys saved together?

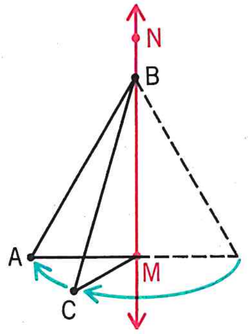
CHAPTER 13

Line symmetry



1. Look at the triangle shown in black at the right. The line shown in red intersects the triangle in point B and in point M.

- Measure $\overline{AB}$ and $\overline{CB}$. Are the lengths of these two segments equal?
- Point M is the middle point of $\overline{AC}$. What can you say about the lengths of $\overline{AM}$ and $\overline{MC}$?



2. Suppose you were able to "fold" triangle ABC along the red line.

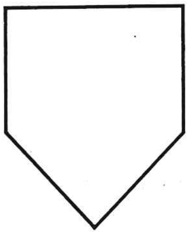
- Would point C fall on point A?
- Would $\overline{CB}$ fall on $\overline{AB}$?
- In triangle ABC, is the part to the right of the red line the exact reflection, or opposite, of the part to the left?

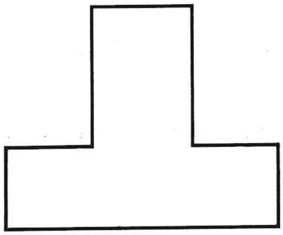
If a line can be drawn through a plane figure so that the part on one side of the line is an exact reflection of the part on the other side, the plane figure has line symmetry. The line that separates the plane figure in this way is called the line of symmetry.

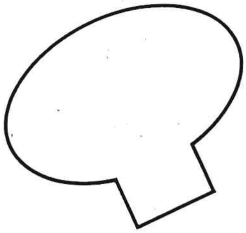
---

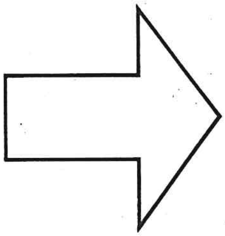
3. Does triangle ABC in Ex. 1 have line symmetry? What line is the line of symmetry?

4. Trace the four closed paths shown below. On your copies, draw the line of symmetry for each.

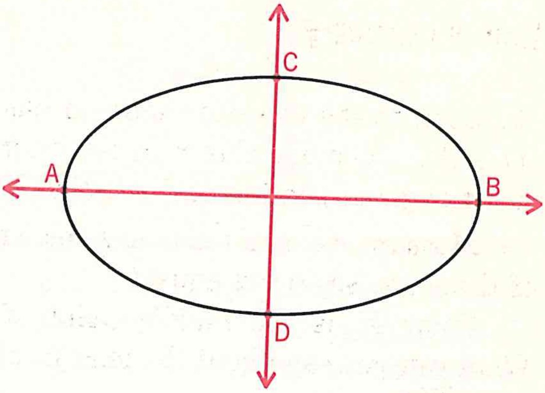






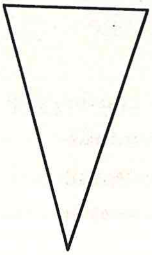


5. The closed path shown in black at the right is called an **ellipse**. How is an ellipse different from a circle?

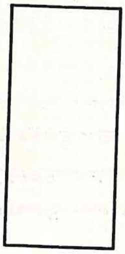


- Is the part of the ellipse below $\overleftrightarrow{AB}$ a reflection of the part of the ellipse above $\overleftrightarrow{AB}$?
- Is the part of the ellipse to the right of $\overleftrightarrow{CD}$ a reflection of the part to the left?
- Then an ellipse has two lines of symmetry. The lines of symmetry for the ellipse shown above are named ? and ?.

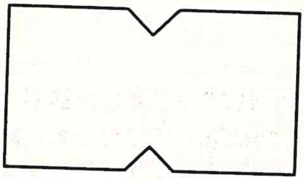
6. Copy the closed paths shown below. Then on your copies show all lines of symmetry for each closed path. Be careful — one of the closed paths does not have line symmetry.



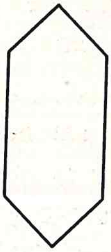
A



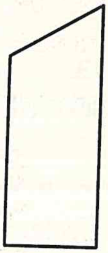
B



C

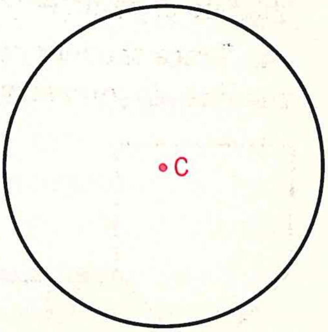


D



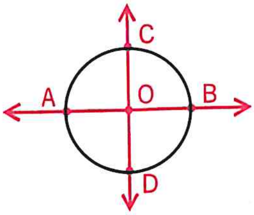
E

7. Shown at the right is a closed path called a ?. Point C is the center.



- Trace the drawing including point C.
- Draw a line through C. Is this line a line of symmetry for the circle?
- Draw another line through C. Is this line also a line of symmetry for the circle?
- How many lines of symmetry do you think a circle has? What point do all these lines of symmetry have in common?

Arc-degrees



1. Shown above is a circle with its center at point O. Lines AB and CD are perpendicular and intersect the circle at points A, C, B, and D.

- Do $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at O? Is $\overline{AB}$ a diameter of the circle? Name another diameter shown.

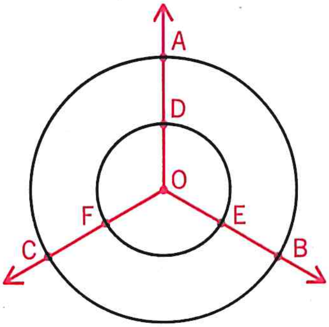
2. Lines AB and CD separate the circle into 4 arcs of the same size. One of these arcs is arc AC. Name the other 3 arcs.

3. Imagine a circle separated into 360 arcs of the same size. Any one of these arcs can be used as a unit to measure any other arc of the circle. This unit is called an **arc-degree.** Every circle measures 360 arc-degrees.

- Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ separate the circle shown above into 4 arcs of the same size, each of these arcs has a measurement of $\underline{?}$ arc-degrees.
- Arc ACB and arc BDA each have a measurement of $\underline{?}$ arc-degrees.

4. Using an arc-degree as a unit, what is the measure of $\frac{1}{2}$ of a circle? $\frac{1}{4}$ of a circle? $\frac{1}{3}$? $\frac{1}{6}$? $\frac{1}{10}$?

5. The drawing below shows two circles that have the same center, point O. Rays OA, OB, and OC separate each of the circles into 3 arcs of the same size.

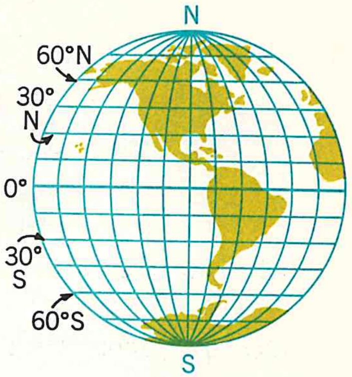


- Is arc DE one third of the smaller circle? Then in arc-degrees, its measure is $\underline{?}$. What is the measure of arc EF? of arc FD?
- Is arc AB one third of the larger circle? Then in arc-degrees, its measure is $\underline{?}$. What is the measure of arc BC? of arc CA?
- Imagine another larger circle with its center at O. Do $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ separate this circle into 3 arcs with measurements of 120 arc-degrees? Explain.

6. Every circle measures 360 arc-degrees. If an arc of a circle measures 36 arc-degrees, is this arc $\frac{36}{360}$, or $\frac{1}{10}$, of the whole circle? What part of a circle is an arc that measures 90 arc-degrees? 40 arc-degrees? 120 arc-degrees?

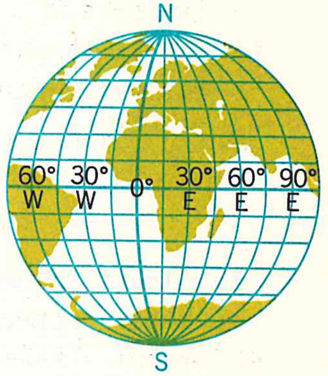
Latitude and longitude

1. What special type of closed surface does the earth resemble? The two pictures at the right show closed paths drawn around the earth. What kind of closed paths are they?



- Find the North Pole in the picture. Find the South Pole. Are all the circles shown running through these poles the same size?
- Are the circles shown running around the earth but not through the poles the same size?

2. The circles that run through the North and South poles are called circles of **longitude.** Is the measurement of each of these circles 360 arc-degrees (360°)? Then the measurement of each arc from pole to pole is ?°. Each of these half-circles from pole to pole is called a **meridian.**



3. The circles that run around the earth are called circles of **latitude.** The one that runs around the middle of the earth is called the **equator.** Find the equator in the picture above; on your classroom globe.

4. The circles of longitude and of latitude are separated into arc-degree units. The equator is called 0° latitude. Find the circle in the picture or on a globe that is 30° north latitude (30° N); 60° south latitude (60° S); 75° north latitude (75° N).

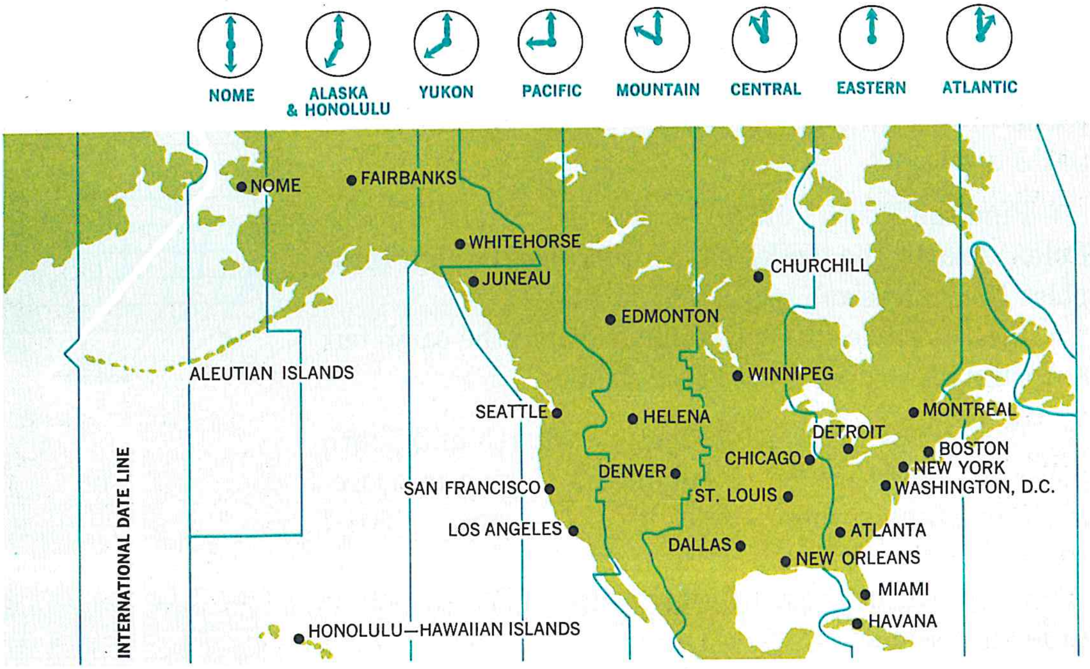
5. The meridian for 0° longitude is called the **prime meridian.** It passes through Greenwich, England. Find the meridian in the picture or on your classroom globe that is 30° east longitude (30° E); 60° west longitude (60° W); 45° east longitude (45° E).

6. We can locate a place on the earth by knowing its latitude and longitude. For example, Philadelphia, Pennsylvania, is located where the circle for 40° north latitude intersects the meridian for 75° west longitude (40° N, 75° W). See if you can find it on your classroom globe.

7. What national park is located at 45° N, 110° W?

8. Give the latitude and longitude for Los Angeles, California; for New Orleans, Louisiana; for Denver, Colorado; for Hilo, Hawaii.

Time zones



1. Because the earth rotates on its axis, the sun appears to move around the earth. Does it really move around the earth? Explain.

2. Suppose on a certain day sunrise in New York is at 6 A.M. Sunrise in California is 3 hours later. Does this mean that it will be 9 A.M. in California when the sun rises there?

The sun will rise in Hawaii 5 hours after it rises in New York. Does this mean it will be 11 A.M. in Hawaii before the Hawaiians see the sun?

So that clocks around the world will show about the same time for sunrise and for sunset, the world is separated into **time zones** like those shown on the map above.

3. Look at the map shown above. As you move from east to west, from New York to Hawaii, there is a difference of one hour between neighboring time zones. This means that when it is 7 A.M. in New York City, it is only ? A.M. in Chicago.

4. Each time zone is given a name as shown on the map. Washington, D.C., is in the Eastern Time Zone. In what time zone is St. Louis? Denver? San Francisco? Fairbanks?

5. When it is 10 o'clock in Miami, what time is it in Chicago? Seattle? Atlanta? Denver? Honolulu? Nome?

6. The difference between the time in Montreal, Canada, and the time in San Francisco is ? hours.

7. Fred and his family live in Denver. They spent their vacation in Miami. Their plane took off at 10 A.M. Denver time. They arrived at Miami 4 hours later. What time did the clock in the Miami airport show upon their arrival?

8. A baseball game between the New York Mets and the Los Angeles Dodgers is scheduled to start at 8 P.M. in New York City. To listen to the game in California, at what time should a Dodger fan turn on his radio?

9. If you live in New Orleans, at what time should you turn on your television set to see the Rose Bowl football game that begins at 2 P.M. in California?

10. The Cotton Bowl football game in Dallas begins at 3 P.M. What time is that in Washington, D.C.? in Winnipeg, Canada?

11. A businessman in Fairbanks, Alaska, wishes to call his associate in Boston. If he knows his associate will be in his office at 9 A.M. Boston time, at what time in Fairbanks should he make the telephone call?

12. What time is it in Boston when it is 9 A.M. in Fairbanks?

13. Mr. Hunter, in New York, wants to talk to a friend in Denver at 9:30 P.M. Mountain time. At what time should he telephone from New York?

14. Dick and his father drove from San Francisco by way of the Alaska Highway to Fairbanks. How many times did they have to change their watches to show the correct time where they were?

15. If you were flying from Juneau, Alaska, to Nome, Alaska, would you set your watch backward or forward? how many hours?

16. If you were flying from Honolulu to Washington, D.C., would you set your watch backward or forward? how many hours?

17. An airline schedule shows it takes 2 hours and 35 minutes to travel from Chicago to New York. If your watch shows 7 o'clock when you leave Chicago, what time will it show when you arrive in New York? What time should it show?

18. When Susan reached Atlanta on her way to camp, she sent a telegram to her mother in New Orleans. The telegram was stamped as having been sent at 2:30 P.M., June 21. Her mother received the telegram at 2:15 P.M., June 21. Explain how this situation is possible.

19. An experimental jet took off from New York just before 1 P.M. It arrived in California just before noon the same day. About how long did it take the plane to fly across the country?

Keeping up in mathematics

Oral

— Find the errors in Ex. 1–3.

$$\begin{array}{lrl}
 & \text{a} & \text{b} \\
\textbf{1.} & \begin{array}{r} 4\frac{2}{3} = 4\frac{6}{9} \\ \underline{-\ 3\frac{1}{9} = 3\frac{1}{9}} \\ 7\frac{7}{9} \end{array} & \begin{array}{r} 5\frac{5}{6} = \frac{10}{12} \\ \underline{+\ 3\frac{1}{12} = \frac{1}{12}} \\ \frac{11}{12} \end{array} \\
\textbf{2.} & \begin{array}{r} 4\frac{2}{3} = 4\frac{4}{9} \\ \underline{+\ 5\frac{1}{9} = 5\frac{1}{9}} \\ 9\frac{5}{9} \end{array} & \begin{array}{r} 3\frac{1}{8} = 3\frac{1}{8} \\ \underline{-\ 2\frac{3}{4} = 2\frac{6}{8}} \\ 1\frac{5}{8} \end{array} \\
\textbf{3.} & \begin{array}{r} 4\frac{2}{3} = 4\frac{4}{6} \\ \underline{-\ 1\frac{1}{2} = 1\frac{3}{6}} \\ 5\frac{1}{6} \end{array} & \begin{array}{r} 2\frac{1}{2} = 2\frac{2}{4} \\ \underline{+\ 3\frac{1}{4} = 3\frac{1}{4}} \\ 5\frac{3}{8} \end{array}
\end{array}$$

4. 324_{six} = $\underline{?}$ thirty-sixes $\underline{?}$ sixes $\underline{?}$ ones

5. 4 thirty-sixes 3 sixes 1 one = $\underline{?}_{six}$

6. What decimal numeral names the same number as 21_{six}?

— Tell the answers.

	a	b
7.	$57 + 9 = \underline{?}$	$86 + 5 = \underline{?}$
8.	$63 + 7 = \underline{?}$	$98 + 6 = \underline{?}$
9.	$200 \times 32 = \underline{?}$	$3{,}000 \times 32 = \underline{?}$
10.	$43 - 4 = \underline{?}$	$62 + \underline{?} = 74$
11.	$25 \times \underline{?} = 200$	$\underline{?} \times 14 = 1{,}400$

12. For each sentence below, name the set of whole numbers that make it true.

$3 \times 2 > n$ $\qquad$ $14 - n > 5$

Written

$$\begin{array}{lrrr}
 & \text{a} & \text{b} & \text{c} \\
\textbf{1.} & \begin{array}{r} 658 \\ 204 \\ \underline{+\ 470} \end{array} & \begin{array}{r} \$469.80 \\ 27.54 \\ \underline{+\ 308.00} \end{array} & \begin{array}{r} 45{,}983 \\ 679 \\ \underline{+\ 8{,}000} \end{array} \\
\textbf{2.} & \begin{array}{r} 409 \\ \underline{\times\ 608} \end{array} & \begin{array}{r} 58{,}679 \\ \underline{-\ 49{,}876} \end{array} & \begin{array}{r} 1{,}587 \\ \underline{\times\ 946} \end{array} \\
\textbf{3.} & \begin{array}{r} 456 \\ \underline{\times\ 280} \end{array} & \begin{array}{r} \$3{,}162.00 \\ \underline{-\ 1{,}874.98} \end{array} & 95\overline{)4{,}370} \\
\textbf{4.} & 46\overline{)1{,}610} & 45\overline{)12{,}035} & 8\overline{)17{,}954}
\end{array}$$

5. A train traveled a distance of 912 miles in 16 hours. Its average speed was $\underline{?}$ miles an hour.

6. At an average rate of 46 miles an hour, how far will a bus travel in 6 hours?

7. Barbara needs 4 yards of gingham and a 45-cent roll of tape to make a laundry bag. If she pays 69¢ a yard for the gingham, how much will it cost to make the bag?

8. Chuck helped his father box and sell 500 tomato plants. They put 25 plants in each box and sold them for $2.00 a box. How much money did they get for the 500 tomato plants?

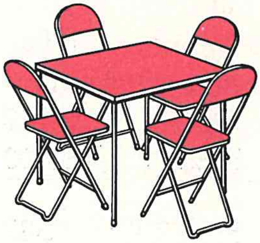
9. Solve this sentence.

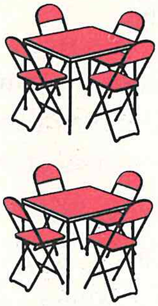
$$n + 56\frac{1}{2} = 100\frac{3}{4}$$

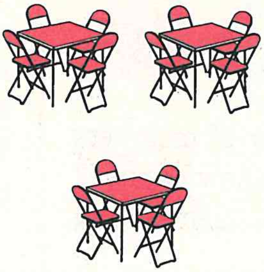
10. The following are models of what kinds of closed surfaces?

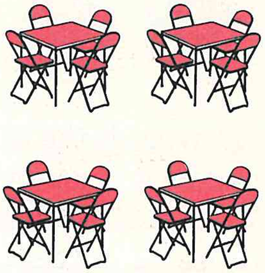
a shoe box $\qquad$ a tin can $\qquad$ a globe

Comparing numbers


A


B


C


D

1. For each picture shown above, name the number of members in the set of tables; in the set of chairs.

2. What is the difference between the number of tables and the number of chairs in A? in B? C? D?

What operation do you use to find the difference between two numbers?

One way to compare two numbers is to find the difference between the two numbers.

3. Is the difference between the number of chairs and the number of tables the same for each picture shown above?

4. Suppose that the difference between the number of chairs and the number of tables is 15 and that the number of chairs is 20. Which of the following statements can be true? Explain.

- The number of tables is 5.
- The number of tables is 35.

5. In A, the number of chairs is ? times the number of tables.

Did you subtract or did you divide to find the answer?

Another way to compare two numbers is to find the quotient of one number divided by the other number.

6. In B, the number of chairs is ? times the number of tables.

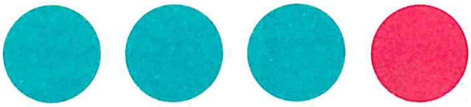
In the same way, compare the number of chairs and the number of tables in C; in D.

7. In each picture shown above, is the number of chairs 4 times the number of tables?

8. Suppose that the number of chairs is 4 times the number of tables and that the number of chairs is 20. Which of these statements can be true? Explain.

- The number of tables is 5.
- The number of tables is 80.

Ratio



1. What is the number of members in the set of blue disks shown above? What is the number of members in the set of red disks?

2. What is the quotient of the number of blue disks divided by the number of red disks? The number of blue disks is $\underline{\ ?\ }$ times the number of red disks.

3. Does the fraction $\frac{1}{3}$ name the quotient of the number of red disks divided by the number of blue disks? The number of red disks is $\underline{\ ?\ }$ of the number of blue disks.

4. When two numbers are compared by finding the quotient of one number divided by the other, the quotient is called a **ratio.** For example, the quotient 3 is called the ratio of 3 to 1. The quotient $\frac{1}{3}$ is called the ratio of 1 to $\underline{\ ?\ }$.

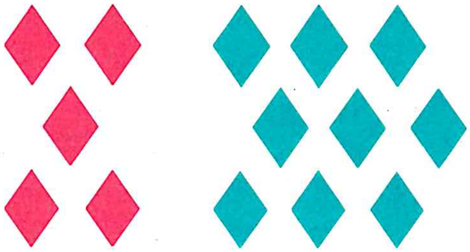
5. In the picture of the boxes shown below, the ratio of the number of blue boxes to the number of red boxes is 4 to 3, or $\frac{4}{3}$. What is the ratio of the number of red boxes to the number of blue boxes?

6. The ratio of one number to another is named by a fraction or by a numeral for a whole number. What fraction or numeral for a whole number names the ratio of 5 to 8? 8 to 5? 12 to 6? 6 to 7? 9 to 1?

— Write a numeral to name each of the following ratios.

	a	b	c
7.	4 to 5	6 to 3	1 to 9
8.	8 to 3	3 to 8	4 to 1
9.	3 to 14	6 to 7	12 to 5
10.	22 to 23	2 to 1	5 to 16

11. Name the ratio of the number of red diamonds to the number of blue diamonds shown below.

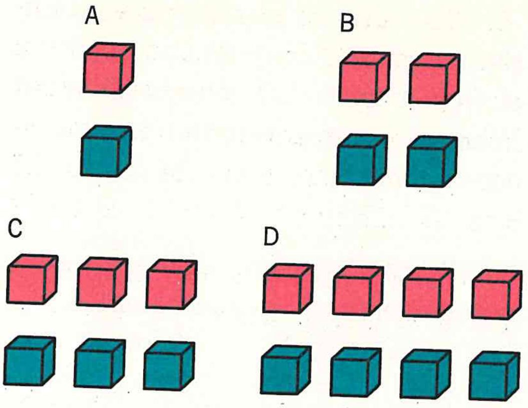


12. Name the ratio of the number of blue diamonds to the number of red diamonds shown in Ex. 11.

13. Name the ratio of the number of red diamonds to the total number of diamonds shown in Ex. 11.

14. Name the ratio of the number of blue diamonds to the total number of diamonds shown in Ex. 11.

Naming ratios with fractions

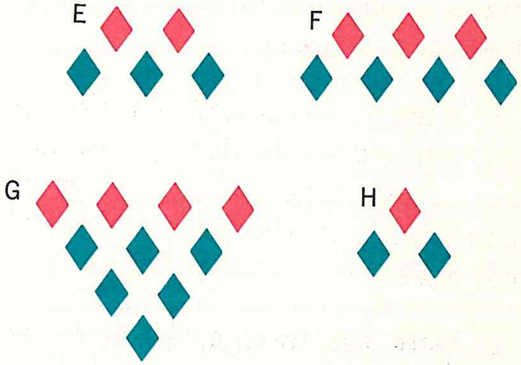


1. Each of the four sets of blocks shown above contains a number of red blocks and a number of blue blocks.

- How many blocks are shown in set A? How many are red? What fraction in lowest terms names the ratio of the number of red blocks to the total number of blocks?
- Look at set B. What fraction in lowest terms names the ratio of the number of red blocks to the total number of blocks?
- What fraction in lowest terms names the ratio of the number of red blocks to the total number of blocks in set C? in set D?
- Is the ratio of the number of red blocks to the total number of blocks the same for each set shown above?

2. What fraction in lowest terms names the ratio of 1 to 2? the ratio of 4 to 8? 9 to 18? 12 to 24? 50 to 100?

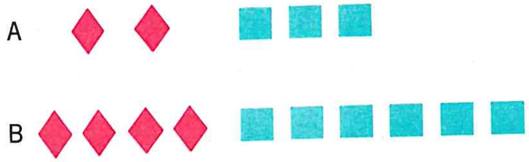
3. Look at the four sets of diamonds shown below. For which two sets is the ratio of the number of red diamonds to the number of blue diamonds the same?



4. The chart below shows five pairs of sets. Copy it. Then draw additional diamonds or boxes on your chart so that the ratio of the number of members in A to the number of members in B is $\frac{3}{4}$.

Set A	Set B
◆ ◆ ◆	■ ■ ■
◆ ◆ ◆	■
◆	■ ■ ■ ■
◆ ◆ ◆ ◆ ◆ ◆	■ ■ ■ ■ ■ ■
◆ ◆ ◆	■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■

Equivalent fractions and ratios



1. Look at sets A and B shown above. What is the ratio of the number of diamonds to the number of boxes in A? What is the ratio of the number of diamonds to the number of boxes in B? Is the ratio the same for both sets?

2. Why can you use either $\frac{2}{3}$ or $\frac{4}{6}$ to name the ratio of the number of diamonds to the number of boxes in B?

3. Look at the set of fractions shown below.

$$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \cdots\right\}$$

- Do these fractions all name the same number? What do we call different fractions that name the same number?

- Do all the fractions shown in the set above name the same ratio? Explain.

4. Suppose the ratio of the number of diamonds to the number of boxes is $\frac{2}{3}$. If a set contains 14 diamonds, how many boxes does it contain? What fraction that shows a numerator of 14 is equivalent to $\frac{2}{3}$?

$$\frac{2}{3} = \frac{14}{?}$$

5. On a can of frozen orange juice, the instructions say to mix each can of frozen juice with 4 cans of water. What is the ratio of the number of cans of frozen juice to the number of cans of water?

- What fraction that shows a numerator of 2 is equivalent to $\frac{1}{4}$? $\frac{1}{4} = \frac{2}{?}$ Then if 2 cans of frozen juice are used, how many cans of water should be used?

- What fraction that shows a denominator of 16 is equivalent to $\frac{1}{4}$? $\frac{1}{4} = \frac{?}{16}$ Then if 16 cans of water are used, how many cans of frozen juice should be used?

- Explain how you can use the idea of equivalent fractions to find how many cans of water should be mixed with a given number of cans of frozen juice; to find how many cans of frozen juice should be mixed with a given number of cans of water.

6. Use the ratio given in Ex. 5 and the idea of equivalent fractions to find how many cans of water should be mixed with 5 cans of frozen juice; with 8 cans of frozen juice; with 12 cans; with 25 cans; with 50 cans; with 100 cans.

7. How many cans of frozen juice should be mixed with 8 cans of water? with 12 cans of water? with 40 cans? with 100 cans?

Using ratios

1. United States nickels are coined from an alloy of copper and nickel called cupronickel. If 1 pound of nickel were used to make this alloy, then 3 pounds of copper would be used. What is the ratio of the number of pounds of nickel to the number of pounds of copper in cupronickel?

2. If 2 pounds of nickel were used to make cupronickel, how many pounds of copper would be used?

3. How many pounds of copper would be used to make cupronickel if 5 pounds of nickel were used? if 25 pounds of nickel were used? 50 pounds of nickel? 100 pounds? 500 pounds?

4. If 27 pounds of copper were used to make cupronickel, how many pounds of nickel would be used?

5. How many pounds of nickel would be used to make cupronickel if 36 pounds of copper were used? if 72 pounds of copper were used? 180 pounds of copper? 450 pounds? 900 pounds?

6. A recipe for chocolate brownies requires 2 cups of sugar and 4 eggs. Find the number of cups of sugar and the number of eggs that would be needed if you double the recipe; if you triple it; if you make only $\frac{1}{2}$ of the recipe.

7. What fraction in lowest terms names the ratio of the number of ears on a cow to the number of hoofs? What is the ratio of ears to hoofs in a herd of 100 cows? 1,000 cows? 10,000 cows?

8. In a certain fuel mixture, the ratio of the number of cups of oil to the number of gallons of gasoline is $\frac{2}{5}$. Use this ratio to find how many gallons of gasoline should be mixed with 4 cups of oil; 6 cups of oil; 8 cups; 10 cups; 100 cups.

9. To obtain the fuel mixture described in Ex. 8, how many cups of oil should be mixed with 10 gallons of gasoline? 25 gallons of gasoline? 100 gallons? 500 gallons?

10. In a recipe for doughnuts, 4 eggs are required to make 3 dozen doughnuts. How many eggs are needed to make 6 dozen doughnuts? 9 dozen doughnuts? 12 dozen? 30 dozen? 120 dozen?

11. For the recipe given in Ex. 10, how many doughnuts can you make with 8 eggs? 12 eggs? 24 eggs? 144 eggs?

Think twice!

12. Using the recipe given in Ex. 10, how many doughnuts can you make with 2 eggs? with 6 eggs? with 10 eggs?

Ratio and money

1. If Harry earns $1.20 for every hour he works, how much will he earn if he works 2 hours? 7 hours? 8 hours? 40 hours?

2. Suppose you wish to find how much Harry earns in a given number of minutes, such as 10 minutes.

- You can think of $1.20 as $\underline{?}$ cents and 1 hour as $\underline{?}$ minutes. Does $\frac{120}{60}$ name the ratio of the number of cents to the number of minutes? What fraction in lowest terms or numeral for a whole number names the same ratio?
- The ratio of the number of cents to the number of minutes tells you that the number of cents is always $\underline{?}$ times the number of minutes. In 10 minutes Harry would earn $\underline{?} \times 10$, or $\underline{?}$, cents.

3. How much will Harry earn in 30 minutes? 45 minutes? $1\frac{1}{2}$ hours? $1\frac{3}{4}$ hours?

4. If wire costs 30¢ a yard, what is the ratio of the number of cents to the number of yards? the number of cents to the number of feet? the number of cents to the number of inches?

5. Use your answers to Ex. 4 to find the cost of 1 foot of wire; 2 feet; 5 feet; $\frac{1}{2}$ foot; $4\frac{1}{2}$ feet.

6. At 60¢ a yard, what is the cost of 1 foot of rope? $1\frac{1}{2}$ feet? $2\frac{1}{2}$ feet?

7. Suppose doughnuts sell for 90¢ a dozen. What is the ratio of the number of cents to the number of dozens? the number of cents to the number of doughnuts?

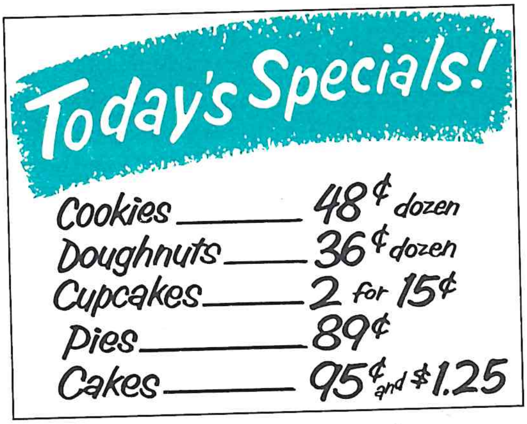
8. Use your answers to Ex. 7 to find the cost of 6 dozen doughnuts; the cost of 6 doughnuts; 18 doughnuts; $1\frac{1}{2}$ dozen doughnuts; 30 doughnuts.

9. Mary can earn 50¢ an hour baby-sitting. How much can she earn in 2 hours? $2\frac{1}{2}$ hours?

10. Tom earns 75¢ an hour for mowing lawns. How much will he earn in 20 minutes? 40 minutes? 1 hour and 20 minutes? 2 hours and 40 minutes?

11. If plums sell at 3 for 10¢, how much will 6 plums cost? 12 plums? 2 dozen plums?

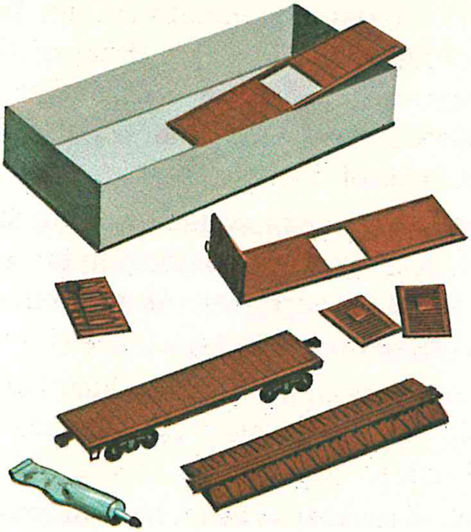
12. Use the sign in the bakery-shop window shown below to find the total cost of 3 dozen cookies, $1\frac{1}{2}$ dozen doughnuts, and 2 dozen cupcakes.



Today's Specials!

Cookies	48¢ dozen
Doughnuts	36¢ dozen
Cupcakes	2 for 15¢
Pies	89¢
Cakes	95¢ and $1.25

Ratio and scale models



1. Bob is building a scale model of a boxcar to add to his model railroad. He says the ratio of the number of inches on the model to the number of feet on the real boxcar is 1 to 4, or $\frac{1}{4}$. This means that 1 inch on the model represents $\underline{?}$ feet on the real boxcar.

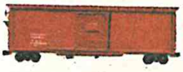
2. The model boxcar Bob is building is 9 inches long. How long is the real boxcar?

3. The door on the real boxcar is 8 feet high. How high is the door on Bob's model?

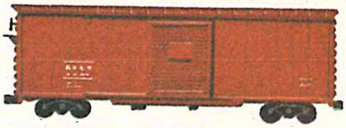
4. Sam is also building a scale model of the 36-foot boxcar. The ratio of the number of inches on Sam's model to the number of feet on the real boxcar is $\frac{1}{10}$. Will Sam's model of the boxcar be larger or smaller than Bob's model? Explain.

5. Jerry is also making a scale model of the boxcar. His model will be $4\frac{1}{2}$ inches long when finished. If the real boxcar is 36 feet long, what is the ratio of the number of inches on Jerry's model to the number of feet on the real boxcar?

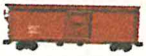
6. Shown below are the three models built by Bob, Sam, and Jerry, but not in that order. For which model is the ratio of the number of inches on the model to the number of feet on the real boxcar the greatest? the least? Explain.



A



B



C

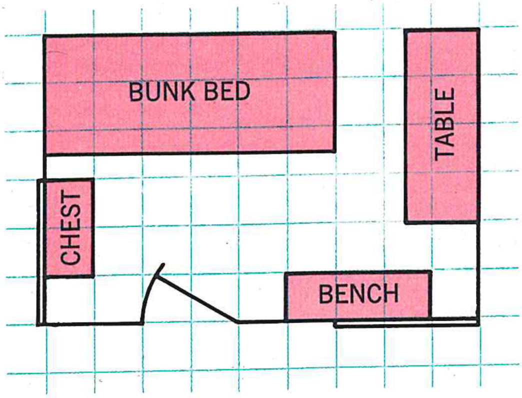
7. Mary's father made her a scale-model dollhouse. The ratio of the number of inches on the dollhouse to the number of feet on a real house is $\frac{2}{3}$. What is the ratio of the number of inches on the dollhouse to the number of inches on a real house?

Think twice!

8. If the doors of a real house are $7\frac{1}{2}$ feet high, how high are the doors on Mary's dollhouse (Ex. 7)?

Scale drawings

1. Jack and his friends are building a clubhouse in the back yard. Before they started, Jack drew the clubhouse plans shown below.



SCALE: $\frac{1}{4}$ IN. TO 1 FT.

- The plans show that Jack used a scale of $\frac{1}{4}$ inch to 1 foot. How is this scale related to the ratio of $\frac{1}{4}$ to 1?
- On the plans, the clubhouse is ? fourths inches long and ? fourths inches wide.
- The actual clubhouse will be ? feet long and ? feet wide.

2. The ratio of $\frac{1}{4}$ to 1 is the quotient of $\frac{1}{4}$ divided by 1. What is the quotient of any number divided by 1? Then $\frac{1}{4} \div 1$ equals ?. The fraction in lowest terms that names the ratio of $\frac{1}{4}$ to 1 is ?.

3. On the floor plan, how is the door marked? How wide is the door on the plans? How wide will it be in the clubhouse?

4. On the floor plan, how are the two windows marked? How wide is each window on the plan? Then how wide will each window be in the clubhouse?

5. If Jack changed his original plans to show a door 3 feet wide, the segment that represents the door on his drawing should be how long?

6. Jack and his friends also planned for the furnishings shown on his drawing. Use the scale shown to find the actual dimensions of each piece.

- Bunk bed: ? ft. by ? ft.
- Table: ? ft. by ? ft.
- Chest: ? ft. by ? ft.
- Bench: ? ft. by ? ft.

7. How long would Jack's drawing be if he had used the scale of $\frac{1}{2}$ inch to 1 foot? how wide?

8. How long would Jack's drawing be if he had used the scale of 1 inch to 1 foot? If this scale were used, could you show the plans on this page? Explain.

9. Jack's friend also made a set of plans for the clubhouse. He used a scale of $\frac{1}{2}$ inch to 1 foot. Tell the dimensions on these plans for each item of furniture.

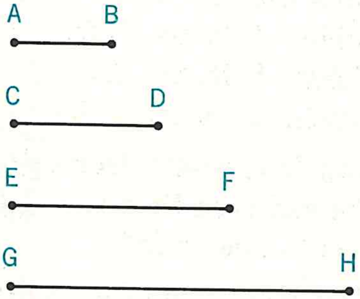
- Bunk bed: ? in. by ? in.
- Table: ? in. by ? in.
- Chest: ? in. by ? in.
- Bench: ? in. by ? in.

10. Using a scale of $\frac{1}{2}$ inch to 1 foot, draw a line segment that represents 2 feet; 3 feet; 5 feet.

11. Using a scale of $\frac{1}{10}$ inch to 1 foot, what distance would be represented by a line segment $\frac{1}{2}$ inch long? 2 inches long? $2\frac{1}{2}$ inches long?

12. Using a scale of 1 inch to 100 miles, what distance would be represented by a line segment $\frac{1}{2}$ inch long? 2 inches long? $1\frac{1}{2}$ inches long?

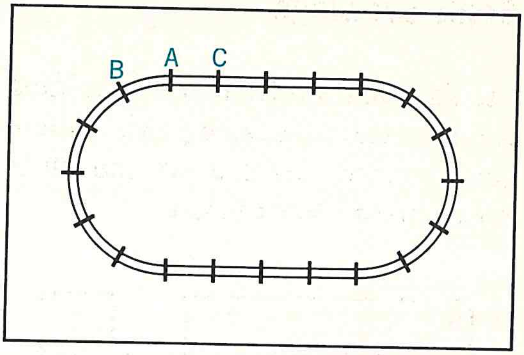
13. Using the scale of $\frac{1}{8}$ inch to 1 mile, tell what distance each of the following line segments represents.



14. Using the scale of $\frac{1}{8}$ inch to 1 foot, draw a line segment to represent 3 feet; 5 feet; 12 feet; 16 feet; 20 feet.

15. Using the scale of $\frac{1}{2}$ inch to 1 yard, draw a line segment to represent 1 yard; 2 yards; 3 yards; 5 yards.

16. Using a scale of $\frac{1}{4}$ inch to 1 yard, what distance would be represented by a line segment $\frac{1}{2}$ inch long? 3 inches long? $3\frac{1}{2}$ inches long? $\frac{1}{8}$ inch long?



SCALE: 1 IN. TO 40 IN.

17. Shown above is a picture of Jack's model railroad. The picture has been drawn to a scale of 1 inch to 40 inches.

- From C to A is one section of straight track, and from A to B is one section of curved track. How many sections of straight track does Jack have? how many sections of curved track?
- How long is each straight section of track?
- What is the total length of the straight sections of track?
- Each curved section of track is $10\frac{3}{4}$ inches long. See if you can find the distance around the model railroad track.

Think twice!

18. Suppose each $\frac{1}{4}$ inch of model track shown in the drawing above represents 1 foot of real railroad track. What would be the distance around this much track of a real railroad?

Another look at comparing numbers

1. Three girls and five boys play the violin in the school orchestra.

- There are ? more boys than girls.
- There are ? fewer girls than boys.
- The ratio of the number of boys to the number of girls is ?.
- The ratio of the number of girls to the number of boys is ?.

2. In Ex. 1, you showed four different comparisons for the numbers 3 and 5. In which of the comparisons did you find the difference between the numbers? In which of the comparisons did you show a ratio of one number to another?

3. How many pupils play the violin in the school orchestra (Ex. 1)?

- What is the difference between the number of boys and the total number of pupils?
- Write a fraction in lowest terms to name the ratio of the number of boys to the total number of pupils; the total number of pupils to the number of boys.
- What is the difference between the number of girls and the total number of pupils?
- Write a fraction in lowest terms to name the ratio of the number of girls to the total number of pupils; the total number of pupils to the number of girls.

4. Peter has 5¢ and Rusty has 30¢. Write four true sentences comparing 5¢ with 30¢.

5. How many girls are there in your classroom? how many boys? Write four true sentences comparing the number of girls with the number of boys.

6. Compare the number of girls with the total number of pupils in your classroom.

7. Compare the number of boys with the total number of pupils in your classroom.

8. Randy and Cliff weighed themselves on the school scales. Randy's weight was 90 pounds. Cliff weighed 100 pounds.

- What is the difference between their weights?
- Write a fraction in lowest terms to name the ratio of Randy's weight to Cliff's weight; of Cliff's weight to Randy's weight.

9. A new bicycle costs about $60. Charles can buy a used one for $20. This is about ? dollars less than the cost of a new one. Does $\frac{1}{3}$ name the ratio of the approximate cost of a new bicycle to the cost of a used one, or the cost of a used bicycle to the approximate cost of a new one?

Comparing large numbers

The table below shows the seven states in the United States having the largest populations according to the 1960 census.

California	15,717,204
Illinois	10,081,158
Michigan	7,823,194
New York	16,782,304
Ohio	9,706,397
Pennsylvania	11,319,366
Texas	9,579,677

1. Read the population figure for the state that has the greatest number of people. Read the population figure for each of the other states shown in the table.

2. What is the difference between the number of people living in Texas and the number of people living in Ohio? the number living in California and the number living in New York? the number living in Pennsylvania and the number living in Michigan?

3. Arrange the names of the states according to the size of population. Begin with the state that has the greatest population.

4. What is the difference between the number of people living in the first state and the number of people living in the last state on your list?

5. What state is the middle state on your list? Its population is how much greater than that of the state with the smallest population on your list? how much less than that of the state with the greatest population?

6. When a ratio is used to compare large numbers like those shown in the table, approximations for the numbers are often used to make the comparison easier.

Make a new table for the seven states having the greatest populations. For this table, approximate each of the numbers shown in the first table to the nearest million.

7. One way to name the ratio of Ohio's population to Michigan's population is shown below.

$$\frac{10{,}000{,}000}{8{,}000{,}000}$$

Another way to name the ratio of 10 million to 8 million is to use the fraction $\frac{10}{8}$. What fraction in lowest terms also names the ratio of 10 million to 8 million?

8. Using the numbers shown in the table you made for Ex. 6, write a fraction in lowest terms or a numeral for a whole number to name the ratio of:

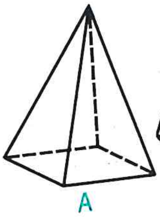
- New York's population to Ohio's
- Texas' population to Illinois's
- Pennsylvania's to California's

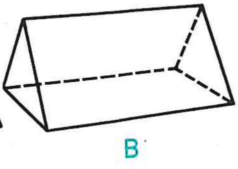
SELF-HELP TEST 11

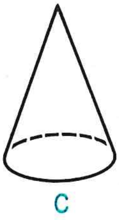
1. Write a Roman numeral for each number named below. [**6–7**]

17 23 32 55 102

2. Name each figure shown below. [**204–05**]







3. Which gives a more precise measurement, measuring to the nearest half inch or measuring to the nearest quarter inch? [**84–85**]

4. List the whole numbers less than 20 that are prime numbers. [**180**]

5. Write a decimal that names 4 tens 6 ones 2 tenths 5 hundredths. [**240**]

6. Write a decimal that names seven thousand and seven thousandths. [**261**]

7. Write a decimal to name each number shown below. [**243, 244, 261**]

$3\frac{3}{10}$ $12\frac{14}{100}$ $24\frac{1}{2}$ $5\frac{1}{4}$ $6\frac{17}{1,000}$

8. 4 sixes 2 ones $= \underline{\ ?\ }_{six}$ [**228–29**]

9. Which numerals below name the same number as $6\frac{2}{3}$? [**253**]

$6\frac{4}{6}$ $6\frac{3}{2}$ $\frac{20}{3}$ $5\frac{5}{3}$ $\frac{40}{6}$ $\frac{40}{3}$

10. Write a fraction in lowest terms or a numeral for a whole number to name each ratio. [**280–81**]

3 to 7 4 to 6 8 to 2

SELF-HELP TEST 12

— Copy, find the answers, and check.

1. $\begin{array}{r} 3\frac{1}{5} \\ +\ 2\frac{7}{10} \\ \hline \end{array}$ [**249**]

2. $\begin{array}{r} 12\frac{4}{5} \\ +\ 34\frac{2}{3} \\ \hline \end{array}$ [**252**]

3. $\begin{array}{r} 7\frac{7}{8} \\ -\ 4\frac{3}{4} \\ \hline \end{array}$ [**251**]

4. $\begin{array}{r} 39\frac{1}{3} \\ -\ 19\frac{7}{8} \\ \hline \end{array}$ [**255**]

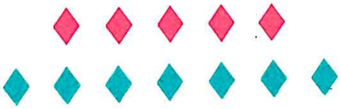
5. $\begin{array}{r} 9.4 \\ +\ 5.9 \\ \hline \end{array}$ [**264**]

6. $\begin{array}{r} 8.62 \\ -\ 7.83 \\ \hline \end{array}$ [**266**]

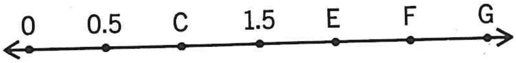
7. $66\overline{)3,157}$ [**143**]

8. $42\overline{)\$16.75}$ [**146**]

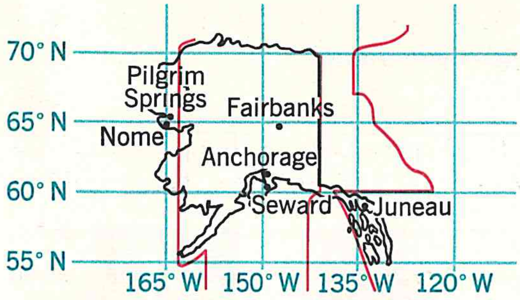
9. What is the ratio of the number of red diamonds to the number of blue diamonds shown below? [**280**]



10. What number should be matched with point *C* on the number line shown below? What number should be matched with point *E*? *F*? *G*? [**245**]



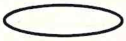
IT'S CHECK-UP TIME

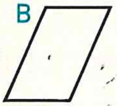


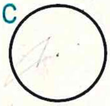
1. The red paths on the map above indicate time zones. What time is it in Juneau when it is 7:00 in Nome?

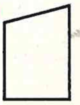
2. Use the blue paths on the map to find the latitude and longitude for Seward; for Pilgrim Springs.

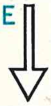
3. Which figures shown below have line symmetry?

A


B


C


D


E


4. How many lines of symmetry can be shown for each figure in Ex. 3?

5. Arc *DE* is $\frac{1}{4}$ of a circle. The measure of arc *DE* is ? arc-degrees.

6. The ratio of the number of blue boxes shown below to the total number of boxes is ?.

🟥 🟦 🟦 🟦 🟦 🟥 🟥 🟦 🟦

7. The ratio of blue boxes to red boxes shown above is ?.

— Name each ratio with a numeral.

8. 5 to 2

9. 4 to 1

10. 3 to 5

11. 4 to 12

12. Which numerals name the same ratio? $\frac{3}{2}$ $\frac{18}{9}$ $\frac{15}{5}$ $\frac{1}{3}$ 3

— Using a scale of $\frac{1}{4}$ inch to 1 foot, find the distance represented by each line segment described below.

13. A segment 2 inches long

14. A segment $2\frac{1}{4}$ inches long

15. A segment $\frac{3}{4}$ inch long

16. Using a scale of $\frac{1}{10}$ inch to 1 mile, how long a line segment would be drawn to represent 15 miles?

17. Rose earns 75¢ an hour baby-sitting. If she baby-sits for 3 hours and 20 minutes, she will earn ?.

18. There are 15 girls and 18 boys in a fifth-grade class. Compare the number of boys to the number of girls in two ways.

19. The population of Illinois was 4,821,550 in 1900 and 10,081,158 in 1960. To the nearest million, find the ratio of Illinois's population in 1900 to its population in 1960.

20. At $1.20 a yard, what is the cost of $2\frac{1}{2}$ feet of material?

If you have time, try these —

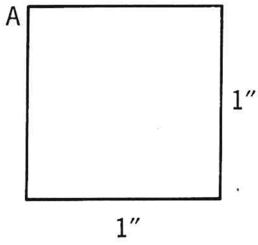
— The ratio of the number of inches on a model plane to the number of feet on a real plane is $\frac{2}{5}$.

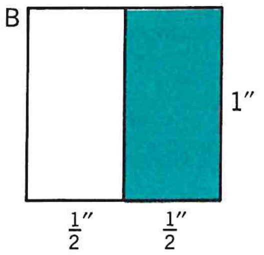
21. If the real plane is $42\frac{1}{2}$ feet long, how long is the model plane?

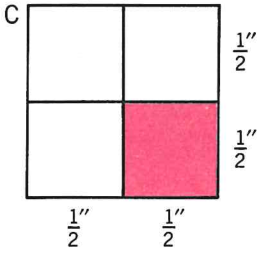
22. If the model plane is $5\frac{3}{5}$ inches high, how high is the real plane?

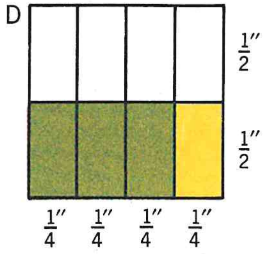
CHAPTER 14

Finding the product of two fractional numbers









1. You have discovered that to find the area of a rectangular region, you multiply length times width. Look at figure A shown above. What is the area of this region?

2. The same rectangular region is shown in figure B above. A line segment separates it into two smaller regions of the same size. Each of these smaller regions is what part of the larger region? Then is the area of each smaller region $\frac{1}{2}$ square inch?

3. In B, what is the area of the region shaded blue? What are the dimensions of this region? Then is this sentence true? $\frac{1}{2} \times 1 = \frac{1}{2}$

4. The same region is shown in C, this time separated into $\underline{?}$ smaller regions of the same size. Each of these regions is $\underline{?}$ of the larger region. The area of the large region is $\underline{?}$ square inch. Then the area of each of the four small regions is $\underline{?}$ square inch.

5. In C, what is the area of the region shaded red? What are the dimensions of this region? Then is this sentence true? $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

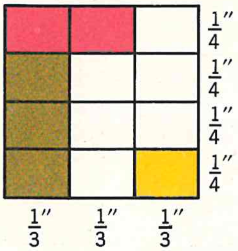
6. Look at the region shown in D. It has been separated into $\underline{?}$ smaller regions of the same size. Is each of these smaller regions $\frac{1}{8}$ of the larger region? Then the area of each smaller region is $\underline{?}$ square inch.

7. What are the dimensions of the region shaded yellow in D? Then is the following number sentence true? $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

8. Since the area of each smaller region shown in D is $\frac{1}{8}$ square inch, what is the area of 3 of these smaller regions?

9. What is the area of the region shaded brown in D? What are the dimensions of this region? Then what number makes this sentence true? $\frac{3}{4} \times \frac{1}{2} = n$

10. The dimensions of the square region shown below are 1″ by 1″. What is the area of this square region?



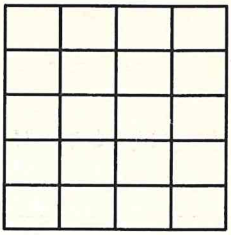
11. The square region is separated into $\underline{?}$ smaller rectangular regions of the same size. The area of each smaller region is what part of the area of the square region? Then is the area of each smaller region $\frac{1}{12}$ square inch?

12. What is the area of the region shaded yellow? What are its dimensions? Then what number makes the following number sentence true?
$\frac{1}{3} \times \frac{1}{4} = n$

13. If 1 smaller region has an area of $\frac{1}{12}$ square inch, what is the area of 3 of these smaller regions?

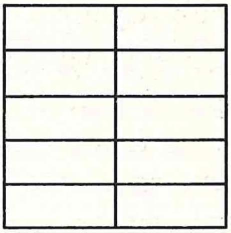
14. What are the dimensions of the region shaded brown? What is its area? Then is this sentence true?
$\frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$

15. What is the area of the region shaded red? What are its dimensions? Then what number makes the following number sentence true?
$\frac{2}{3} \times \frac{1}{4} = n$



— Use the square region shown above to help you solve the following number sentences.

	a	b
16.	$\frac{1}{4} \times \frac{1}{5} = n$	$\frac{1}{4} \times \frac{2}{5} = n$
17.	$\frac{3}{4} \times \frac{1}{5} = n$	$\frac{1}{4} \times \frac{3}{5} = n$
18.	$\frac{1}{4} \times \frac{4}{5} = n$	$\frac{3}{4} \times \frac{3}{5} = n$



— Use the square region shown above to help you solve these number sentences.

19. $\frac{1}{2} \times \frac{1}{5} = n$ $\frac{1}{2} \times \frac{2}{5} = n$

20. $\frac{4}{5} \times \frac{1}{2} = n$ $\frac{3}{5} \times \frac{1}{2} = n$

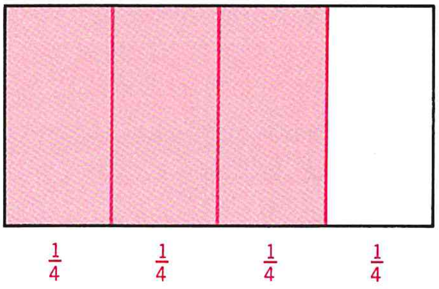
Think twice!

— Draw and separate a square region to help you solve these number sentences.

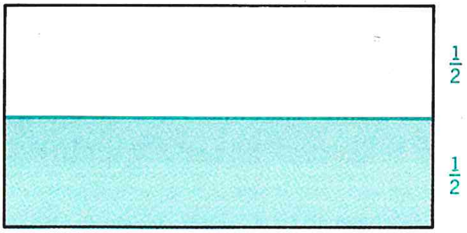
21. $\frac{1}{4} \times \frac{5}{8} = n$ $\frac{3}{4} \times \frac{3}{8} = n$

22. $\frac{7}{8} \times \frac{1}{4} = n$ $\frac{5}{8} \times \frac{3}{4} = n$

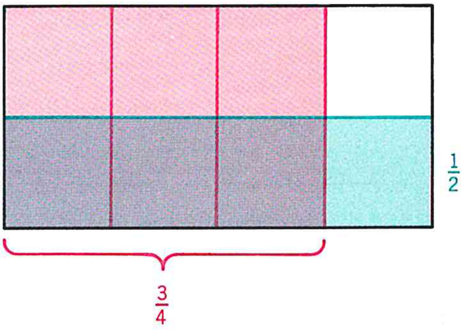
Fractional parts of a fractional number



1. The red line segments separate the rectangular region shown at the right into ? parts of the same size. Is the part of the region that is shaded red $\frac{3}{4}$ of the whole rectangular region?



2. The same rectangular region is shown in the second drawing at the right. This time the region has been separated by a blue line segment into ? parts of the same size. The part of the rectangular region that is shaded blue is ? of the whole rectangular region.

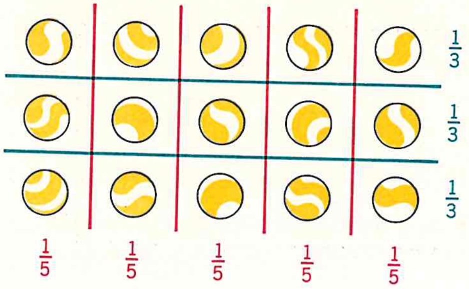


3. In the third drawing, the same rectangular region is shown once again. Look at the intersection of the part of the region that is shaded red with the part that is shaded blue. Is this intersection $\frac{1}{2}$ of $\frac{3}{4}$ of the whole rectangular region?

4. Look at the third drawing again. Together, the red line segments and the blue line segment separate the rectangular region into eight parts of the same size. Each of these parts is ? of the whole rectangular region. Then the part of the region that is purple is ? of the whole rectangular region.

5. In Ex. 3 you discovered that the part shaded purple is $\frac{1}{2}$ of $\frac{3}{4}$ of the whole rectangular region. In Ex. 4 you discovered that the part shaded purple is $\frac{3}{8}$ of the rectangular region. Then is this number sentence true? $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$

6. Look at this set of 15 marbles.



$\frac{1}{3}$
$\frac{1}{3}$
$\frac{1}{3}$
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$

- The red paths separate the set into 5 subsets with the same number of members. Is each of these subsets $\frac{1}{5}$ of the set of marbles?
- The blue paths separate the set into 3 subsets with the same number of members. Is each of these subsets $\frac{1}{3}$ of the set of marbles?
- Together the red and blue paths separate the set of marbles into $\underline{?}$ subsets with the same number of members. Each marble is what part of the set of 15 marbles?

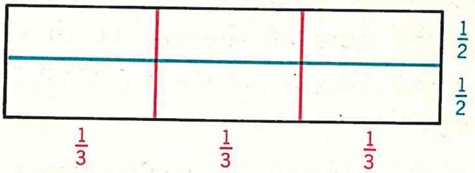
7. Using the drawing above, tell how many marbles are in the subset of $\frac{1}{3}$ of $\frac{1}{5}$ of the set of marbles. Is this sentence true? $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$

8. How many marbles are in $\frac{1}{3}$ of $\frac{4}{5}$ of the set of marbles? Then $\frac{1}{3}$ of $\frac{4}{5}$ equals $\frac{?}{15}$.

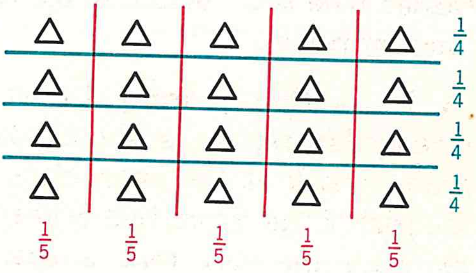
— Copy and complete these sentences, using the diagram in Ex. 6 to help you.

	a	**b**
9.	$\frac{2}{3}$ of $\frac{1}{5} = \underline{?}$	$\frac{2}{3}$ of $\frac{4}{5} = \underline{?}$
10.	$\frac{1}{3}$ of $\frac{2}{5} = \underline{?}$	$\frac{2}{3}$ of $\frac{2}{5} = \underline{?}$

— Use the drawings below to help you as you copy and complete the number sentences shown under each drawing.

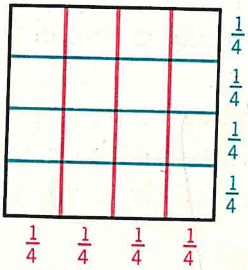


	a	**b**
11.	$\frac{1}{2}$ of $\frac{1}{3} = \underline{?}$	$\frac{1}{2}$ of $\frac{2}{3} = \underline{?}$
12.	$\frac{1}{3}$ of $\frac{1}{2} = \underline{?}$	$\frac{2}{3}$ of $\frac{1}{2} = \underline{?}$



$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$

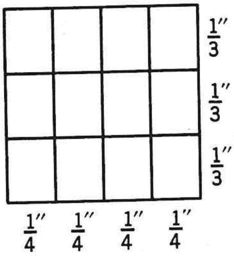
13.	$\frac{1}{4}$ of $\frac{1}{5} = \underline{?}$	$\frac{1}{4}$ of $\frac{3}{5} = \underline{?}$
14.	$\frac{3}{4}$ of $\frac{1}{5} = \underline{?}$	$\frac{3}{5}$ of $\frac{3}{4} = \underline{?}$



15.	$\frac{1}{4}$ of $\frac{1}{4} = \underline{?}$	$\frac{1}{4}$ of $\frac{3}{4} = \underline{?}$
16.	$\frac{3}{4}$ of $\frac{1}{4} = \underline{?}$	$\frac{2}{4}$ of $\frac{3}{4} = \underline{?}$

Finding a fractional part and multiplication

1. The region shown below has an area of 1 square inch. It is separated into $\underline{\ ?\ }$ regions of the same size.

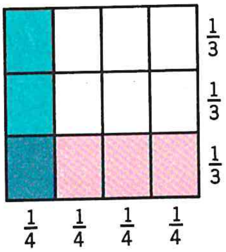


- Since each smaller region is $\frac{1}{12}$ of the square region, the area of each smaller region is $\underline{\ ?\ }$ square inch.
- Is this sentence true? $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ Explain.

2. Use the square region above to solve these sentences.

$\frac{3}{4} \times \frac{1}{3} = n \qquad \frac{1}{4} \times \frac{2}{3} = n$

3. The region below is separated into $\underline{\ ?\ }$ regions of the same size.



- Is the blue region $\frac{1}{4}$ of the whole rectangular region? What part of the rectangular region is red?
- The intersection of the blue region and the red region is what part of the whole region? Then $\frac{1}{4}$ of $\frac{1}{3}$ equals $\frac{?}{12}$. Explain.

4. Use the region shown in Ex. 3 to solve these sentences.

$\frac{3}{4} \text{ of } \frac{1}{3} = n \qquad \frac{1}{4} \text{ of } \frac{2}{3} = n$

5. Look at your answers for Ex. 2 and Ex. 4. Does $\frac{3}{4} \times \frac{1}{3}$ name the same number as $\frac{3}{4}$ of $\frac{1}{3}$? How are $\frac{1}{4} \times \frac{2}{3}$ and $\frac{1}{4}$ of $\frac{2}{3}$ related?

Your discovery

Tell your discovery

How is finding the fractional part of a number related to the multiplication of fractional numbers?

Use your discovery

- If $\frac{5}{8} \times \frac{7}{9}$ equals $\frac{35}{72}$, what number is $\frac{5}{8}$ of $\frac{7}{9}$?
- If $\frac{3}{5}$ of $\frac{4}{7}$ equals $\frac{12}{35}$, what number is $\frac{3}{5} \times \frac{4}{7}$?

- Solve each of these sentences. If you need help, use the drawings of regions shown on pages 292–96.

	a	b
6.	$\frac{1}{4} \times \frac{1}{2} = n$	$\frac{1}{4} \text{ of } \frac{1}{2} = n$
7.	$\frac{3}{4} \times \frac{1}{2} = n$	$\frac{3}{4} \text{ of } \frac{1}{2} = n$
8.	$\frac{1}{3} \times \frac{3}{4} = \frac{1}{3} \text{ of } n$	$\frac{2}{3} \text{ of } \frac{1}{4} = n \times \frac{1}{4}$
9.	$\frac{1}{4} \times n = \frac{1}{4} \text{ of } \frac{1}{5}$	$n \times \frac{4}{5} = \frac{3}{4} \text{ of } \frac{4}{5}$
10.	$\frac{3}{5} \times \frac{1}{2} = n$	$\frac{1}{2} \text{ of } \frac{3}{5} = n$

A pattern for multiplication

1. On the previous pages you discovered that all of the number sentences below are true.

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ $\qquad \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ $\qquad \frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$

Study these number sentences and then answer the following questions.

- In each sentence, how is the numerator of the product related to the numerators of the factors?
- In each sentence, how is the denominator of the product related to the denominators of the factors?

2. Shown below are four more true number sentences.

$\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$ $\qquad \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$ $\qquad \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

- In each of these sentences, how is the numerator of the product related to the numerators of the factors?
- In each of these sentences, how is the denominator of the product related to the denominators of the factors?

3. Are the following number sentences true?

- $\frac{1}{3} \times \frac{1}{4} = \frac{1 \times 1}{3 \times 4} = \frac{1}{12}$
- $\frac{3}{4} \times \frac{3}{5} = \frac{3 \times 3}{4 \times 5} = \frac{9}{20}$

Do you see a pattern you can use to find the product of two fractional numbers?

4. Copy and solve these sentences.

- $\frac{4}{5} \times \frac{2}{3} = \frac{a \times 2}{b \times 3} = \frac{8}{15}$
- $\frac{3}{4} \times \frac{5}{8} = \frac{3 \times n}{4 \times m} = \frac{15}{32}$
- $\frac{5}{6} \times \frac{7}{8} = \frac{5 \times c}{d \times 8} = \frac{35}{48}$

Your discovery

Tell your discovery

In the multiplication of fractional numbers, how are the numerator and denominator of the product related to the numerators and denominators of the factors?

Use your discovery

- Find the product of $\frac{1}{7}$ and $\frac{2}{3}$.
- Find the product of $\frac{3}{5}$ and $\frac{4}{7}$.

5. If $\frac{a}{b}$ and $\frac{c}{d}$ represent any two fractional numbers, how does the following sentence show you the pattern for finding the product of any two fractional numbers?

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

— Use the pattern you have discovered to solve these sentences.

	a	b
6.	$\frac{1}{5} \times \frac{3}{8} = n$	$\frac{1}{4} \times \frac{2}{3} = n$
7.	$\frac{3}{8} \times \frac{5}{7} = n$	$\frac{1}{9} \times \frac{1}{5} = n$
8.	$\frac{4}{5} \times \frac{3}{5} = n$	$\frac{9}{10} \times \frac{7}{8} = n$

Using the pattern for multiplying fractional numbers

1. Is the number sentence shown below true?

$$\frac{3}{4} \times \frac{5}{9} = \frac{3 \times 5}{4 \times 9} = \frac{15}{36}$$

Look at the numerator and denominator of the product. Do these terms have any common factor other than 1? Then is $\frac{15}{36}$ in lowest terms? Name the product of $\frac{3}{4}$ and $\frac{5}{9}$ with a fraction in lowest terms.

— Solve each of the following number sentences. Name each answer with a fraction in lowest terms.

	a	**b**
2.	$\frac{1}{8} \times \frac{2}{3} = n$	$\frac{3}{4} \times \frac{1}{7} = n$
3.	$\frac{5}{6} \times \frac{3}{4} = n$	$\frac{1}{9} \times \frac{6}{7} = n$
4.	$\frac{1}{2} \times \frac{3}{5} = n$	$\frac{1}{7} \times \frac{3}{4} = n$
5.	$\frac{2}{3} \times \frac{3}{5} = n$	$\frac{5}{9} \times \frac{3}{10} = n$
6.	$\frac{7}{10} \times \frac{3}{5} = n$	$\frac{3}{8} \times \frac{1}{6} = n$

7. To find the product of $\frac{3}{5}$ and $\frac{7}{2}$, Bob and Joan both used the pattern you have discovered. But Bob said the product is $\frac{21}{10}$ and Joan said the product is $2\frac{1}{10}$.

- Is this number sentence true?

$$\frac{3}{5} \times \frac{7}{2} = \frac{3 \times 7}{5 \times 2} = \frac{21}{10}$$

Then is Bob's answer correct?

- Is this number sentence true?

$$\frac{21}{10} = \frac{20}{10} + \frac{1}{10} = 2 + \frac{1}{10} = 2\frac{1}{10}$$

Is Joan's answer correct also?

8. Bob said the product of $\frac{5}{4}$ and $\frac{3}{2}$ is $\frac{15}{8}$. Joan said it is $1\frac{7}{8}$. Explain why both answers are correct.

— Solve each of these sentences. Express each answer in two ways.

	a	**b**
9.	$\frac{3}{7} \times \frac{9}{2} = n$	$\frac{6}{5} \times \frac{7}{5} = n$
10.	$\frac{1}{2} \times \frac{11}{3} = n$	$\frac{4}{7} \times \frac{9}{5} = n$
11.	$\frac{7}{2} \times \frac{3}{5} = n$	$\frac{9}{10} \times \frac{3}{2} = n$
12.	$\frac{3}{2} \times \frac{5}{4} = n$	$\frac{7}{8} \times \frac{5}{4} = n$

— Solve these sentences. You may name products greater than 1 using a mixed form if you wish, but express all answers in lowest terms.

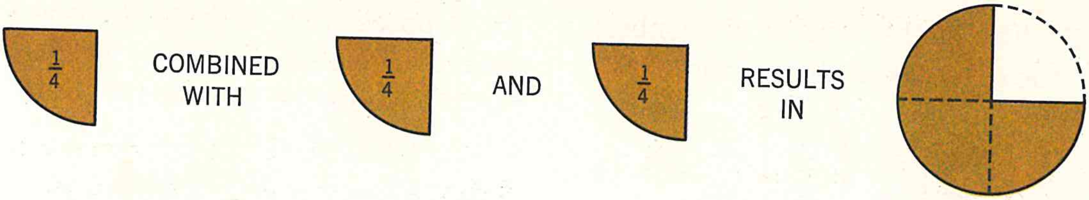
13.	$\frac{5}{8} \times \frac{2}{3} = n$	$\frac{4}{5} \times \frac{7}{8} = n$
14.	$\frac{9}{2} \times \frac{3}{4} = n$	$\frac{3}{8} \times \frac{7}{10} = n$
15.	$\frac{15}{8} \times \frac{2}{3} = n$	$\frac{9}{4} \times \frac{6}{7} = n$
16.	$\frac{3}{5} \times \frac{3}{5} = n$	$\frac{5}{3} \times \frac{4}{5} = n$
17.	$\frac{3}{2} \times \frac{1}{6} = n$	$\frac{2}{3} \times \frac{3}{8} = n$

Think twice!

18. Mike is making a hot-plate holder for his mother by gluing $\frac{1}{4}$-inch square pieces of tile on a rectangular-shaped piece of wood. If the piece of wood is $6\frac{1}{2}$ inches long and $4\frac{1}{2}$ inches wide, how many pieces of tile does Mike need?

If you need more practice, do Practice Set 32, page 334.

Multiplying a fractional number by a whole number



1. The drawing above shows that when you combine three equal parts that are each $\frac{1}{4}$ of a whole, the result is $\underline{\ ?\ }$ of a whole.

What number is $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$? Is $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ the same as three $\frac{1}{4}$'s, or $3 \times \frac{1}{4}$? Then what number makes this sentence true?

$$3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = n$$

2. Is this a true sentence? $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{1+1+1}{4}$ What operation other than addition can be used to find the sum of the 3 ones in the numerator of $\frac{1+1+1}{4}$? Then is this number sentence true? $3 \times \frac{1}{4} = \frac{3 \times 1}{4} = \frac{3}{4}$

3. Is $\frac{3}{1}$ another way to express the number 3? Then is this number sentence true? $3 \times \frac{1}{4} = \frac{3}{1} \times \frac{1}{4}$ Using the pattern for multiplying fractional numbers, what is the product for $\frac{3}{1} \times \frac{1}{4}$?

4. Suppose you wish to solve this number sentence. $4 \times \frac{2}{3} = n$

- Does $4 \times \frac{2}{3}$ name the same number as $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$? What is the sum for $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$? Then what does $4 \times \frac{2}{3}$ equal?
- Does $4 \times \frac{2}{3}$ name the same number as $\frac{4}{1} \times \frac{2}{3}$? What number is named by $\frac{4}{1} \times \frac{2}{3}$? Then $4 \times \frac{2}{3}$ equals $\underline{\ ?\ }$.

5. Does $2 \times \frac{5}{7}$ name the same number as $\frac{2}{1} \times \frac{5}{7}$? What number is named by $\frac{2}{1} \times \frac{5}{7}$? Then $2 \times \frac{5}{7}$ equals $\underline{\ ?\ }$.

What number makes this sentence true? $2 \times \frac{5}{7} = \frac{2 \times 5}{7} = \frac{n}{7}$

6. What numbers make this sentence true? $3 \times \frac{4}{5} = \frac{a \times 4}{5} = \frac{b}{5}$

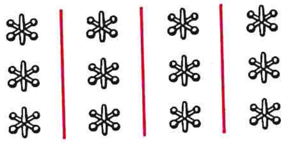
— Solve these sentences. You may express your answers in mixed form if you wish, but be sure all answers are in lowest terms.

	a	b
7.	$2 \times \frac{1}{8} = n$	$3 \times \frac{2}{7} = n$
8.	$4 \times \frac{2}{3} = n$	$6 \times \frac{3}{4} = n$
9.	$5 \times \frac{1}{2} = n$	$2 \times \frac{3}{5} = n$
10.	$7 \times \frac{3}{8} = n$	$3 \times \frac{4}{5} = n$

If you need more practice, do Practice Set 33, page 334.

Multiplying a whole number by a fractional number

1. What is the number of members in the set of jacks shown below?



- Red line segments are shown separating the set of jacks into ? subsets of the same number.
- What part of the whole set is each of these subsets? How many members are in each subset? Then is this sentence true? $\frac{1}{4}$ of $12 = 3$
- What part of the whole set is 3 of these subsets? How many members are in 3 subsets? Then is this sentence true? $\frac{3}{4}$ of $12 = 9$

2. You have discovered that another way to express $\frac{1}{4}$ of 12 and $\frac{3}{4}$ of 12 is $\frac{1}{4} \times 12$ and $\frac{3}{4} \times 12$. What numbers make these sentences true?

$\frac{1}{4} \times 12 = n$ $\qquad$ $\frac{3}{4} \times 12 = n$

3. Is $\frac{12}{1}$ another way to express the number 12? Are the sentences shown below true?

- $\frac{1}{4} \times \frac{12}{1} = \frac{1 \times 12}{4 \times 1} = \frac{12}{4} = 3$
- $\frac{3}{4} \times \frac{12}{1} = \frac{3 \times 12}{4 \times 1} = \frac{36}{4} = 9$

4. Does the pattern for multiplying fractional numbers work when one of the factors is a whole number? Is every whole number also a fractional number?

5. Suppose you wish to find the product of $\frac{3}{5}$ and 17. Is $\frac{17}{1}$ another way to express the number 17? Then is the following sentence true?

$$\frac{3}{5} \times 17 = \frac{3}{5} \times \frac{17}{1} = \frac{3 \times 17}{5 \times 1} = \frac{51}{5}$$

How would you name $\frac{51}{5}$ using a mixed form? The product of $\frac{3}{5}$ and 17 can be named by $\frac{51}{5}$ or by ?.

6. Copy and complete, naming the product in two ways.

$$\frac{2}{3} \times 19 = \frac{2}{3} \times \frac{19}{1} = \frac{2 \times ?}{3 \times ?} = ?$$

— Name each whole-number factor shown below as a fraction with a denominator of 1. Then find the product.

	a	b
7.	$\frac{3}{5} \times 45 = n$	$\frac{5}{8} \times 56 = n$
8.	$\frac{2}{3} \times 25 = n$	$\frac{3}{7} \times 20 = n$

9. What can you tell about a product when one of its two factors is 1? Then is the second sentence below a shorter way to find the product of $\frac{3}{5}$ and 45?

$$\frac{3}{5} \times 45 = \frac{3}{5} \times \frac{45}{1} = \frac{3 \times 45}{5 \times 1} = \frac{135}{5} = 27$$

$$\frac{3}{5} \times 45 = \frac{3 \times 45}{5} = \frac{135}{5} = 27$$

— Use the shorter form shown in Ex. 9 to solve each sentence.

10. $\frac{2}{3} \times 15 = n$ $\qquad$ $\frac{3}{4} \times 2 = n$

11. $\frac{4}{5} \times 21 = n$ $\qquad$ $\frac{3}{8} \times 16 = n$

If you need more practice, do Practice Set 34, page 334.

Properties of multiplication of fractional numbers

— Solve these number sentences.

a **b**

1. $\frac{3}{5} \times \frac{1}{2} = n$ $\qquad \frac{1}{2} \times \frac{3}{5} = n$
2. $\frac{2}{3} \times \frac{4}{7} = n$ $\qquad \frac{4}{7} \times \frac{2}{3} = n$
3. $5 \times \frac{3}{4} = n$ $\qquad \frac{3}{4} \times 5 = n$

4. In each exercise shown above, did you find the same product for sentence **b** as you did for sentence **a**? Then does the commutative property apply to the multiplication of fractional numbers?

— For each sentence below, tell the fractional number that makes the sentence true.

5. $\frac{2}{3} \times \frac{1}{7} = n \times \frac{2}{3}$
6. $\frac{1}{9} \times \frac{4}{5} = \frac{4}{5} \times n$
7. $\frac{3}{8} \times 5 = 5 \times n$
8. $27 \times \frac{2}{3} = n \times 27$

9. What do the parentheses mean in the sentence shown below?

$$(\tfrac{1}{2} \times \tfrac{3}{4}) \times \tfrac{3}{5} = n$$

What is the product of $\frac{1}{2}$ and $\frac{3}{4}$? What is the product of this number and $\frac{3}{5}$? Then what is the solution set for the sentence shown above?

10. Now look at this sentence.

$$\tfrac{1}{2} \times (\tfrac{3}{4} \times \tfrac{3}{5}) = n$$

What is the product of $\frac{3}{4}$ and $\frac{3}{5}$? What is the product of $\frac{1}{2}$ and this number? Then what is the solution set for the sentence shown above?

11. Compare the sentences shown in Ex. 9 and 10. How are they alike? How do they differ?

Compare the solution sets for these two sentences. What do you discover?

12. Solve the two sentences shown below and compare the solution sets.

$(\frac{2}{3} \times \frac{1}{5}) \times \frac{4}{7} = n \qquad \frac{2}{3} \times (\frac{1}{5} \times \frac{4}{7}) = n$

Is the following sentence true?

$$(\tfrac{2}{3} \times \tfrac{1}{5}) \times \tfrac{4}{7} = \tfrac{2}{3} \times (\tfrac{1}{5} \times \tfrac{4}{7})$$

— Solve the following sentences and compare the solution set for **b** with the solution set for **a**.

a **b**

13. $(\frac{1}{7} \times \frac{3}{4}) \times \frac{1}{2} = n$ $\qquad \frac{1}{7} \times (\frac{3}{4} \times \frac{1}{2}) = n$
14. $(\frac{2}{3} \times \frac{4}{5}) \times \frac{7}{9} = n$ $\qquad \frac{2}{3} \times (\frac{4}{5} \times \frac{7}{9}) = n$

15. Does the associative property apply to the multiplication of fractional numbers?

16. Look at the following sentence.

$$\tfrac{1}{7} \times \tfrac{3}{4} \times \tfrac{1}{2} = \frac{1 \times 3 \times 1}{7 \times 4 \times 2}$$

Is the number named at the left of the equals sign the same as the number named at the right? Use this discovery to tell another way to find the product of three or more fractional numbers.

— Solve the following sentences.

17. $\frac{2}{3} \times \frac{5}{8} \times \frac{1}{2} = n$ $\qquad \frac{3}{5} \times \frac{4}{7} \times \frac{1}{2} = n$
18. $\frac{1}{8} \times \frac{3}{4} \times \frac{2}{3} = n$ $\qquad \frac{4}{7} \times \frac{2}{3} \times \frac{1}{5} = n$

The distributive property and fractional numbers

— Copy and complete the following number sentences.

1. $$\begin{aligned} \tfrac{1}{7} \times (\tfrac{3}{4} + \tfrac{1}{2}) &= \tfrac{1}{7} \times (\tfrac{3}{4} + \tfrac{?}{4}) \\ &= \tfrac{1}{7} \times \quad \tfrac{?}{4} \\ &= \underline{\ ?\ } \end{aligned}$$

2. $$\begin{aligned} (\tfrac{1}{7} \times \tfrac{3}{4}) + (\tfrac{1}{7} \times \tfrac{1}{2}) &= \tfrac{?}{28} + \tfrac{1}{14} \\ &= \tfrac{?}{28} + \tfrac{?}{28} \\ &= \underline{\ ?\ } \end{aligned}$$

3. Did you get the same answer for Ex. 1 and 2? Then is this number sentence true?

$$\tfrac{1}{7} \times (\tfrac{3}{4} + \tfrac{1}{2}) = (\tfrac{1}{7} \times \tfrac{3}{4}) + (\tfrac{1}{7} \times \tfrac{1}{2})$$

4. Is this number sentence true?

$$\tfrac{3}{4} \times (\tfrac{1}{5} + \tfrac{7}{8}) = (\tfrac{3}{4} \times \tfrac{1}{5}) + (\tfrac{3}{4} \times \tfrac{7}{8})$$

5. What fractional number makes this sentence true?

$$\tfrac{1}{3} \times (\tfrac{2}{5} + \tfrac{4}{9}) = (\tfrac{1}{3} \times \tfrac{2}{5}) + (\tfrac{1}{3} \times n)$$

What fractional number makes this sentence true?

$$\tfrac{5}{7} \times (\tfrac{1}{2} + \tfrac{2}{3}) = (n \times \tfrac{1}{2}) + (n \times \tfrac{2}{3})$$

6. Does the distributive property apply to fractional numbers? Explain.

7. Is $4 + \frac{3}{5}$ another name for $4\frac{3}{5}$? Copy and complete the following sentences to find the product of $\frac{1}{2}$ and $4\frac{3}{5}$.

$$\begin{aligned} \tfrac{1}{2} \times 4\tfrac{3}{5} &= \tfrac{1}{2} \times (4 + \tfrac{3}{5}) \\ &= (\tfrac{1}{2} \times 4) + (\tfrac{1}{2} \times \tfrac{3}{5}) \\ &= \quad \underline{\ ?\ } \quad + \quad \tfrac{3}{10} \\ &= \underline{\ ?\ } \end{aligned}$$

— Copy and complete the following number sentences.

8. $$\begin{aligned} \tfrac{2}{3} \times 9\tfrac{1}{4} &= \tfrac{2}{3} \times (9 + \tfrac{1}{4}) \\ &= (\tfrac{2}{3} \times \underline{\ ?\ }) + (\tfrac{2}{3} \times \tfrac{1}{4}) \\ &= \quad 6 \quad + \quad \underline{\ ?\ } \\ &= \underline{\ ?\ } \end{aligned}$$

9. $$\begin{aligned} \tfrac{1}{5} \times 3\tfrac{2}{7} &= \tfrac{1}{5} \times (3 + \tfrac{2}{7}) \\ &= (\underline{\ ?\ } \times 3) + (\underline{\ ?\ } \times \tfrac{2}{7}) \\ &= \quad \tfrac{3}{5} \quad + \quad \tfrac{2}{35} \\ &= \underline{\ ?\ } \end{aligned}$$

10. Explain how you can use the distributive property to find the product of $\frac{1}{2}$ and $6\frac{3}{4}$. What is the product?

11. How can you use the distributive property to find the product of 4 and $2\frac{1}{2}$? What is the product?

— Solve each of these sentences.

	a	b
12.	$\frac{2}{5} \times 10\frac{1}{2} = n$	$\frac{1}{4} \times 3\frac{1}{2} = n$
13.	$\frac{1}{3} \times 8\frac{3}{4} = n$	$\frac{3}{8} \times 2\frac{3}{4} = n$
14.	$5 \times 4\frac{1}{5} = n$	$8 \times 2\frac{3}{4} = n$

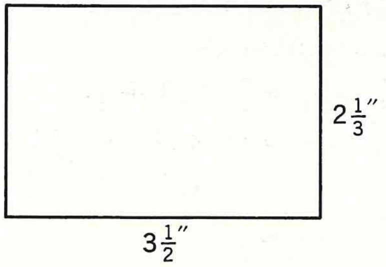
15. Is $\frac{7}{3}$ another name for $2\frac{1}{3}$? Copy and complete these sentences.

$$\begin{aligned} \tfrac{2}{5} \times 2\tfrac{1}{3} &= \tfrac{2}{5} \times \tfrac{7}{3} \\ &= \tfrac{2 \times 7}{5 \times 3} \\ &= \tfrac{?}{15} \end{aligned}$$

— Use the way shown in Ex. 15 to solve each of these sentences.

16.	$\frac{1}{2} \times 1\frac{2}{3} = n$	$\frac{1}{8} \times 2\frac{1}{4} = n$
17.	$\frac{2}{5} \times 2\frac{1}{7} = n$	$\frac{3}{7} \times 1\frac{1}{3} = n$

Multiplying fractional numbers in mixed form



1. To find the area in square inches of the rectangular region shown above, you find the product of $3\frac{1}{2}$ and $2\frac{1}{3}$.

One way to find the product is to use the distributive property. Is the following sentence true?

$$3\frac{1}{2} \times 2\frac{1}{3} = (3\frac{1}{2} \times 2) + (3\frac{1}{2} \times \frac{1}{3})$$

- To find what number is $3\frac{1}{2} \times 2$, first multiply 3 and 2. $3 \times 2 = \underline{?}$ Then multiply $\frac{1}{2}$ and 2. $\frac{1}{2} \times 2 = \underline{?}$ What is the sum of these products? $6 + 1 = \underline{?}$
- Next find what $3\frac{1}{2} \times \frac{1}{3}$ equals. First multiply 3 and $\frac{1}{3}$. $3 \times \frac{1}{3} = \underline{?}$ Then multiply $\frac{1}{2}$ and $\frac{1}{3}$. $\frac{1}{2} \times \frac{1}{3} = \underline{?}$ What is the sum of these products? $1 + \frac{1}{6} = \underline{?}$
- Finally, add the product of $3\frac{1}{2}$ and 2 and the product of $3\frac{1}{2}$ and $\frac{1}{3}$. $7 + 1\frac{1}{6} = \underline{?}$ In square inches, the area of the rectangular region shown above is $\underline{?}$.

2. In the same way, see if you can find the area of a rectangular region that is $2\frac{1}{4}$ inches long and $2\frac{1}{2}$ inches wide.

3. Another way to find the area of the rectangular region shown in Ex. 1 is to rename the length and width.

- Is $\frac{7}{2}$ another name for $3\frac{1}{2}$? Another name for $2\frac{1}{3}$ is $\frac{6}{3} + \frac{1}{3}$, or $\frac{?}{3}$. Find the product of these two numbers. $\frac{7}{2} \times \frac{7}{3} = \underline{?}$
- What is the greatest whole number of 6's in 49? Then another name for $\frac{49}{6}$ is $8 + \frac{1}{6}$, or $\underline{?}$. Is this the same answer you found in Ex. 1?

4. Copy and complete the following sentences to help you find the area of the rectangular region in Ex. 2.

$$\begin{aligned} 2\frac{1}{4} \times 2\frac{1}{2} &= \frac{9}{4} \times \frac{5}{2} \\ &= \frac{9 \times 5}{4 \times 2} \\ &= \frac{?}{8}, \text{ or } \underline{?} \end{aligned}$$

5. Which way of finding the product when both factors are expressed in mixed form is easier for you: using the distributive property or renaming the factors as fractions?

— Use the way you prefer to solve each of the following sentences.

	a	b
6.	$3\frac{1}{2} \times 1\frac{1}{2} = n$	$2\frac{1}{4} \times 3\frac{1}{4} = n$
7.	$4 \times 5\frac{3}{8} = n$	$6 \times 2\frac{3}{4} = n$
8.	$1\frac{1}{2} \times 2\frac{1}{4} = n$	$1\frac{3}{4} \times 4\frac{1}{4} = n$
9.	$3\frac{2}{3} \times 1\frac{1}{2} = n$	$4\frac{1}{3} \times 2\frac{1}{5} = n$

If you need more practice, do Practice Set 35, page 335.

Practice in multiplying fractional numbers

— Solve these number sentences and express your answers in lowest terms.

	a	b
1.	$\frac{1}{8} \times \frac{5}{7} = n$	$\frac{2}{5} \times \frac{4}{10} = n$
2.	$\frac{2}{3} \times \frac{8}{9} = n$	$\frac{2}{7} \times \frac{1}{5} = n$
3.	$\frac{7}{8} \times \frac{4}{5} = n$	$\frac{5}{6} \times \frac{7}{9} = n$
4.	$\frac{9}{10} \times \frac{2}{3} = n$	$\frac{2}{3} \times \frac{5}{8} = n$
5.	$\frac{1}{3} \times \frac{3}{10} = n$	$\frac{3}{5} \times \frac{4}{9} = n$
6.	$\frac{5}{8} \times \frac{4}{10} = n$	$\frac{9}{16} \times \frac{2}{3} = n$

— Find the solution set for each of these sentences. You may show your answers in mixed form if you wish, but be sure all answers are expressed in lowest terms.

7.	$3 \times \frac{4}{5} = n$	$8 \times \frac{5}{6} = n$
8.	$7 \times \frac{3}{8} = n$	$12 \times \frac{5}{7} = n$
9.	$\frac{4}{7} \times 10 = n$	$\frac{5}{9} \times 4 = n$
10.	$\frac{2}{3} \times 16 = n$	$\frac{3}{7} \times 5 = n$
11.	$4 \times \frac{5}{8} = n$	$12 \times \frac{1}{6} = n$
12.	$\frac{3}{5} \times 10 = n$	$\frac{4}{7} \times 14 = n$
13.	$5 \times 1\frac{7}{8} = n$	$7 \times 2\frac{3}{4} = n$
14.	$6 \times 3\frac{2}{5} = n$	$8 \times 12\frac{1}{2} = n$
15.	$16\frac{2}{3} \times 6 = n$	$3\frac{3}{4} \times 9 = n$
16.	$3\frac{1}{2} \times 6 = n$	$3\frac{7}{8} \times 6 = n$
17.	$2 \times 13\frac{1}{3} = n$	$4 \times 24\frac{1}{6} = n$
18.	$18\frac{1}{4} \times 6 = n$	$16\frac{2}{3} \times 5 = n$

19. Miss Park's class is going to make paper streamers to decorate the classroom for a party. Each streamer will be $5\frac{3}{4}$ yards long. How many yards of paper ribbon will be needed to make 6 streamers?

20. How many yards of ribbon will be needed to make 3 streamers if each streamer is $5\frac{3}{4}$ yards long?

21. Harvey needs $1\frac{2}{3}$ yards of wire screening to cover each window in his clubhouse. If the clubhouse has 4 windows, how many yards of screening does he need?

22. What is the area of a rectangular region that is 6 inches long and $4\frac{2}{3}$ inches wide?

23. If frozen dog food costs 40¢ a pound, how much will $6\frac{3}{8}$ pounds cost?

24. A jet airliner is traveling at an average speed of 480 miles per hour. At this rate, how far will the airliner travel in $4\frac{4}{5}$ hours?

25. How far will an airliner travel in $4\frac{4}{5}$ hours if its average speed is 520 miles per hour?

BOEING

Keeping up in mathematics

Oral

1. Which is greater, $\frac{5}{8}$ or $\frac{3}{4}$? Explain how you can tell without drawing a diagram or using a ruler.

2. Tell which of the following fractions are equivalent.

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{4}{8}$ $\frac{5}{10}$ $\frac{2}{4}$

3. Which of the fractions in Ex. 2 are not in lowest terms?

4. If 3,687 is increased by 400, what will the result be?

5. Read this numeral. 3,402.64

6. Which of the following is nearest to $\frac{1}{2}$?

0.40 0.90 0.05 0.52 0.16

— Tell the numbers that make these sentences true.

	a	b
7.	$10 - \frac{1}{4} = n$	$3 - \frac{3}{8} = n$
8.	$6 - \frac{1}{10} = n$	$5 - \frac{9}{10} = n$
9.	$\frac{1}{2} + \frac{1}{4} = n$	$\frac{1}{2} - \frac{1}{4} = n$
10.	$10 \times 88 = n$	$1{,}000 \times 88 = n$

11. Suppose sheets of cardboard are 0.1 inch thick. How many sheets are there in a pile 1.0 inch thick?

— Estimate to find which number in the set is the quotient.

12. $24\overline{)12{,}048}$ $\{52, 502, 520\}$

13. $98\overline{)5{,}096}$ $\{52, 62, 72\}$

14. $8\overline{)3{,}936}$ $\{392, 492, 592\}$

Written

— Copy, find the answers, and check.

	a	b	c
1.	$\begin{array}{r}70{,}509\\ -\ 10{,}860\\ \hline\end{array}$	$\begin{array}{r}\$893.97\\ -\ 209.88\\ \hline\end{array}$	$\begin{array}{r}70{,}098\\ +\ 1{,}567\\ \hline\end{array}$
2.	$\begin{array}{r}869\\ \times\ 407\\ \hline\end{array}$	$\begin{array}{r}908\\ \times\ 950\\ \hline\end{array}$	$\begin{array}{r}8{,}375\\ \times\ 684\\ \hline\end{array}$

3. Divide 11,578 by 96. Divide 24,352 by 48.

— Arrange in order beginning with the name for the least number.

4. $\{2\frac{1}{2}, \frac{9}{4}, \frac{21}{8}, 2, 2\frac{3}{4}\}$

5. {0.50, 0.05, 0.15, 1.50, 0.55}

6. A refreshment committee spent 75¢ for candy, 40¢ for a cake, and $1.05 for ice cream. If 6 members of the committee wish to share the cost equally, how much should each pay?

7. At 60¢ a dozen, how much will 18 ears of corn cost?

8. A piece of ribbon is $15\frac{1}{6}$ yards long. How much will be left if $3\frac{1}{4}$ yards of ribbon are cut off?

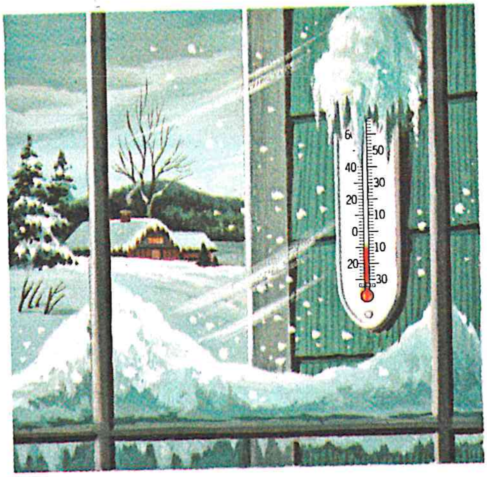
9. Find the perimeter of a triangle whose sides are $2\frac{3}{4}$ inches long, $2\frac{5}{8}$ inches long, and $3\frac{1}{2}$ inches long.

10. How far will a missile travel in 24 hours at an average rate of 5,780 miles an hour?

Inquiry into number

Numbers shown on a thermometer

Some numbers are shown twice on each of the thermometers pictured below. Do you know why? Can you tell what each means? Can you tell what the new temperature will be if during some night the temperature falls 18° from 6° above zero? if during some day the temperature rises 24° from 8° below zero?



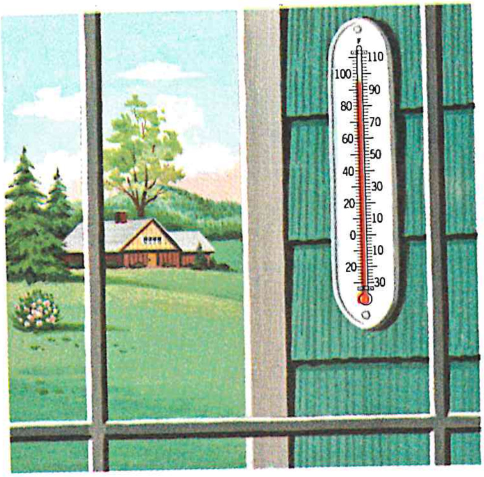
▶ Find two 10's on the thermometer in this picture. Find two 20's. Find two 30's. Which 10 represents the colder temperature? Which 20? Which 30?

What point on the thermometer is exactly between the two 10's? the two 20's? 30's?

To distinguish between the two 10's, 20's, or 30's, we say that one is *above zero* and the other is *below zero*. Which indicates colder weather, temperatures above zero or temperatures below zero?

What is the difference in degrees between 10° above zero and 10° below zero? between 20° above zero and 20° below zero? between 30° above and 30° below? between 6° above and 6° below?

During any given period, the temperature may rise or fall. If it rises, does this mean the weather is getting colder or warmer? What does it mean if the temperature falls?



▶ Shown below are two thermometers similar to the one pictured outside the window on the opposite page. What temperature is shown on thermometer A? on thermometer B?

Would you count up or down on these thermometers to find where the top of the liquid would be if the temperature rises some number of degrees? In which direction would you count if the temperature falls some number of degrees?

Look at thermometer A. Suppose the temperature rises 18°. At what point will the top of the liquid be then? What temperature is represented by this point on the thermometer?

Looking at thermometer A again, suppose the temperature falls 18°. At what point will the top of the liquid be then? What temperature is represented by this point?

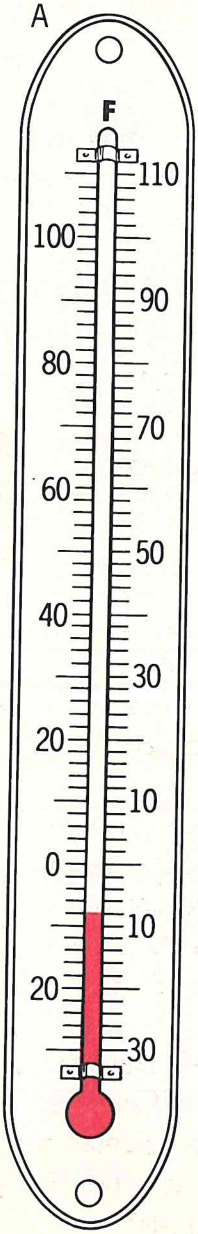
Look at thermometer B. Suppose the temperature rises 24°. At what point will the top of the liquid be then? What temperature is represented by this point?

At what point will the top of the liquid be in thermometer B if the temperature falls 18°? What temperature is represented by this point?

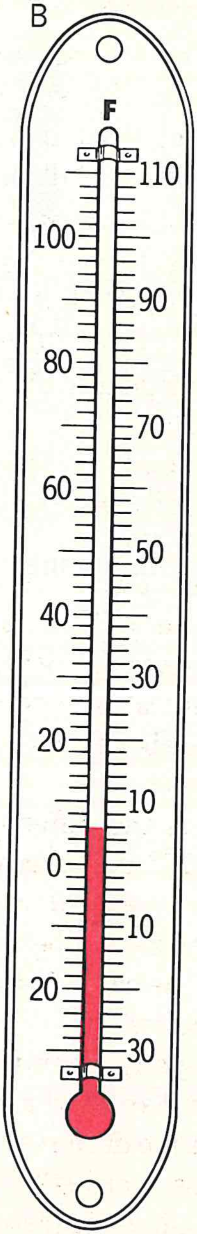
▶ Copy this chart and complete it. Use one of these thermometers if you need help getting started.

ORIGINAL TEMPERATURE	CHANGE	NEW TEMPERATURE
20° above 0	42° rise	?
20° above 0	42° fall	?
20° below 0	42° rise	?
20° below 0	42° fall	?

A

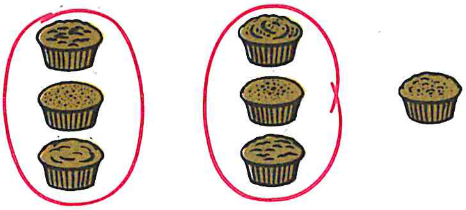


B



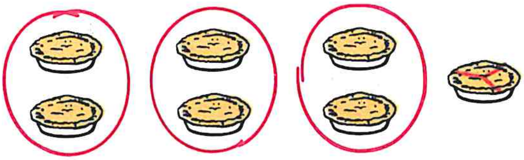
Dividing the remainder

1. Suppose 7 cupcakes are to be shared equally by 2 boys. Each boy will get $\underline{\ ?\ }$ cakes, with $\underline{\ ?\ }$ cake left over. Is this number sentence true? $(3 \times 2) + 1 = 7$



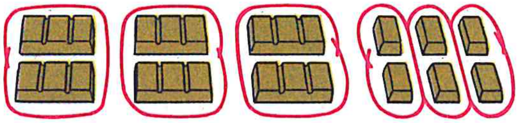
2. The rings drawn around the cupcakes show how many each boy will receive. Suppose one of the boys suggests cutting the cupcake left over into 2 equal parts. Each part will be $\underline{\ ?\ }$ of a cupcake. Then each boy will receive 3 plus $\frac{1}{2}$ or $\underline{\ ?\ }$ cupcakes. Is this number sentence true? $7 \div 2 = \frac{7}{2}$, or $3\frac{1}{2}$

3. Seven small pies are to be shared equally by 3 girls. How many pies will each girl receive?



Is $7 \div 3$ equal to 2 with a remainder of 1? Suppose the pie left over is separated into 3 equal parts. Then each girl will receive 2 plus $\frac{1}{3}$ or $\underline{\ ?\ }$ pies. Is this number sentence true? $7 \div 3 = \frac{7}{3}$, or $2\frac{1}{3}$

4. Look at the picture below to see how 8 candy bars can be shared equally by 3 boys.



- First, each boy receives $\underline{\ ?\ }$ whole candy bars. There are 2 candy bars left over to share.
- The 2 remaining candy bars are separated into thirds with each boy receiving $\underline{\ ?\ }$ thirds of a candy bar. $2 \div 3 = \frac{2}{3}$
- Is this number sentence true? $8 \div 3 = \frac{8}{3}$, or $2\frac{2}{3}$ Each boy should receive $\underline{\ ?\ }$ candy bars.

5. If we say the quotient for $8 \div 3$ is $2\frac{2}{3}$, then $2\frac{2}{3} \times 3$ must equal 8. Is the following number sentence true?

$$2\tfrac{2}{3} \times 3 = \tfrac{8}{3} \times 3 = \tfrac{24}{3}, \text{ or } 8$$

6. Look at this division sentence.

$$15 \div 4 = \tfrac{15}{4}, \text{ or } 3\tfrac{3}{4}$$

If $3\frac{3}{4}$ is the quotient for $15 \div 4$, then $3\frac{3}{4} \times 4$ must equal 15. Copy and complete this sentence.

$$3\tfrac{3}{4} \times 4 = \tfrac{15}{4} \times 4 = \tfrac{?}{4}, \text{ or } \underline{\ ?\ }$$

— Name each quotient using a mixed form. Check your answers by multiplying.

	a	b
7.	$12 \div 5 = n$	$5 \div 4 = n$
8.	$14 \div 3 = n$	$25 \div 2 = n$

9. Suppose you wish to find the quotient for 17 ÷ 5. The box at the right shows how you might write an example for this exercise. Explain the work as far as it is shown. What is the remainder?

$$\boxed{\begin{array}{r} 3 \\ 5\overline{)17} \\ \underline{15} \\ 2 \end{array}}$$

Rather than writing the remainder in the answer, continue to divide; that is, divide the remainder by 5.

$2 \div 5 = \underline{\ ?\ }$

$$\boxed{\begin{array}{r} 3\frac{2}{5} \\ 5\overline{)17} \\ \underline{15} \\ 2 \end{array}}$$

The fraction for your answer is shown as part of the quotient. Is this sentence true? $3\frac{2}{5} \times 5 = 17$ Then the quotient for 17 ÷ 5 is $\underline{\ ?\ }$.

— Explain each of the division examples shown below. Then multiply to see if the quotients shown in the book are correct.

	a	b	c
10.	$\begin{array}{r} 2\frac{1}{3} \\ 3\overline{)7} \\ \underline{6} \\ 1 \end{array}$	$\begin{array}{r} 4\frac{1}{2} \\ 4\overline{)18} \\ \underline{16} \\ 2 \end{array}$	$\begin{array}{r} 9\frac{3}{5} \\ 5\overline{)48} \\ \underline{45} \\ 3 \end{array}$
11.	$\begin{array}{r} 4\frac{5}{6} \\ 6\overline{)29} \\ \underline{24} \\ 5 \end{array}$	$\begin{array}{r} 5\frac{1}{2} \\ 8\overline{)44} \\ \underline{40} \\ 4 \end{array}$	$\begin{array}{r} 4\frac{1}{3} \\ 9\overline{)39} \\ \underline{36} \\ 3 \end{array}$
12.	$\begin{array}{r} 5\frac{1}{5} \\ 5\overline{)26} \\ \underline{25} \\ 1 \end{array}$	$\begin{array}{r} 3\frac{1}{2} \\ 4\overline{)14} \\ \underline{12} \\ 2 \end{array}$	$\begin{array}{r} 9\frac{2}{3} \\ 9\overline{)87} \\ \underline{81} \\ 6 \end{array}$

— Copy and divide. Name each quotient using a mixed form. Multiply to check your answers.

	a	b	c	d
13.	$9\overline{)20}$	$6\overline{)32}$	$8\overline{)27}$	$4\overline{)30}$
14.	$7\overline{)30}$	$4\overline{)37}$	$8\overline{)59}$	$3\overline{)28}$
15.	$7\overline{)69}$	$9\overline{)31}$	$6\overline{)25}$	$8\overline{)19}$
16.	$6\overline{)50}$	$7\overline{)25}$	$6\overline{)57}$	$4\overline{)26}$
17.	$4\overline{)34}$	$7\overline{)60}$	$9\overline{)42}$	$6\overline{)20}$

18. Study the example at the right to see if you can explain each step. Then copy only the part of the example shown in black. Find the quotient and check your work by comparing it with the work shown in the example above in color. Check also by multiplying the divisor by the quotient.

$$\boxed{\begin{array}{r} 16\frac{1}{4} \\ 4\overline{)65} \\ \underline{40} \\ 25 \\ \underline{24} \\ 1 \end{array}}$$

19. Explain each example below. Then check the quotients by multiplying.

$$\begin{array}{r} 14\frac{1}{6} \\ 6\overline{)85} \\ \underline{60} \\ 25 \\ \underline{24} \\ 1 \end{array} \qquad \begin{array}{r} 28\frac{2}{3} \\ 3\overline{)86} \\ \underline{60} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

— Copy, divide, and check.

20.	$2\overline{)35}$	$5\overline{)64}$	$3\overline{)76}$	$8\overline{)92}$
21.	$4\overline{)93}$	$7\overline{)81}$	$2\overline{)53}$	$6\overline{)73}$

When to divide the remainder

1. David and two of his friends had 7 candy bars to share equally. Each boy took 2 whole bars. There was 1 bar left over. David broke the bar that was left over into 3 parts of the same size.

$$\boxed{\begin{array}{r} 2\frac{1}{3} \\ 3\overline{)7} \end{array}}$$

Each boy received 2 whole bars and $\frac{1}{3}$ of a bar, or $\underline{\ ?\ }$ candy bars. In this case, did David divide the remainder 1 by the divisor 3?

2. David also had 7 marbles he wished to share equally with his friends. He said, "Each of you can have 2 marbles, but I can't share the 1 that's left over." Explain why in this case it would make little sense for David to divide the remainder 1 by the divisor 3.

$$\boxed{\begin{array}{r} 2\text{ r}1 \\ 3\overline{)7} \end{array}}$$

3. Whether or not you divide the remainder depends upon the situation. In which of the situations described below would it make sense to divide the remainder?

- 5 cupcakes shared by 2 boys
- 10 roses shared by 3 girls
- 50¢ shared by 8 girls
- 24 doughnuts shared by 16 boys
- 16 theater tickets shared by 5 girls
- $21 shared by 6 boys

4. Can 3 boys share 25¢ equally? Explain. What is the best they can do?

5. Can 8 girls share 12 yards of material equally? Explain.

6. Jane has a piece of ribbon that is 15 inches long. If she cuts it into 2 pieces of the same length, how long will each piece be?

7. Johnny has 16 ounces of orange juice. How many ounces can he serve to each person if he serves 3 people? if he serves 5 people?

— Find the numbers that make these sentences true. Name each answer using a mixed form. Check by multiplying.

	a	b
8.	$36 \div 8 = n$	$45 \div 6 = n$
9.	$33 \div 5 = n$	$67 \div 8 = n$
10.	$19 \div 4 = n$	$56 \div 9 = n$

11. If one factor of 42 is 5, which number in the following set is the other factor? $\{8, 8\frac{2}{5}, 9\}$ How can you check your answer?

— Find the factors that make these sentences true.

12. $8 \times n = 36$ $\quad n \times 9 = 78$

13. $n \times 6 = 27$ $\quad 3 \times n = 25$

14. $4 \times n = 31$ $\quad n \times 9 = 67$

15. $n \times 8 = 50$ $\quad 8 \times n = 57$

16. The perimeter of a square is 62 inches. What is the length of each side of the square?

Finding equivalent measurements

1. An oil pipe is 64 inches long. How many feet long is the pipe?

- To find how many feet 64 inches make, you divide 64 by _?_.

$$\begin{array}{l} \quad\;\; 5\frac{4}{12} = 5\frac{1}{3} \\ 12\overline{)64} \\ \quad\; \underline{60} \\ \qquad 4 \end{array}$$

- The greatest whole number of 12's in 64 is _?_ with a remainder of _?_.
- If you divide the remainder by 12 and express the result in lowest terms as part of the quotient, the quotient for 64 ÷ 12 is _?_.
- Check. $5\frac{1}{3} \times 12 =$ _?_

2. A piece of pipe is 80 inches long. This length is the same as _?_ feet.

3. How many feet long is a steel rod that measures 90 inches?

4. Copy and complete the following table, using *multiply* or *divide* in place of each question mark.

TO EXPRESS A NUMBER OF	AS A NUMBER OF	YOU
feet	inches	_?_ by 12
feet	yards	_?_ by 3
minutes	hours	_?_ by 60
yards	inches	_?_ by 36
minutes	seconds	_?_ by 60
ounces	pounds	_?_ by 16
pounds	ounces	_?_ by 16

5. To express a number of larger units as a number of smaller units, do you multiply or divide?

6. To express a number of smaller units as a number of larger units, do you multiply or divide?

— Copy and complete. Name your answers using mixed forms.

7. 18 inches make _?_ feet

8. 28 inches make _?_ feet

9. 20 ounces make _?_ pounds

10. 90 minutes make _?_ hours

11. 31 days make _?_ weeks

12. 42 inches make _?_ yards

13. 24 ounces make _?_ pounds

14. 128 seconds make _?_ minutes

15. 25 feet make _?_ yards

16. 60 days make _?_ weeks

17. 36 ounces make _?_ pounds

18. 54 inches make _?_ yards

19. At an average rate of 40 miles per hour, you can go _?_ miles in $3\frac{1}{4}$ hours.

20. A car traveled 362 miles in 8 hours. The average rate of travel was _?_ miles per hour.

21. How many dozen doughnuts are there in 28 doughnuts? 39 doughnuts? 56 doughnuts?

22. How many years are there in 40 months? 45 months? 52 months?

23. At 60¢ a gallon, find the cost of 10 quarts of cider.

Practice with fractional numbers

— First estimate the answers. Then copy and compute. Express all answers in lowest terms. Be sure to check.

	a	b	c
1.	$\begin{array}{r} 5\frac{1}{2} \\ +\ 6\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{1}{4} \\ +\ 4\frac{7}{8} \\ \hline \end{array}$	$\begin{array}{r} 3\frac{1}{12} \\ +\ 9\frac{1}{3} \\ \hline \end{array}$
2.	$\begin{array}{r} 7\frac{3}{10} \\ +\ 3\frac{4}{15} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{1}{3} \\ +\ 5\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 13\frac{3}{5} \\ +\ 6\frac{1}{4} \\ \hline \end{array}$
3.	$\begin{array}{r} 3\frac{5}{6} \\ -\ 1\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 4\frac{5}{8} \\ -\ 2\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{1}{2} \\ -\ 3\frac{7}{8} \\ \hline \end{array}$
4.	$\begin{array}{r} 12\frac{2}{3} \\ -\ 4\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 16\frac{3}{8} \\ -\ 9\frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} 22\frac{1}{9} \\ -\ 10\frac{5}{6} \\ \hline \end{array}$

— Solve each of these sentences.

	a	b
5.	$\frac{5}{5} + \frac{5}{6} = n$	$\frac{1}{3} + \frac{1}{6} + \frac{1}{4} = n$
6.	$\frac{3}{4} - \frac{3}{8} = n$	$\frac{3}{4} - \frac{1}{3} = n$
7.	$5\frac{3}{8} + 1\frac{4}{5} = n$	$7\frac{5}{12} + 1\frac{1}{4} = n$
8.	$6\frac{3}{4} - 4\frac{1}{3} = n$	$9\frac{4}{5} - 3\frac{7}{8} = n$
9.	$\frac{5}{8} + n = \frac{7}{8}$	$n + \frac{7}{9} = \frac{8}{9}$
10.	$\frac{7}{8} - n = \frac{3}{8}$	$\frac{3}{5} - n = \frac{1}{2}$
11.	$n - \frac{1}{2} = 3\frac{1}{4}$	$n - \frac{5}{8} = 2\frac{1}{3}$
12.	$n - 8\frac{5}{8} = 20$	$12\frac{1}{2} - n = 7\frac{5}{12}$
13.	$n - 7\frac{3}{4} = 2\frac{1}{3}$	$6\frac{1}{8} + n = 8\frac{1}{12}$
14.	$4\frac{3}{4} + n = 10$	$18\frac{5}{8} - n = 6\frac{1}{2}$

— Solve each of these sentences.

	a	b
15.	$\frac{1}{2}$ of $12 = n$	$\frac{3}{4}$ of $24 = n$
16.	$\frac{5}{8}$ of $22 = n$	$\frac{6}{7}$ of $24 = n$
17.	$\frac{1}{2}$ of $12\frac{1}{4} = n$	$\frac{5}{6}$ of $2\frac{1}{3} = n$

— What fractional numbers make these sentences true?

18. $\frac{5}{8} \times \frac{3}{4} = n$ $\qquad$ $\frac{6}{5} \times \frac{1}{2} = n$

19. $\frac{3}{8} \times 2\frac{1}{4} = n$ $\qquad$ $\frac{7}{10} \times 4\frac{1}{2} = n$

20. $3\frac{1}{3} \times 4\frac{1}{10} = n$ $\qquad$ $5\frac{1}{3} \times 1\frac{1}{2} = n$

— Copy each sentence and replace the ● with >, <, or =, to make the sentence true.

21. $\frac{1}{2} + \frac{1}{2}$ ● 1 $\qquad$ $\frac{3}{4} + \frac{1}{2}$ ● 1

22. $\frac{2}{3} - \frac{1}{2}$ ● $\frac{1}{4}$ $\qquad$ $\frac{4}{5} - \frac{1}{3}$ ● $\frac{1}{2}$

23. $\frac{3}{4} \times \frac{1}{2}$ ● $\frac{3}{8}$ $\qquad$ $\frac{1}{2} \times \frac{3}{4}$ ● $\frac{1}{2}$

24. $2\frac{1}{2} - \frac{5}{8}$ ● $1\frac{1}{2}$ $\qquad$ $3\frac{1}{4} - 2\frac{2}{3}$ ● $\frac{1}{4}$

25. $1\frac{1}{2} \times 1\frac{1}{2}$ ● 3 $\qquad$ $3\frac{1}{4} \times 1\frac{1}{2}$ ● $4\frac{3}{8}$

— Name the members in each set below using like fractions.

26. $\{\frac{1}{8}, \frac{3}{4}, \frac{1}{2}\}$ $\qquad$ $\{\frac{2}{3}, \frac{1}{5}, \frac{1}{2}\}$

27. $\{\frac{5}{6}, \frac{4}{9}, \frac{1}{3}\}$ $\qquad$ $\{\frac{3}{4}, \frac{1}{8}, \frac{5}{6}\}$

Think twice!

28. On a number line, the point for $\frac{5}{8}$ is halfway between $\frac{9}{16}$ and _?_.

29. On a number line, the point for _?_ is halfway between $\frac{3}{4}$ and $\frac{7}{8}$.

Problems involving fractional numbers

1. The first satellite launched by the United States was Explorer I. It weighed $30\frac{4}{5}$ pounds. Explorer XVII, successfully launched five years after Explorer I, weighed 405 pounds.

- Explorer XVII weighed ? pounds more than Explorer I.
- If Explorer I had been 13 times as heavy, would it have weighed more than, as much as, or less than Explorer XVII?

2. Explorer VII was designed to send back important information on radiation and magnetic storms. It weighed $91\frac{1}{2}$ pounds.

- Explorer VII was ? pounds heavier than Explorer I.
- Explorer XVII was ? pounds heavier than the combined weight of Explorers I and VII.

3. The table below shows how long it took each satellite to travel once around the earth.

SATELLITE	ORBIT TIME
Explorer I	$114\frac{1}{5}$ minutes
Explorer VII	$101\frac{1}{3}$ minutes
Explorer XVII	$96\frac{2}{5}$ minutes

Find the difference between the orbit times of:

- Explorers I and XVII
- Explorers I and VII
- Explorers VII and XVII

4. Using the table shown in Ex. 3, express the orbit time of each satellite in hours, minutes, and seconds.

5. How many minutes would it take Explorer I to complete 6 orbits? 12 orbits? 20 orbits? 108 orbits?

6. The weight devoted to scientific instruments is sometimes called the *scientific payload* of a satellite. Given in the table below are the scientific payloads of another important set of satellites, the Vanguards.

SATELLITE	SCIENTIFIC PAYLOAD
Vanguard I	$3\frac{1}{2}$ pounds
Vanguard II	$23\frac{4}{5}$ pounds
Vanguard III	$52\frac{1}{2}$ pounds

- Vanguard II carried ? pounds more payload than Vanguard I.
- How many more pounds payload did Vanguard III carry into space than Vanguard II?
- What is the difference between the payloads of Vanguard I and Vanguard III?
- The total scientific payload carried into space by the three satellites was ? pounds.
- Three times the payload of Vanguard II is ? pounds more than the payload of Vanguard III.
- Express each payload in the table as a decimal.

Numbers and their reciprocals

— Find the product of each pair of fractional numbers named below.

	a	b	c
1.	$\frac{3}{5}, \frac{5}{3}$	$\frac{2}{3}, \frac{3}{2}$	$\frac{6}{5}, \frac{5}{6}$
2.	$\frac{5}{12}, \frac{12}{5}$	$\frac{7}{8}, \frac{8}{7}$	$\frac{7}{16}, \frac{16}{7}$
3.	$\frac{4}{7}, \frac{7}{4}$	$\frac{10}{19}, \frac{19}{10}$	$\frac{11}{9}, \frac{9}{11}$

4. How are the products of the pairs of numbers shown in Ex. 1–3 related?

If the product of two numbers is 1, the two numbers are called reciprocals. For example, since $\frac{2}{3} \times \frac{3}{2}$ equals 1, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ and the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$.

5. What is the product of $\frac{4}{9}$ and $\frac{9}{4}$? Then are $\frac{4}{9}$ and $\frac{9}{4}$ reciprocals?

6. What is the product of $\frac{3}{8}$ and $\frac{8}{3}$? Then the reciprocal of $\frac{3}{8}$ is $\underline{\ ?\ }$ and the reciprocal of $\frac{8}{3}$ is $\underline{\ ?\ }$.

7. Study each pair of reciprocals shown in Ex. 1–6. How is the numerator of one reciprocal related to the denominator of the other reciprocal in each pair?

— Use what you discovered in Ex. 7 to complete each of these sentences.

	a	b
8.	$\frac{4}{5} \times \frac{?}{4} = 1$	$\frac{3}{2} \times \frac{2}{?} = 1$
9.	$\frac{?}{7} \times \frac{7}{6} = 1$	$\frac{8}{?} \times \frac{5}{8} = 1$

10. Name the reciprocal of each of these numbers. $\frac{3}{10}$ $\frac{9}{4}$ $\frac{7}{8}$ $\frac{12}{5}$ $\frac{3}{4}$

11. Is the following a true number sentence? $\frac{1}{3} \times \frac{3}{1} = 1$ Then are $\frac{1}{3}$ and $\frac{3}{1}$ reciprocals?

Is $\frac{3}{1}$ another name for 3? Then the reciprocal of $\frac{1}{3}$ is $\underline{\ ?\ }$, and the reciprocal of 3 is $\underline{\ ?\ }$.

12. Is this number sentence true? $\frac{1}{5} \times \frac{5}{1} = 1$ Then the reciprocal of 5 is $\underline{\ ?\ }$, and the reciprocal of $\frac{1}{5}$ is $\underline{\ ?\ }$.

13. What whole number is the reciprocal of $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{4}$? $\frac{1}{2}$? $\frac{1}{15}$? $\frac{1}{18}$?

14. What fractional number is the reciprocal of 6? 9? 10? 14? 32?

15. Suppose you wish to find the reciprocal of $2\frac{1}{3}$. How would knowing that $\frac{7}{3}$ is another name for $2\frac{1}{3}$ help? Is $\frac{3}{7}$ the reciprocal of $\frac{7}{3}$? of $2\frac{1}{3}$?

16. Why is $\frac{2}{3}$ the reciprocal of $1\frac{1}{2}$?

17. What is the reciprocal of $2\frac{1}{2}$? of $3\frac{2}{3}$? of $2\frac{7}{8}$? of $4\frac{3}{4}$? of $3\frac{4}{5}$?

— Solve these sentences.

	a	b
18.	$n \times \frac{3}{8} = 1$	$\frac{7}{5} \times n = 1$
19.	$2\frac{1}{5} \times n = 1$	$n \times 3\frac{1}{4} = 1$
20.	$n \times 5 = 1$	$7 \times n = 1$

21. Tell the reciprocal of each.

$\frac{3}{4}$	$\frac{5}{14}$	$\frac{1}{2}$	$\frac{3}{8}$	9	17
23	104	$3\frac{1}{8}$	$4\frac{2}{3}$	$5\frac{1}{4}$	$8\frac{1}{6}$

Another kind of numeral

1. You already know that a fraction is a numeral for a fractional number. The numerator shown in a fraction can be any whole number and the denominator can be any counting number.

What are equivalent fractions?

2. The numerator shown in the fraction at the right is $\underline{\ ?\ }$ and the denominator is $\underline{\ ?\ }$. What fractional number is named by $\frac{2}{3}$? Show three other fractions that are equivalent to $\frac{2}{3}$.

$$\boxed{\frac{2}{3}}$$

3. Is $\frac{4}{2}$ another name for 2? Suppose we rename the numerator shown in $\frac{2}{3}$ as $\frac{4}{2}$. Do you think the result, shown at the right, still names the fractional number $\frac{2}{3}$?

$$\boxed{\frac{\frac{4}{2}}{3}}$$

4. Is $\frac{6}{2}$ another name for 3? Suppose we rename the denominator shown in $\frac{2}{3}$ as $\frac{6}{2}$. Do you think the result, shown at the right, is equivalent to the fraction $\frac{2}{3}$?

$$\boxed{\frac{2}{\frac{6}{2}}}$$

5. Suppose we rename the numerator shown in $\frac{2}{3}$ as $\frac{4}{2}$ and the denominator as $\frac{6}{2}$. Since $\frac{4}{2}$ is another name for 2 and $\frac{6}{2}$ is another name for 3, is the numeral shown at the right equivalent to the fraction $\frac{2}{3}$? Is this sentence true? $\frac{2}{3} = \frac{\frac{4}{2}}{\frac{6}{2}}$

$$\boxed{\frac{\frac{4}{2}}{\frac{6}{2}}}$$

6. You know that the numerator and the denominator shown in a fraction can be multiplied by the same number to get an equivalent fraction. Look at the number sentence below.

$$\frac{2}{3} = \frac{\frac{1}{5} \times 2}{\frac{1}{5} \times 3} = \frac{\frac{2}{5}}{\frac{3}{5}}$$

By what number has the numerator of $\frac{2}{3}$ been multiplied? Has the denominator of $\frac{2}{3}$ been multiplied by the same number? Then is the result equivalent to the fraction $\frac{2}{3}$?

7. Write a fraction for the number that makes the following number sentence true.

$$\frac{2}{3} = \frac{\frac{3}{7} \times 2}{\frac{3}{7} \times 3} = \frac{n}{\frac{9}{7}}$$

8. Look at the following set of numerals.

$$\left\{ \frac{\frac{4}{2}}{3},\ \frac{2}{\frac{6}{2}},\ \frac{\frac{4}{2}}{\frac{6}{2}},\ \frac{\frac{2}{5}}{\frac{3}{5}},\ \frac{\frac{6}{7}}{\frac{9}{7}} \right\}$$

In the previous exercises, you have discovered that all of these numerals are equivalent to $\frac{2}{3}$. How are these numerals similar to a fraction? How do they differ from a fraction?

Numerals such as those shown above are called **complex fractions** because they show a fraction for the numerator, or for the denominator, or for both the numerator and the denominator.

Simplifying complex fractions

As you discovered on the preceding page, complex fractions and fractions both show a numerator and a denominator.

There is another way in which complex fractions and fractions are similar.

If the numerator and the denominator shown in a complex fraction are multiplied by the same number, the result is an equivalent fraction or an equivalent complex fraction.

1. Look at this number sentence.

$$\frac{\frac{3}{5}}{\frac{2}{3}} = \frac{\frac{1}{2}\times\frac{3}{5}}{\frac{1}{2}\times\frac{2}{3}} = \frac{\frac{3}{10}}{\frac{1}{3}}$$

- By what number have the numerator and the denominator shown in the first complex fraction been multiplied?
- What two complex fractions are shown to be equivalent by the true number sentence above?

2. Now look at this sentence.

$$\frac{\frac{3}{7}}{\frac{5}{2}} = \frac{2\times\frac{3}{7}}{2\times\frac{5}{2}} = \frac{\frac{6}{7}}{5}$$

- Have the numerator and the denominator shown in the complex fraction been multiplied by the same number? What is that number?
- What two numerals are shown to be equivalent by this true number sentence?

— Copy and complete each of the following number sentences to make it true. Then tell what two numerals are shown to be equivalent by each true sentence.

3. $\frac{\frac{3}{4}}{\frac{5}{7}} = \frac{\frac{1}{2}\times\frac{3}{4}}{\frac{1}{2}\times\frac{5}{7}} = \frac{?}{\frac{5}{14}}$

4. $\frac{\frac{2}{3}}{\frac{5}{8}} = \frac{3\times\frac{2}{3}}{3\times\frac{5}{8}} = \frac{2}{?}$

5. $\frac{\frac{1}{2}}{\frac{3}{5}} = \frac{5\times\frac{1}{2}}{5\times\frac{3}{5}} = \frac{\frac{5}{2}}{?}$

6. $\frac{4}{\frac{5}{9}} = \frac{\frac{2}{3}\times 4}{\frac{2}{3}\times\frac{5}{9}} = \frac{?}{?}$

7. $\frac{\frac{3}{8}}{6} = \frac{\frac{1}{5}\times\frac{3}{8}}{\frac{1}{5}\times 6} = \frac{?}{?}$

8. $\frac{\frac{2}{7}}{\frac{1}{2}} = \frac{2\times\frac{2}{7}}{2\times\frac{1}{2}} = \frac{?}{1}$, or $\underline{?}$

9. $\frac{\frac{1}{2}}{3} = \frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times 3} = \frac{?}{1}$, or $\underline{?}$

10. $\frac{\frac{2}{3}}{\frac{3}{5}} = \frac{\frac{5}{3}\times\frac{2}{3}}{\frac{5}{3}\times\frac{3}{5}} = \frac{?}{?}$, or $\underline{?}$

11. $\frac{\frac{1}{7}}{\frac{1}{4}} = \frac{4\times\frac{1}{7}}{4\times\frac{1}{4}} = \frac{?}{?}$, or $\underline{?}$

12. $\frac{\frac{3}{4}}{\frac{2}{3}} = \frac{\frac{3}{2}\times\frac{3}{4}}{\frac{3}{2}\times\frac{2}{3}} = \frac{?}{?}$, or $\underline{?}$

13. $\frac{4}{\frac{5}{7}} = \frac{\frac{7}{5}\times 4}{\frac{7}{5}\times\frac{5}{7}} = \frac{?}{?}$, or $\underline{?}$

14. In Ex. 8–13, did you find that your answers were named by fractions rather than complex fractions? Study each sentence to see if you can discover why.

By what number did you multiply the numerator and denominator shown in the complex fraction? What relationship does this number have to the denominator shown in the complex fraction?

15. Look at this sentence.

$$\frac{\frac{4}{5}}{\frac{7}{8}} = \frac{\frac{8}{7} \times \frac{4}{5}}{\frac{8}{7} \times \frac{7}{8}} = \frac{\frac{32}{35}}{1} = \frac{32}{35}$$

- By what number were the numerator and denominator multiplied?
- What relationship does this number have to the denominator shown in the complex fraction?

Finding a fraction equivalent to a complex fraction is called **simplifying** the complex fraction.

— In simplifying the following complex fractions, see if you can tell how the number used as a factor is related to the denominator shown in the complex fraction.

16. $\frac{\frac{5}{6}}{\frac{2}{7}} = \frac{\frac{7}{2} \times \frac{5}{6}}{\frac{7}{2} \times \frac{2}{7}} = \frac{\frac{35}{12}}{1} = \underline{\ ?\ }$

17. $\frac{\frac{4}{5}}{9} = \frac{\frac{1}{9} \times \frac{4}{5}}{\frac{1}{9} \times 9} = \frac{\frac{4}{45}}{1} = \underline{\ ?\ }$

18. $\frac{5}{\frac{2}{5}} = \frac{\frac{5}{2} \times 5}{\frac{5}{2} \times \frac{2}{5}} = \frac{\frac{25}{2}}{?} = \underline{\ ?\ }$

— What numbers make these sentences true?

19. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{a \times \frac{5}{9}}{a \times \frac{3}{4}} = \frac{n}{1} = n$

20. $\frac{\frac{2}{7}}{\frac{1}{8}} = \frac{a \times \frac{2}{7}}{a \times \frac{1}{8}} = \frac{n}{1} = n$

21. $\frac{\frac{3}{4}}{2} = \frac{a \times \frac{3}{4}}{a \times 2} = \frac{n}{1} = n$

22. $\frac{3}{\frac{2}{5}} = \frac{a \times 3}{a \times \frac{2}{5}} = \frac{n}{1} = n$

23. $\frac{\frac{1}{3}}{7} = \frac{a \times \frac{1}{3}}{a \times 7} = \frac{n}{1} = n$

★ Your discovery

Tell your discovery

Tell how reciprocals can be used to simplify complex fractions.

Use your discovery

Simplify the following complex fractions.

$\frac{\frac{3}{4}}{\frac{1}{5}}$ $\frac{\frac{2}{7}}{\frac{3}{8}}$ $\frac{\frac{4}{5}}{3}$ $\frac{9}{\frac{5}{6}}$

— Write a fraction in lowest terms that is equivalent to each of the following complex fractions.

	a	b	c	d
24.	$\frac{\frac{4}{5}}{\frac{7}{9}}$	$\frac{\frac{3}{8}}{4}$	$\frac{\frac{1}{2}}{\frac{5}{6}}$	$\frac{\frac{2}{3}}{2}$
25.	$\frac{8}{\frac{3}{4}}$	$\frac{\frac{9}{2}}{\frac{1}{3}}$	$\frac{\frac{3}{4}}{\frac{1}{2}}$	$\frac{6}{\frac{5}{7}}$

PROBLEM TEST 7

1. Tony is going to enlarge a photograph for framing. The ratio of the width of the photograph to the length is $\frac{2}{3}$. If Tony enlarges the photograph to 12 inches long, how wide will it be?

How long will the photograph be if Tony enlarges it to 6 inches wide?

2. The scale for a drawing of a house is 1 inch to 4 feet. On the drawing, the living room is $3\frac{1}{2}$ inches by 5 inches. What are the dimensions of the actual living room?

3. Mary earns \$.90 an hour as a mother's helper. How much money does she earn in a half hour? in $4\frac{1}{3}$ hours?

4. Look at the table below. To the nearest million square miles, what is the area of the Pacific Ocean? of the Atlantic Ocean?

OCEAN	AREA IN SQUARE MILES
Pacific	63,801,668
Atlantic	31,839,306

Using the approximations, write a fraction in lowest terms or numeral for a whole number to name the ratio of the area of the Atlantic to the area of the Pacific; the ratio of the area of the Pacific to the area of the Atlantic.

5. What is the area of a square region that measures $\frac{3}{8}$ inch on one side?

6. Mrs. Jonas roasts turkey 16 minutes for every pound. How many minutes should she roast an $11\frac{1}{4}$-pound turkey? How many hours should she roast it?

7. What is the area of a rectangular region whose dimensions are $7\frac{3}{4}$ yards by $9\frac{1}{2}$ yards?

8. Dave has a 26-inch board that he wants to saw into 4 pieces of the same length. How long should each piece be?

9. Curtain material costs \$1.08 a yard. What is the price of 54 inches of this material?

10. There are 14 boys and 21 girls in the school drama club.

How many more girls than boys are there?

What is the ratio of the number of girls to the number of boys in the drama club?

What is the difference between the number of boys and the total number of members in the club?

What is the ratio of the number of boys to the total number of members in the club?

Write your score on your Problem Test Record.

IT'S CHECK-UP TIME

— Solve these sentences.

1. $\frac{3}{4} \times \frac{1}{2} = \frac{3 \times a}{4 \times 2}$

2. $\frac{2}{3} \times \frac{5}{8} = x \times \frac{2}{3}$

3. $\frac{5}{7} \times (\frac{1}{12} \times \frac{1}{3}) = (\frac{5}{7} \times \frac{1}{12}) \times d$

4. $\frac{1}{6} \times c = 1$

5. $6 \times t = 43$

— Copy and complete. Name your answers in mixed form.

6. 25 days make ? weeks.

7. 30 inches make ? feet.

— Copy and solve these sentences. Name all answers in lowest terms.

a b

8. $\frac{3}{5} \times \frac{1}{7} = n$ $\qquad$ $\frac{1}{2} \times \frac{3}{9} = n$

9. $\frac{3}{4}$ of $\frac{1}{8} = n$ $\qquad$ $\frac{5}{8}$ of $\frac{1}{4} = n$

10. $\frac{2}{7} \times 3 = n$ $\qquad$ $\frac{1}{6} \times 12 = n$

11. $4 \times \frac{1}{2} = n$ $\qquad$ $7 \times \frac{3}{14} = n$

12. $\frac{1}{8} \times \frac{3}{4} \times \frac{2}{3} = n$ $\qquad$ $\frac{2}{5} \times \frac{4}{5} \times \frac{1}{2} = n$

13. $6\frac{1}{2} \times 2\frac{3}{4} = n$ $\qquad$ $7\frac{2}{5} \times 4\frac{1}{4} = n$

14. $1\frac{2}{3} \times 2\frac{1}{3} = n$ $\qquad$ $1\frac{3}{4} \times 1\frac{2}{5} = n$

— Write a fraction in lowest terms that is equivalent to each of the following complex fractions.

a b

15. $\frac{\frac{3}{8}}{\frac{2}{3}}$ $\qquad$ $\frac{\frac{2}{5}}{2}$

16. $\frac{6}{\frac{5}{6}}$ $\qquad$ $\frac{\frac{3}{4}}{\frac{1}{6}}$

— Copy and divide. Name each quotient in mixed form.

17. $7\overline{)33}$ $\qquad$ $5\overline{)19}$

18. $6\overline{)27}$ $\qquad$ $9\overline{)60}$

19. A train traveling at an average rate of 65 miles per hour will go ? miles in $3\frac{1}{5}$ hours.

20. If 4 girls share 6 yards of material equally, each girl will have ? yards of the material.

If you have time, try these —

— Copy and solve these sentences.

21. $\frac{3}{5} \times \frac{6}{7} = \frac{18}{35}$, so $\frac{18}{35} \div \frac{6}{7} = n$

22. $\frac{2}{3} \times 12 = 8$, so $8 \div \frac{2}{3} = n$

Mathamusements

Joe wrote a numeral on a piece of paper and noticed that it had line symmetry. He added 10 to the number named by this numeral, and the numeral for the sum had two lines of symmetry. When he added 100 to the sum, the numeral for the new sum had two lines of symmetry. When he multiplied the new sum by 8, the numeral for the product had two lines of symmetry. What were the four numerals?

TESTS ON NUMBER FACTS

ADDITION FACTS TEST

	a	b	c	d	e	f	g	h
1.	5 + 4	1 + 3	6 + 8	9 + 6	5 + 3	9 + 8	2 + 2	8 + 9
2.	2 + 0	9 + 5	7 + 8	0 + 1	4 + 1	6 + 7	9 + 9	5 + 2
3.	4 + 2	6 + 9	1 + 4	6 + 6	8 + 7	1 + 5	0 + 3	8 + 8
4.	6 + 5	0 + 0	3 + 9	5 + 1	3 + 2	8 + 6	5 + 6	4 + 3
5.	3 + 1	7 + 5	5 + 5	6 + 4	7 + 6	4 + 4	3 + 3	5 + 0
6.	8 + 5	4 + 5	0 + 2	1 + 6	3 + 0	9 + 3	7 + 0	2 + 3
7.	2 + 6	3 + 4	6 + 3	5 + 7	7 + 9	4 + 6	2 + 5	0 + 8
8.	1 + 8	7 + 2	4 + 7	8 + 1	2 + 4	8 + 4	8 + 0	3 + 5
9.	7 + 1	0 + 9	5 + 8	2 + 7	1 + 0	0 + 7	6 + 2	7 + 4
10.	4 + 9	2 + 8	6 + 0	7 + 3	4 + 8	6 + 1	3 + 7	9 + 1
11.	1 + 1	8 + 2	3 + 6	9 + 0	0 + 6	1 + 2	9 + 7	9 + 2
12.	0 + 4	3 + 8	7 + 7	1 + 9	8 + 3	5 + 9	2 + 9	9 + 4

SUBTRACTION FACTS TEST

	a	b	c	d	e	f	g	h
1.	10 − 2	13 − 7	2 − 1	12 − 4	9 − 2	16 − 9	12 − 5	5 − 4
2.	1 − 1	7 − 3	9 − 5	8 − 2	4 − 4	11 − 5	9 − 1	13 − 4
3.	7 − 2	11 − 4	5 − 3	7 − 7	12 − 6	1 − 0	10 − 4	6 − 3
4.	10 − 5	12 − 3	9 − 7	0 − 0	9 − 4	12 − 7	8 − 5	13 − 6
5.	8 − 7	4 − 3	14 − 9	7 − 5	9 − 6	3 − 1	13 − 8	5 − 0
6.	8 − 4	4 − 1	6 − 5	3 − 3	11 − 3	12 − 8	8 − 1	16 − 7
7.	14 − 6	15 − 7	4 − 0	11 − 7	6 − 1	9 − 9	10 − 3	13 − 5
8.	9 − 3	2 − 0	7 − 4	3 − 2	15 − 6	3 − 0	15 − 8	7 − 6
9.	2 − 2	8 − 6	11 − 2	6 − 4	5 − 5	11 − 9	7 − 1	14 − 8
10.	7 − 0	10 − 7	6 − 6	10 − 9	11 − 8	14 − 7	12 − 9	10 − 6
11.	8 − 8	5 − 2	14 − 5	15 − 9	6 − 2	18 − 9	17 − 8	8 − 3
12.	4 − 2	16 − 8	9 − 8	8 − 0	13 − 9	17 − 9	11 − 6	10 − 8

MULTIPLICATION FACTS TEST

	a	b	c	d	e	f	g	h
1.	4 × 6	3 × 5	6 × 6	5 × 9	1 × 1	7 × 5	5 × 1	0 × 5
2.	0 × 6	8 × 2	0 × 0	9 × 8	6 × 2	4 × 5	2 × 9	5 × 2
3.	3 × 4	4 × 4	7 × 4	2 × 8	5 × 3	2 × 1	9 × 9	8 × 0
4.	3 × 9	2 × 7	6 × 1	0 × 4	6 × 5	8 × 7	6 × 8	3 × 6
5.	5 × 0	8 × 1	1 × 9	6 × 7	3 × 8	7 × 9	1 × 0	4 × 3
6.	2 × 6	1 × 2	3 × 7	8 × 6	2 × 3	0 × 3	8 × 8	4 × 9
7.	0 × 1	7 × 3	6 × 0	3 × 2	4 × 8	9 × 0	5 × 6	1 × 8
8.	6 × 9	4 × 7	9 × 6	0 × 2	7 × 2	1 × 7	9 × 4	6 × 4
9.	3 × 0	5 × 5	9 × 7	4 × 0	2 × 2	8 × 4	4 × 2	1 × 3
10.	8 × 5	1 × 4	5 × 8	7 × 8	5 × 4	9 × 3	2 × 5	7 × 0
11.	1 × 6	9 × 2	6 × 3	3 × 1	8 × 3	4 × 1	9 × 5	0 × 9
12.	7 × 7	0 × 7	2 × 4	9 × 1	5 × 7	3 × 3	8 × 9	7 × 6

DIVISION FACTS TEST

	a	b	c	d	e	f	g	h
1.	$9\overline{)27}$	$4\overline{)20}$	$9\overline{)81}$	$6\overline{)42}$	$8\overline{)48}$	$1\overline{)4}$	$5\overline{)40}$	$7\overline{)42}$
2.	$5\overline{)45}$	$9\overline{)36}$	$1\overline{)2}$	$8\overline{)40}$	$7\overline{)28}$	$2\overline{)18}$	$4\overline{)16}$	$8\overline{)64}$
3.	$1\overline{)1}$	$5\overline{)30}$	$2\overline{)16}$	$5\overline{)35}$	$3\overline{)9}$	$6\overline{)36}$	$2\overline{)6}$	$3\overline{)0}$
4.	$2\overline{)4}$	$6\overline{)48}$	$3\overline{)6}$	$7\overline{)35}$	$6\overline{)6}$	$2\overline{)0}$	$5\overline{)25}$	$4\overline{)12}$
5.	$7\overline{)56}$	$4\overline{)4}$	$7\overline{)49}$	$2\overline{)2}$	$3\overline{)27}$	$5\overline{)20}$	$8\overline{)0}$	$2\overline{)14}$
6.	$3\overline{)3}$	$9\overline{)18}$	$1\overline{)0}$	$5\overline{)15}$	$4\overline{)8}$	$2\overline{)12}$	$6\overline{)30}$	$4\overline{)36}$
7.	$6\overline{)12}$	$1\overline{)9}$	$6\overline{)54}$	$7\overline{)0}$	$8\overline{)16}$	$1\overline{)7}$	$7\overline{)63}$	$3\overline{)15}$
8.	$8\overline{)56}$	$5\overline{)10}$	$1\overline{)6}$	$4\overline{)24}$	$3\overline{)12}$	$8\overline{)24}$	$2\overline{)10}$	$7\overline{)14}$
9.	$4\overline{)0}$	$9\overline{)63}$	$4\overline{)32}$	$8\overline{)8}$	$5\overline{)0}$	$7\overline{)7}$	$3\overline{)24}$	$8\overline{)72}$
10.	$7\overline{)21}$	$9\overline{)0}$	$9\overline{)45}$	$3\overline{)21}$	$4\overline{)28}$	$9\overline{)9}$	$6\overline{)24}$	$1\overline{)8}$
11.	$2\overline{)8}$	$9\overline{)72}$	$6\overline{)18}$	$1\overline{)5}$	$9\overline{)54}$	$5\overline{)5}$	$3\overline{)18}$	$8\overline{)32}$

PRACTICE SETS

PRACTICE SET 1, Addition (page 23)

	a	b	c	d
1.	$37 + 2 = n$	$91 + 7 = n$	$44 + 3 = n$	$70 + 5 = n$
2.	$16 + 3 = n$	$47 + 0 = n$	$11 + 3 = n$	$34 + 1 = n$
3.	$125 + 4 = n$	$342 + 4 = n$	$572 + 5 = n$	$424 + 4 = n$
4.	$723 + 2 = n$	$213 + 5 = n$	$116 + 2 = n$	$573 + 3 = n$
5.	$624 + 5 = n$	$142 + 7 = n$	$310 + 9 = n$	$403 + 4 = n$

PRACTICE SET 2, Addition (page 23)

	a	b	c	d	e	f	g	h
1.	75 + 13	24 + 32	89 + 10	42 + 57	30 + 43	85 + 4	16 + 52	27 + 42
2.	428 + 351	732 + 155	612 + 384	117 + 572	308 + 361	644 + 310	959 + 30	536 + 213

PRACTICE SET 3, Addition (page 25)

	a	b	c	d	e	f	g
1.	49 + 65	28 + 10	26 + 77	53 + 69	17 + 45	48 + 52	34 + 88
2.	274 + 433	762 + 976	453 + 878	155 + 796	897 + 394	586 + 872	172 + 259
3.	419 + 927	511 + 697	893 + 882	976 + 675	668 + 376	895 + 145	708 + 499
4.	7,260 + 5,899	2,386 + 9,954	9,448 + 3,562	3,564 + 8,484	8,039 + 9,875	4,556 + 7,036	3,420 + 5,763

PRACTICE SET 4, Addition (page 28)

	a	b	c	d	e	f
1.	✓774 897 + 314	✓878 563 + 249	✓375 906 + 693	764 975 + 348	418 357 + 697	543 618 + 379
2.	\$5.31 4.28 + 3.19	\$6.78 3.45 + 5.89	\$3.62 7.40 + 7.89	\$32.79 9.24 + 78.47	\$47.95 90.72 + 87.56	\$65.00 29.69 + 37.84
3.	9,483 3,521 765 + 4,918	8,296 7,508 76 + 5,325	3,291 6,477 8,202 + 9,953	17,582 4,611 36,079 + 57,719	42,187 77,318 479 + 69,532	75,638 42,080 13,946 + 54,875

PRACTICE SET 5, Addition (page 39)

	a	b	c	d
1.	3 ft. 6 in. + 5 ft. 3 in.	8 lb. 4 oz. + 7 lb. 11 oz.	15 min. 10 sec. + 22 min. 8 sec.	2 qt. 1 pt. + 4 qt. 1 pt.
2.	8 yd. 1 ft. + 2 yd. 2 ft.	4 lb. 12 oz. + 7 lb. 7 oz.	35 min. 20 sec. + 12 min. 55 sec.	2 ft. 9 in. + 4 ft. 10 in.
3.	1 hr. 28 min. + 2 hr. 45 min.	2 gal. 3 qt. + 4 gal. 2 qt.	6 ft. 4 in. + 5 ft. 9 in.	3 hr. 11 min. + 2 hr. 49 min.

PRACTICE SET 6, Addition (page 187)

1. $\frac{3}{5}+\frac{3}{5}=n$ $\frac{5}{8}+\frac{1}{8}=n$ $n=\frac{1}{2}+\frac{3}{2}$ $n=\frac{6}{10}+\frac{3}{10}$

	a	b	c	d	e	f	g	h
2.	$\frac{1}{6}$ $+\frac{3}{6}$	$\frac{1}{7}$ $+\frac{4}{7}$	$\frac{2}{5}$ $+\frac{4}{5}$	$\frac{5}{9}$ $+\frac{7}{9}$	$\frac{5}{10}$ $\frac{1}{10}$ $+\frac{7}{10}$	$\frac{7}{12}$ $\frac{9}{12}$ $+\frac{1}{12}$	$\frac{3}{5}$ $\frac{4}{5}$ $+\frac{1}{5}$	$\frac{8}{16}$ $\frac{5}{16}$ $+\frac{3}{16}$

20
17
3

PRACTICE SET 7, Addition (page 195)

	a	b	c	d
1.	$\frac{1}{6} + \frac{2}{3} = n$	$\frac{2}{3} + \frac{1}{9} = n$	$\frac{1}{12} + \frac{3}{4} = n$	$\frac{1}{6} + \frac{1}{2} = n$
2.	$\frac{1}{12} + \frac{2}{3} = n$	$\frac{1}{2} + \frac{3}{5} = n$	$\frac{3}{10} + \frac{2}{5} = n$	$\frac{5}{6} + \frac{2}{3} = n$

	a	b	c	d	e	f	g	h
3.	$\frac{1}{3}$ $+\frac{4}{5}$	$\frac{3}{4}$ $+\frac{1}{3}$	$\frac{1}{2}$ $+\frac{2}{3}$	$\frac{3}{5}$ $+\frac{1}{4}$	$\frac{5}{6}$ $+\frac{3}{4}$	$\frac{2}{3}$ $+\frac{1}{7}$	$\frac{7}{8}$ $+\frac{1}{3}$	$\frac{4}{5}$ $+\frac{1}{2}$
4.	$\frac{1}{2}$ $\frac{1}{3}$ $+\frac{1}{4}$	$\frac{2}{5}$ $\frac{1}{2}$ $+\frac{3}{4}$	$\frac{2}{3}$ $\frac{1}{4}$ $+\frac{1}{6}$	$\frac{3}{5}$ $\frac{2}{3}$ $+\frac{1}{2}$	$\frac{4}{5}$ $\frac{1}{2}$ $+\frac{3}{4}$	$\frac{1}{3}$ $\frac{1}{2}$ $+\frac{3}{10}$	$\frac{2}{3}$ $\frac{1}{4}$ $+\frac{3}{8}$	$\frac{3}{4}$ $\frac{2}{3}$ $+\frac{1}{9}$

PRACTICE SET 8, Addition (page 248)

1.	$3\frac{1}{3}$ $+6\frac{1}{3}$	$2\frac{1}{6}$ $+9\frac{4}{6}$	$7\frac{2}{7}$ $+5\frac{1}{7}$	$9\frac{1}{11}$ $+8\frac{3}{11}$	$7\frac{4}{10}$ $+6\frac{5}{10}$	$6\frac{2}{4}$ $+5\frac{1}{4}$	$4\frac{3}{8}$ $+7\frac{4}{8}$	$8\frac{2}{5}$ $+4\frac{1}{5}$
2.	$5\frac{5}{13}$ $+8\frac{4}{13}$	$6\frac{3}{7}$ $+9\frac{2}{7}$	$2\frac{1}{9}$ $+7\frac{4}{9}$	$1\frac{10}{20}$ $+8\frac{7}{20}$	$8\frac{5}{10}$ $+3\frac{2}{10}$	$4\frac{1}{7}$ $+6\frac{5}{7}$	$7\frac{4}{11}$ $+3\frac{6}{11}$	$9\frac{4}{15}$ $+4\frac{7}{15}$
3.	$8\frac{2}{9}$ $+8\frac{2}{9}$	$5\frac{1}{5}$ $+4\frac{3}{5}$	$4\frac{3}{17}$ $+3\frac{9}{17}$	$3\frac{8}{15}$ $+9\frac{5}{15}$	$5\frac{5}{9}$ $+2\frac{2}{9}$	$7\frac{2}{5}$ $+9\frac{2}{5}$	$1\frac{7}{13}$ $+2\frac{3}{13}$	$5\frac{2}{8}$ $+5\frac{1}{8}$

PRACTICE SET 9, Addition (page 249)

1.	$9\frac{1}{8}$ $+1\frac{1}{4}$	$7\frac{2}{3}$ $+7\frac{1}{5}$	$3\frac{1}{6}$ $+5\frac{1}{4}$	$4\frac{1}{7}$ $+8\frac{1}{3}$	$2\frac{3}{8}$ $+3\frac{2}{5}$	$6\frac{4}{9}$ $+4\frac{1}{2}$	$5\frac{5}{16}$ $+6\frac{5}{8}$	$8\frac{3}{7}$ $+7\frac{1}{4}$
2.	$7\frac{3}{4}$ $+4\frac{1}{8}$	$4\frac{1}{7}$ $+5\frac{3}{5}$	$3\frac{5}{6}$ $+8\frac{1}{12}$	$9\frac{1}{2}$ $+3\frac{2}{11}$	$1\frac{1}{9}$ $+1\frac{1}{3}$	$6\frac{1}{6}$ $+7\frac{5}{8}$	$8\frac{1}{4}$ $+9\frac{1}{2}$	$5\frac{2}{5}$ $+7\frac{1}{2}$
3.	$2\frac{1}{3}$ $+6\frac{1}{2}$	$5\frac{3}{8}$ $+1\frac{1}{3}$	$6\frac{1}{2}$ $+3\frac{2}{7}$	$8\frac{5}{12}$ $+5\frac{3}{8}$	$3\frac{1}{6}$ $+4\frac{1}{9}$	$9\frac{3}{5}$ $+6\frac{1}{3}$	$7\frac{1}{5}$ $+2\frac{7}{10}$	$4\frac{2}{9}$ $+9\frac{1}{6}$

PRACTICE SET 10, Addition (page 253)

	a	b	c	d	e	f	g	h
1.	$7\frac{2}{3}$ $+ 3\frac{3}{4}$	$5\frac{7}{8}$ $+ 6\frac{1}{2}$	$3\frac{1}{2}$ $+ \frac{7}{10}$	$4\frac{5}{6}$ $+ 8\frac{2}{3}$	$9\frac{2}{5}$ $+ 7\frac{3}{4}$	$8\frac{9}{10}$ $+ 2\frac{2}{5}$	$5\frac{3}{4}$ $+ 3\frac{5}{8}$	$4\frac{5}{7}$ $+ 2\frac{1}{3}$
2.	$6\frac{3}{5}$ $+ 9\frac{2}{3}$	$8\frac{11}{12}$ $+ 7\frac{1}{4}$	$6\frac{6}{11}$ $+ 4\frac{1}{3}$	$9\frac{5}{8}$ $+ 5\frac{7}{10}$	$5\frac{5}{6}$ $+ 7\frac{5}{9}$	$3\frac{1}{2}$ $+ 4\frac{3}{4}$	$7\frac{1}{4}$ $+ 6\frac{7}{12}$	$5\frac{3}{8}$ $+ 1\frac{5}{6}$
3.	$\frac{5}{6}$ $\frac{1}{3}$ $+ \frac{3}{4}$	$\frac{1}{7}$ $\frac{1}{2}$ $+ \frac{3}{7}$	$\frac{3}{8}$ $\frac{1}{4}$ $+ \frac{2}{3}$	$\frac{4}{9}$ $\frac{5}{6}$ $+ \frac{1}{2}$	$3\frac{1}{5}$ $2\frac{1}{2}$ $+ 5\frac{7}{10}$	$7\frac{3}{4}$ $3\frac{4}{5}$ $+ 1\frac{1}{10}$	$4\frac{3}{4}$ $8\frac{7}{12}$ $+ \frac{2}{3}$	$5\frac{7}{9}$ $2\frac{1}{3}$ $+ 6\frac{1}{2}$

PRACTICE SET 11, Addition (page 263)

	a	b	c	d	e	f	g
1.	0.3 + 0.4	0.1 + 0.1	0.5 + 0.4	0.36 + 0.20	0.18 + 0.30	0.01 + 0.36	0.35 + 0.14
2.	4.8 + 4.1	3.2 + 3.1	4.1 + 5.2	5.47 + 2.31	6.28 + 1.61	2.45 + 4.11	5.37 + 0.42
3.	23.17 + 26.51	27.03 + 50.15	69.61 + 20.38	14.24 + 70.32	78.05 + 21.23	26.39 + 22.50	12.12 + 35.47

PRACTICE SET 12, Addition (page 264)

1.	0.49 + 0.86	0.84 + 0.16	0.08 + 0.13	4.79 + 7.95	6.78 + 9.26	7.19 + 3.19	1.74 + 6.49
2.	91.79 + 99.77	19.07 + 27.38	79.86 + 58.37	18.76 + 37.58	75.08 + 43.28	57.89 + 21.24	49.57 + 56.17
3.	1.965 + 8.248	8.068 + 6.755	6.758 + 6.697	6.048 + 7.549	1.965 + 4.534	6.995 + 4.139	6.083 + 1.696
4.	0.854 + 4.579	9.849 + 3.428	5.035 + 6.897	1.953 + 5.458	6.634 + 5.288	3.032 + 7.945	1.979 + 7.231

PRACTICE SET 13, Subtraction (page 32)

	a	b	c	d
1.	$39 - 7 = n$	$85 - 4 = n$	$17 - 3 = n$	$28 - 6 = n$
2.	$74 - 3 = n$	$18 - 5 = n$	$33 - 1 = n$	$99 - 5 = n$
3.	$29 - 9 = n$	$47 - 8 = n$	$55 - 7 = n$	$73 - 8 = n$
4.	$44 - 7 = n$	$66 - 9 = n$	$32 - 8 = n$	$61 - 8 = n$
5.	$72 - 3 = n$	$84 - 6 = n$	$19 - 9 = n$	$57 - 9 = n$

PRACTICE SET 14, Subtraction (page 32)

	a	b	c	d	e	f	g	h
1.	69 − 53	79 − 57	85 − 34	79 − 29	64 − 22	86 − 40	59 − 24	23 − 12
2.	543 − 132	736 − 314	659 − 322	274 − 113	458 − 214	396 − 332	517 − 306	785 − 434
3.	878 − 762	969 − 532	387 − 283	885 − 563	476 − 106	479 − 348	679 − 416	645 − 323

PRACTICE SET 15, Subtraction (page 34)

	a	b	c	d	e	f	g
1.	840 − 184	701 − 359	900 − 203	408 − 229	400 − 112	830 − 486	260 − 218
2.	930 − 398	500 − 391	620 − 292	305 − 116	900 − 532	507 − 288	437 − 237
3.	9,000 − 6,375	6,070 − 4,494	7,202 − 4,364	7,006 − 5,718	9,604 − 7,776	6,000 − 3,798	8,650 − 8,429
4.	6,102 − 5,658	4,010 − 883	8,000 − 5,296	4,000 − 3,743	4,003 − 1,697	5,003 − 1,515	9,742 − 5,738

PRACTICE SET 16, Subtraction (page 38)

	a	b	c	d	e	f
1.	723 − 347	484 − 159	660 − 279	935 − 673	541 − 398	817 − 356
2.	$5.27 − 2.79	$4.00 − 2.67	$8.14 − 3.75	$73.56 − 29.14	$90.07 − 40.89	$28.42 − 17.53
3.	7,063 − 2,257	17,028 − 8,174	9,614 − 8,738	12,485 − 8,629	19,233 − 5,794	35,476 − 8,925
4.	76,245 − 49,378	81,637 − 14,848	93,947 − 56,284	82,460 − 26,735	48,300 − 16,437	79,213 − 59,584

PRACTICE SET 17, Subtraction (page 38)

	a	b	c
1.	$n + 34 = 76$	$55 + n = 97$	$n + 12 = 82$
2.	$29 + n = 65$	$n + 83 = 101$	$48 + n = 64$
3.	$n + 321 = 416$	$n + 254 = 628$	$545 + n = 711$
4.	$n + 508 = 900$	$727 + n = 743$	$n + 468 = 805$
5.	$4{,}663 + n = 7{,}142$	$n + 2{,}545 = 5{,}004$	$n + 5{,}847 = 5{,}898$
6.	$n + 8{,}364 = 9{,}430$	$n + 4{,}124 = 6{,}257$	$3{,}869 + n = 7{,}447$

PRACTICE SET 18, Subtraction (page 39)

	a	b	c	d
1.	3 hr. 10 min. − 2 hr. 3 min.	5 ft. 11 in. − 2 ft. 8 in.	15 lb. 15 oz. − 5 lb. 15 oz.	7 gal. 2 qt. − 4 gal. 1 qt.
2.	7 lb. 8 oz. − 3 lb. 14 oz.	2 qt. 0 pt. − 1 qt. 1 pt.	30 min. 8 sec. − 12 min. 24 sec.	10 yd. 1 ft. − 8 yd. 2 ft.
3.	4 gal. 0 qt. − 2 gal. 3 qt.	3 ft. 4 in. − 1 ft. 9 in.	8 hr. 13 min. − 3 hr. 54 min.	35 lb. 10 oz. − 16 lb. 15 oz.

PRACTICE SET 19, Subtraction (page 187)

	a	b	c	d
1.	$\frac{7}{10} - \frac{3}{10} = n$	$\frac{5}{14} - \frac{1}{14} = n$	$\frac{9}{4} - \frac{3}{4} = n$	$\frac{5}{7} - \frac{2}{7} = n$
2.	$\frac{3}{5} + n = \frac{7}{5}$	$n + \frac{1}{8} = \frac{5}{8}$	$\frac{7}{15} + n = \frac{11}{15}$	$n + \frac{6}{11} = \frac{10}{11}$
3.	$\frac{2}{3} + n = \frac{4}{3}$	$\frac{9}{12} - \frac{3}{12} = n$	$\frac{3}{9} + n = \frac{8}{9}$	$n + \frac{7}{18} = \frac{17}{18}$

	a	b	c	d	e	f	g	h
4.	$\begin{array}{r} \frac{4}{9} \\ -\frac{1}{9} \\ \hline \end{array}$	$\begin{array}{r} \frac{9}{10} \\ -\frac{3}{10} \\ \hline \end{array}$	$\begin{array}{r} \frac{12}{17} \\ -\frac{3}{17} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{8} \\ -\frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} \frac{11}{12} \\ -\frac{4}{12} \\ \hline \end{array}$	$\begin{array}{r} \frac{4}{5} \\ -\frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{15}{18} \\ -\frac{11}{18} \\ \hline \end{array}$	$\begin{array}{r} \frac{19}{20} \\ -\frac{12}{20} \\ \hline \end{array}$
5.	$\begin{array}{r} \frac{5}{9} \\ -\frac{1}{9} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{12} \\ -\frac{5}{12} \\ \hline \end{array}$	$\begin{array}{r} \frac{17}{20} \\ -\frac{7}{20} \\ \hline \end{array}$	$\begin{array}{r} \frac{4}{5} \\ -\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{9}{16} \\ -\frac{5}{16} \\ \hline \end{array}$	$\begin{array}{r} \frac{6}{7} \\ -\frac{3}{7} \\ \hline \end{array}$	$\begin{array}{r} \frac{8}{13} \\ -\frac{4}{13} \\ \hline \end{array}$	$\begin{array}{r} \frac{13}{18} \\ -\frac{4}{18} \\ \hline \end{array}$

PRACTICE SET 20, Subtraction (page 195)

1.	$\begin{array}{r} \frac{1}{2} \\ -\frac{1}{7} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{3} \\ -\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{2} \\ -\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{2}{3} \\ -\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ -\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{4}{5} \\ -\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{9} \\ -\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{5} \\ -\frac{1}{3} \\ \hline \end{array}$
2.	$\begin{array}{r} \frac{1}{3} \\ -\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ -\frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{2}{3} \\ -\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ -\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{2}{5} \\ -\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{8} \\ -\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{9} \\ -\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ -\frac{3}{5} \\ \hline \end{array}$

	a	b	c	d
3.	$\frac{2}{7} + n = \frac{3}{4}$	$n + \frac{2}{5} = \frac{7}{8}$	$n = \frac{5}{9} - \frac{1}{4}$	$\frac{2}{3} - \frac{1}{8} = n$
4.	$n + \frac{1}{5} = \frac{5}{6}$	$\frac{1}{10} + n = \frac{1}{2}$	$\frac{5}{8} - \frac{1}{3} = n$	$n = \frac{6}{11} - \frac{1}{2}$
5.	$\frac{4}{9} - \frac{1}{3} = n$	$n = \frac{3}{4} - \frac{1}{12}$	$\frac{9}{10} = n + \frac{1}{3}$	$\frac{2}{3} = \frac{2}{7} + n$

PRACTICE SET 21, Subtraction (page 199)

1.	$1 - \frac{3}{5} = n$	$4 - \frac{1}{3} = n$	$7 - \frac{5}{6} = n$	$9 - \frac{7}{8} = n$
2.	$n = 2 - \frac{5}{7}$	$n = 3 - \frac{2}{3}$	$n = 8 - \frac{4}{9}$	$n = 5 - \frac{1}{2}$
3.	$n + \frac{5}{8} = 6$	$\frac{2}{7} + n = 2$	$n + \frac{11}{12} = 1$	$4 = \frac{2}{5} + n$
4.	$n + \frac{3}{7} = 7$	$n + \frac{4}{5} = 3$	$\frac{7}{10} + n = 9$	$\frac{1}{9} + n = 8$
5.	$8 = n + \frac{3}{4}$	$5 = \frac{3}{11} + n$	$1 = n + \frac{1}{6}$	$\frac{1}{4} + n = 4$

PRACTICE SET 22, Subtraction (page 250)

	a	b	c	d	e	f	g	h
1.	$\begin{array}{r} 8\frac{5}{7} \\ -1\frac{2}{7} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{3}{5} \\ -6\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{7}{8} \\ -4\frac{2}{8} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{3}{4} \\ -5\frac{2}{4} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{5}{9} \\ -8\frac{1}{9} \\ \hline \end{array}$	$\begin{array}{r} 13\frac{6}{12} \\ -4\frac{1}{12} \\ \hline \end{array}$	$\begin{array}{r} 12\frac{2}{3} \\ -9\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{7}{10} \\ -2\frac{4}{10} \\ \hline \end{array}$
2.	$\begin{array}{r} 12\frac{1}{2} \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 14\frac{6}{7} \\ -8\frac{4}{7} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{5}{8} \\ -3\frac{2}{8} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{8}{9} \\ -8\frac{7}{9} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{1}{3} \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 4\frac{7}{11} \\ -4\frac{4}{11} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{7}{12} \\ -3\frac{6}{12} \\ \hline \end{array}$	$\begin{array}{r} 13\frac{9}{20} \\ -5\frac{6}{20} \\ \hline \end{array}$

PRACTICE SET 23, Subtraction (page 251)

1.	$\begin{array}{r} 18\frac{3}{4} \\ -9\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 12\frac{1}{2} \\ -7\frac{1}{8} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{2}{3} \\ -5\frac{1}{12} \\ \hline \end{array}$	$\begin{array}{r} 12\frac{3}{10} \\ -3\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{5}{8} \\ -4\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{5}{9} \\ -7\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} 16\frac{3}{5} \\ -8\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{3}{4} \\ -5\frac{1}{2} \\ \hline \end{array}$
2.	$\begin{array}{r} 13\frac{4}{5} \\ -8\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{2}{3} \\ -3\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} 11\frac{3}{8} \\ -6\frac{1}{12} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{5}{6} \\ -2\frac{4}{5} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{9}{10} \\ -9\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{3}{4} \\ -4\frac{1}{7} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{7}{11} \\ -6\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 15\frac{7}{9} \\ -7\frac{1}{3} \\ \hline \end{array}$
3.	$\begin{array}{r} 4\frac{5}{8} \\ -2\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} 16\frac{2}{3} \\ -9\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{1}{3} \\ -3\frac{2}{7} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{5}{12} \\ -7\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 14\frac{3}{7} \\ -5 \\ \hline \end{array}$	$\begin{array}{r} 11\frac{5}{6} \\ -3\frac{2}{9} \\ \hline \end{array}$	$\begin{array}{r} 15\frac{7}{8} \\ -6\frac{5}{12} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{2}{3} \\ -5\frac{1}{10} \\ \hline \end{array}$

	a	b	c	d
4.	$12\frac{1}{2} - 4\frac{1}{2} = n$	$7\frac{6}{7} - 2\frac{2}{3} = n$	$9\frac{5}{8} - 6\frac{1}{2} = n$	$10\frac{5}{6} - 2\frac{3}{8} = n$

PRACTICE SET 24, Subtraction (page 255)

1.	$7 - 4\frac{1}{8} = n$	$17\frac{1}{6} - 9\frac{1}{2} = n$	$8\frac{2}{3} - 5\frac{7}{8} = n$	$3\frac{1}{12} - 1\frac{3}{4} = n$
2.	$15\frac{1}{3} - 7\frac{2}{3} = n$	$14\frac{5}{8} - 6\frac{9}{10} = n$	$11\frac{7}{12} - 6\frac{3}{4} = n$	$12\frac{1}{2} - 5\frac{1}{6} = n$

	a	b	c	d	e	f	g	h
3.	$\begin{array}{r} 5\frac{1}{5} \\ -3\frac{2}{7} \\ \hline \end{array}$	$\begin{array}{r} 2\frac{5}{6} \\ -1\frac{7}{8} \\ \hline \end{array}$	$\begin{array}{r} 18\frac{5}{12} \\ -9\frac{7}{8} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{3}{4} \\ -6\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 6\frac{3}{7} \\ -3\frac{5}{7} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{2}{9} \\ -4\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 11\frac{3}{8} \\ -7\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{4}{5} \\ -6\frac{2}{15} \\ \hline \end{array}$
4.	$\begin{array}{r} 15 \\ -4\frac{7}{10} \\ \hline \end{array}$	$\begin{array}{r} 16\frac{1}{9} \\ -8\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 7\frac{5}{6} \\ -5\frac{11}{12} \\ \hline \end{array}$	$\begin{array}{r} 3\frac{1}{4} \\ -2\frac{5}{8} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{2}{11} \\ -4\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 12\frac{3}{4} \\ -7\frac{5}{12} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{1}{6} \\ -8\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{5}{6} \\ -8\frac{1}{6} \\ \hline \end{array}$
5.	$\begin{array}{r} 14\frac{4}{9} \\ -8\frac{7}{12} \\ \hline \end{array}$	$\begin{array}{r} 12\frac{2}{5} \\ -11\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{1}{2} \\ -3\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{2}{5} \\ -8\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 11\frac{1}{8} \\ -5\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{7}{8} \\ -8\frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{2}{7} \\ -9\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 13\frac{1}{2} \\ -5\frac{2}{3} \\ \hline \end{array}$

PRACTICE SET 25, Subtraction (page 265)

	a	b	c	d	e	f	g
1.	0.9 − 0.3	0.5 − 0.3	0.7 − 0.2	0.95 − 0.94	0.86 − 0.32	0.94 − 0.52	0.67 − 0.43
2.	6.8 − 6.2	6.7 − 1.4	7.9 − 1.7	8.76 − 1.60	8.95 − 4.82	8.69 − 8.30	8.97 − 6.44
3.	3.798 − 2.727	2.846 − 1.741	4.395 − 3.352	3.849 − 1.517	47.98 − 25.13	95.96 − 31.65	65.87 − 35.65

PRACTICE SET 26, Subtraction (page 266)

1.	8.0 − 5.2	7.4 − 3.9	8.2 − 4.6	9.13 − 5.78	8.70 − 5.41	6.20 − 4.57	9.32 − 2.57
2.	91.05 − 42.49	60.13 − 38.67	83.53 − 33.96	72.52 − 58.39	92.44 − 19.96	94.93 − 35.79	80.51 − 63.68
3.	7.537 − 5.899	8.682 − 2.977	6.315 − 5.857	4.095 − 3.694	3.607 − 2.652	9.504 − 7.595	7.837 − 2.848
4.	4.825 − 2.936	6.740 − 1.684	5.381 − 2.764	8.544 − 1.847	7.961 − 7.883	6.521 − 2.749	8.968 − 5.671

PRACTICE SET 27, Multiplication (page 50)

1.	27 × 2	84 × 5	76 × 8	78 × 6	58 × 2	98 × 8	94 × 6
2.	952 × 3	871 × 7	795 × 5	708 × 3	698 × 7	319 × 9	346 × 4
3.	4,316 × 3	6,325 × 6	4,069 × 2	2,567 × 8	1,326 × 9	7,324 × 7	1,326 × 5
4.	9,097 × 4	8,143 × 8	5,456 × 7	2,185 × 4	5,798 × 9	9,764 × 6	7,845 × 9

PRACTICE SET 28, Multiplication (page 53)

	a	b	c	d	e	f	g
1.	76 × 34	52 × 78	35 × 29	87 × 35	27 × 96	49 × 47	64 × 57
2.	604 × 78	584 × 39	769 × 47	947 × 83	409 × 57	760 × 90	269 × 75
3.	$6.39 × 65	$5.07 × 94	$9.48 × 76	$8.56 × 49	$7.20 × 83	$8.09 × 78	$6.94 × 85

PRACTICE SET 29, Multiplication (page 55)

1.	498 × 573	858 × 796	697 × 284	572 × 834	383 × 796	749 × 653	684 × 749
2.	573 × 709	640 × 769	905 × 398	728 × 605	653 × 370	900 × 407	760 × 800
3.	694 × 578	837 × 426	549 × 937	285 × 768	986 × 437	736 × 509	607 × 805

PRACTICE SET 30, Multiplication (page 59)

	a	b	c	d	e	f
1.	58 × 67	43 × 21	75 × 94	88 × 60	26 × 98	73 × 85
2.	384 × 536	507 × 709	813 × 678	749 × 395	686 × 427	457 × 239
3.	$8.54 × 67	$9.23 × 574	$14.34 × 78	$91.53 × 842	$37.25 × 658	$52.48 × 408
4.	5,472 × 3,689	1,036 × 4,751	8,925 × 7,604	3,176 × 9,258	8,942 × 4,376	12,845 × 6,497

	a	b	c
5.	$72 \times 43 = n$	$n = 56 \times 325$	$742 \times 839 = n$
6.	$n = 475 \times 682$	$670 \times 523 = n$	$437 \times 988 = n$

PRACTICE SET 31, Multiplication (page 69)

	a	b	c	d	e
1.	6 hr. 4 min. × 4	2 lb. 1 oz. × 8	5 ft. 3 in. × 3	3 yd. 1 ft. × 3	3 lb. 4 oz. × 4
2.	3 hr. 20 min. × 3	3 qt. 1 pt. × 7	4 ft. 5 in. × 3	4 min. 12 sec. × 6	5 yd. 1 ft. × 2
3.	2 gal. 3 qt. × 5	4 yd. 2 ft. × 6	6 lb. 3 oz. × 8	12 min. 45 sec. × 3	4 qt. 1 pt. × 6

PRACTICE SET 32, Multiplication (page 298)

	a	b	c	d
1.	$\frac{5}{6} \times \frac{1}{3} = n$	$\frac{1}{4} \times \frac{1}{2} = n$	$\frac{3}{7} \times \frac{2}{5} = n$	$\frac{1}{9} \times \frac{2}{3} = n$
2.	$\frac{3}{4} \times \frac{1}{12} = n$	$\frac{1}{3} \times \frac{5}{8} = n$	$\frac{3}{5} \times \frac{5}{4} = n$	$\frac{3}{8} \times \frac{5}{2} = n$
3.	$\frac{2}{7} \times \frac{2}{3} = n$	$\frac{9}{2} \times \frac{1}{4} = n$	$\frac{5}{7} \times \frac{4}{3} = n$	$\frac{1}{6} \times \frac{2}{3} = n$
4.	$\frac{1}{2} \times \frac{3}{4} = n$	$\frac{12}{5} \times \frac{1}{3} = n$	$\frac{3}{10} \times \frac{5}{3} = n$	$\frac{4}{5} \times \frac{3}{4} = n$

PRACTICE SET 33, Multiplication (page 299)

1.	$5 \times \frac{3}{4} = n$	$3 \times \frac{3}{7} = n$	$6 \times \frac{1}{8} = n$	$8 \times \frac{2}{3} = n$
2.	$7 \times \frac{1}{2} = n$	$9 \times \frac{4}{5} = n$	$12 \times \frac{1}{10} = n$	$4 \times \frac{3}{5} = n$
3.	$18 \times \frac{1}{3} = n$	$5 \times \frac{7}{12} = n$	$3 \times \frac{6}{7} = n$	$4 \times \frac{5}{8} = n$
4.	$16 \times \frac{2}{5} = n$	$7 \times \frac{4}{9} = n$	$2 \times \frac{5}{9} = n$	$9 \times \frac{4}{11} = n$

PRACTICE SET 34, Multiplication (page 300)

1.	$\frac{3}{5} \times 12 = n$	$\frac{5}{9} \times 7 = n$	$\frac{4}{7} \times 24 = n$	$\frac{2}{5} \times 10 = n$
2.	$\frac{3}{4} \times 15 = n$	$\frac{1}{2} \times 35 = n$	$\frac{7}{8} \times 16 = n$	$\frac{2}{3} \times 25 = n$
3.	$\frac{5}{6} \times 18 = n$	$\frac{3}{7} \times 20 = n$	$\frac{4}{9} \times 36 = n$	$\frac{7}{8} \times 3 = n$
4.	$\frac{2}{3} \times 9 = n$	$\frac{3}{5} \times 75 = n$	$\frac{1}{7} \times 14 = n$	$\frac{1}{3} \times 6 = n$

PRACTICE SET 35, Multiplication (page 303)

	a	b	c	d
1.	$2\frac{1}{2} \times 3\frac{1}{3} = n$	$4\frac{3}{5} \times 1\frac{1}{4} = n$	$6\frac{1}{8} \times 4 = n$	$5\frac{2}{5} \times 2\frac{1}{3} = n$
2.	$4\frac{3}{4} \times 3\frac{1}{2} = n$	$1\frac{1}{2} \times 2\frac{4}{9} = n$	$\frac{4}{5} \times 1\frac{1}{6} = n$	$4\frac{2}{3} \times 6\frac{1}{3} = n$
3.	$3\frac{1}{5} \times \frac{5}{7} = n$	$5 \times 4\frac{1}{4} = n$	$3\frac{1}{4} \times 5\frac{1}{4} = n$	$2\frac{1}{5} \times 3\frac{2}{7} = n$
4.	$2\frac{3}{4} \times 6\frac{1}{3} = n$	$3\frac{2}{3} \times 2\frac{3}{7} = n$	$1\frac{3}{8} \times 4\frac{1}{5} = n$	$1\frac{3}{4} \times 1\frac{1}{2} = n$
5.	$3\frac{1}{5} \times 4\frac{1}{4} = n$	$1\frac{1}{4} \times 1\frac{7}{8} = n$	$2\frac{5}{6} \times 2\frac{1}{3} = n$	$7\frac{2}{3} \times 3\frac{1}{2} = n$

PRACTICE SET 36, Division (page 105)

	a	b	c	d	e	f
1.	$7\overline{)455}$	$6\overline{)247}$	$4\overline{)631}$	$3\overline{)893}$	$5\overline{)255}$	$8\overline{)249}$
2.	$8\overline{)428}$	$7\overline{)473}$	$3\overline{)585}$	$2\overline{)167}$	$9\overline{)504}$	$6\overline{)473}$
3.	$5\overline{)1{,}933}$	$6\overline{)5{,}627}$	$9\overline{)6{,}708}$	$8\overline{)3{,}831}$	$6\overline{)5{,}140}$	$9\overline{)4{,}374}$
4.	$6\overline{)3{,}831}$	$9\overline{)7{,}105}$	$8\overline{)3{,}980}$	$3\overline{)1{,}374}$	$5\overline{)2{,}391}$	$4\overline{)3{,}885}$
5.	$7\overline{)3{,}429}$	$4\overline{)2{,}328}$	$2\overline{)1{,}173}$	$6\overline{)5{,}060}$	$8\overline{)4{,}626}$	$9\overline{)1{,}512}$

PRACTICE SET 37, Division (page 106)

	a	b	c	d
1.	$5\overline{)\text{20 min. 10 sec.}}$	$3\overline{)\text{6 gal. 3 qt.}}$	$2\overline{)\text{8 ft. 4 in.}}$	$4\overline{)\text{12 lb. 8 oz.}}$
2.	$4\overline{)\text{5 ft. 8 in.}}$	$6\overline{)\text{7 hr. 6 min.}}$	$5\overline{)\text{11 yd. 2 ft.}}$	$7\overline{)\text{8 lb. 5 oz.}}$
3.	$2\overline{)\text{7 gal. 2 qt.}}$	$3\overline{)\text{4 qt. 1 pt.}}$	$8\overline{)\text{10 min. 40 sec.}}$	$4\overline{)\text{1 ft. 4 in.}}$
4.	$3\overline{)\text{1 qt. 1 pt.}}$	$5\overline{)\text{1 hr. 15 min.}}$	$6\overline{)\text{2 ft. 6 in.}}$	$2\overline{)\text{3 qt. 0 pt.}}$

PRACTICE SET 38, Division (page 108)

	a	b	c	d	e	f
1.	$4\overline{)1,288}$	$6\overline{)2,739}$	$2\overline{)134}$	$9\overline{)2,947}$	$3\overline{)2,834}$	$8\overline{)934}$
2.	$2\overline{)1,572}$	$9\overline{)575}$	$7\overline{)2,646}$	$8\overline{)4,168}$	$5\overline{)3,728}$	$6\overline{)4,508}$
3.	$8\overline{)\$68.99}$	$5\overline{)\$18.95}$	$9\overline{)\$69.20}$	$7\overline{)\$59.73}$	$3\overline{)\$11.76}$	$4\overline{)\$38.57}$

PRACTICE SET 39, Division (page 108)

	a	b	c
1.	$n = 2,415 \div 7$	$2,732 \div 4 = n$	$n = 762 \div 6$
2.	$113 = (n \times 3) + 2$	$2,309 = (n \times 5) + 4$	$1,578 = (n \times 2) + 0$
3.	$2,871 = (n \times 8) + 7$	$7,521 = (n \times 9) + 6$	$4,828 = (n \times 7) + 5$

PRACTICE SET 40, Division (page 138)

	a	b	c	d	e	f
1.	$63\overline{)5,292}$	$14\overline{)931}$	$44\overline{)3,241}$	$51\overline{)3,770}$	$83\overline{)5,440}$	$23\overline{)1,298}$
2.	$53\overline{)1,337}$	$74\overline{)2,682}$	$84\overline{)3,108}$	$22\overline{)2,012}$	$34\overline{)580}$	$64\overline{)2,403}$
3.	$43\overline{)975}$	$42\overline{)2,184}$	$94\overline{)7,217}$	$93\overline{)2,232}$	$72\overline{)6,047}$	$91\overline{)5,383}$
4.	$13\overline{)1,167}$	$62\overline{)1,238}$	$31\overline{)2,666}$	$82\overline{)1,494}$	$32\overline{)2,228}$	$54\overline{)2,484}$

PRACTICE SET 41, Division (page 140)

1.	$69\overline{)1,720}$	$57\overline{)4,332}$	$87\overline{)5,570}$	$97\overline{)3,244}$	$29\overline{)1,639}$	$79\overline{)3,871}$
2.	$48\overline{)2,160}$	$78\overline{)1,091}$	$17\overline{)583}$	$47\overline{)3,206}$	$89\overline{)2,279}$	$59\overline{)2,546}$
3.	$77\overline{)7,084}$	$38\overline{)2,394}$	$68\overline{)3,636}$	$98\overline{)7,519}$	$58\overline{)870}$	$39\overline{)1,632}$
4.	$37\overline{)1,094}$	$67\overline{)5,511}$	$49\overline{)1,130}$	$88\overline{)3,341}$	$99\overline{)8,712}$	$28\overline{)2,664}$

PRACTICE SET 42, Division (page 143)

	a	b	c	d	e	f
1.	$25\overline{)1{,}075}$	$65\overline{)5{,}390}$	$16\overline{)548}$	$45\overline{)4{,}227}$	$75\overline{)2{,}150}$	$66\overline{)3{,}234}$
2.	$86\overline{)3{,}083}$	$35\overline{)2{,}310}$	$95\overline{)6{,}478}$	$26\overline{)2{,}273}$	$36\overline{)1{,}761}$	$55\overline{)4{,}180}$
3.	$46\overline{)725}$	$15\overline{)1{,}095}$	$76\overline{)7{,}371}$	$56\overline{)1{,}904}$	$85\overline{)4{,}928}$	$96\overline{)3{,}103}$

PRACTICE SET 43, Division (page 144)

1.	$43\overline{)384}$	$91\overline{)286}$	$68\overline{)612}$	$29\overline{)209}$	$95\overline{)575}$	$26\overline{)382}$
2.	$17\overline{)590}$	$76\overline{)4{,}704}$	$52\overline{)936}$	$94\overline{)4{,}381}$	$56\overline{)2{,}991}$	$38\overline{)2{,}736}$
3.	$87\overline{)4{,}002}$	$13\overline{)885}$	$67\overline{)2{,}411}$	$89\overline{)5{,}147}$	$41\overline{)939}$	$37\overline{)1{,}838}$

	a	b	c
4.	$3{,}513 = (n \times 46) + 17$	$2{,}048 = (24 \times 85) + n$	$894 = (n \times 32) + 30$
5.	$85 \times n = 6{,}290$	$n \times 74 = 2{,}146$	$55 \times n = 5{,}335$

PRACTICE SET 44, Division (page 222)

	a	b	c	d	e
1.	$38\overline{)1{,}783}$	$45\overline{)2{,}733}$	$83\overline{)5{,}810}$	$54\overline{)4{,}229}$	$79\overline{)1{,}643}$
2.	$57\overline{)17{,}328}$	$21\overline{)9{,}590}$	$17\overline{)13{,}413}$	$63\overline{)42{,}258}$	$96\overline{)77{,}754}$
3.	$85\overline{)13{,}828}$	$46\overline{)17{,}957}$	$68\overline{)16{,}660}$	$89\overline{)62{,}837}$	$92\overline{)8{,}187}$
4.	$42\overline{)20{,}160}$	$56\overline{)6{,}132}$	$71\overline{)37{,}642}$	$66\overline{)53{,}986}$	$95\overline{)38{,}570}$

PRACTICE SET 45, Equivalent fractions (page 116)

	a	b	c	d	e	f
1.	$\frac{2}{3} = \frac{n}{6}$	$\frac{1}{2} = \frac{8}{n}$ 8	$\frac{n}{4} = \frac{4}{16}$	$\frac{15}{24} = \frac{n}{8}$ 13	$\frac{3}{4} = \frac{12}{n}$	$\frac{7}{15} = \frac{21}{n}$
2.	$\frac{6}{8} = \frac{n}{4}$	$\frac{5}{11} = \frac{15}{n}$ 3	$\frac{3}{21} = \frac{1}{n}$	$\frac{18}{20} = \frac{n}{10}$ 5 18	$\frac{4}{5} = \frac{n}{25}$	$\frac{3}{7} = \frac{12}{n}$
3.	$\frac{4}{9} = \frac{8}{n}$	$\frac{n}{6} = \frac{3}{18}$	$\frac{8}{24} = \frac{n}{3}$	$\frac{9}{30} = \frac{3}{n}$ $\frac{7}{126}$	$\frac{1}{3} = \frac{6}{n}$	$\frac{7}{9} = \frac{n}{27}$
4.	$\frac{20}{n} = \frac{4}{7}$	$\frac{1}{n} = \frac{6}{48}$	$\frac{12}{20} = \frac{3}{n}$	$\frac{n}{12} = \frac{21}{36}$ 3 12 12	$\frac{6}{n} = \frac{3}{7}$	$\frac{3}{4} = \frac{27}{n}$

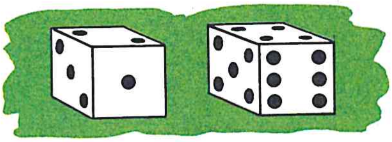
INQUIRY

Probability

You have often heard someone say, "The *chances* are that it will rain tomorrow," or "It is *probable* that our team will win the game."

In mathematics the study of the chances of a certain event happening is called **probability.** Probability is the measure of how often a given event can occur.

1. Shown in the picture below is a pair of dice.



If you were to pick up one die and examine it, you would see that it has 6 sides. Each side has a different number of dots on it, from 1 to 6. When you toss a die, it can land with only 1 of its 6 sides up. So a die can land in $\underline{?}$ of $\underline{?}$ possible ways.

2. The *sample space* for tossing a die is the set of all possible ways the die can land. In this case it has $\underline{?}$ members.

Copy and complete the sample space shown below for tossing a die. Each number in the sample space is the number of dots on the side that lands face up.

$\{1, 2, 3, \underline{?}, \underline{?}, \underline{?}\}$

3. How many members are there in the sample space for tossing a die? In how many ways can 4 dots land face up on a die? Then the probability of a die landing with 4 dots up is 1 out of 6, or $\frac{1}{6}$.

The probability of a die landing with 6 dots up is $\underline{?}$ out of $\underline{?}$, or $\underline{?}$.

4. To "toss a 2" means that you must throw the die so that 2 dots land up. What is the probability that you will toss a 2?

5. What is the probability that you will toss a 1? a 3? a 4?

6. Suppose in throwing a die you toss a 4. When you throw again, do you think the probability of tossing a 4 is still $\frac{1}{6}$?

- How many members are in the sample space for each toss? Is 4 one of these members? Then no matter what you tossed the first time, is the probability of tossing a 4 still 1 out of 6, or $\frac{1}{6}$?
- Assume the die you are using is a "fair" die. You have just tossed a 4 three times in a row. What is the probability that you will toss a 4 on the next toss?

7. Try an experiment. Toss a die 60 times and keep a record of how often you toss a 4. How many times do you expect to toss a 4?

More probability with dice

1. On page 338, you learned that the probability of tossing a 2 is 1 out of 6, or $\frac{1}{6}$. The probability of tossing a 3 is also 1 out of 6, or $\frac{1}{6}$. But suppose you are asked for the probability of tossing *either a* 2 *or a* 3.

▶ How many members are there in the sample space?

▶ You will be successful if you toss a 2. You will also be successful if you toss a 3. So you can be successful in how many ways?

▶ So the probability of tossing a 2 or a 3 is 2 out of 6, or $\underline{?}$.

2. What is the probability of tossing either a 3 or a 5? of tossing either a 1 or a 6?

3. Suppose you are asked to find the probability of tossing either a 4, a 5, or a 6.

▶ How many members are there in the sample space?

▶ In how many ways can you be successful?

▶ Then the probability of tossing either a 4, a 5, or a 6 is 3 out of 6, or $\underline{?}$.

4. What is the probability of tossing either a 1, a 3, or a 5?

5. What is the probability of tossing either a 1, a 2, a 3, a 4, or a 5?

6. What is the probability of *not* tossing a 6?

7. What is the probability of not tossing a 1? a 2? a 4?

8. How many even numbers of dots are in the sample space for one die? Then the probability of tossing an even number of dots is 3 out of 6, or $\underline{?}$.

9. What is the probability of tossing an odd number of dots?

10. How many members of the sample space are not even numbers? Then what is the probability of *not* tossing an even number of dots?

11. What is the probability of not tossing an odd number of dots?

12. How many members in the sample space are either even numbers or odd numbers? Then the probability that you will toss either an even number of dots or an odd number of dots is 6 out of 6, or $\underline{?}$.

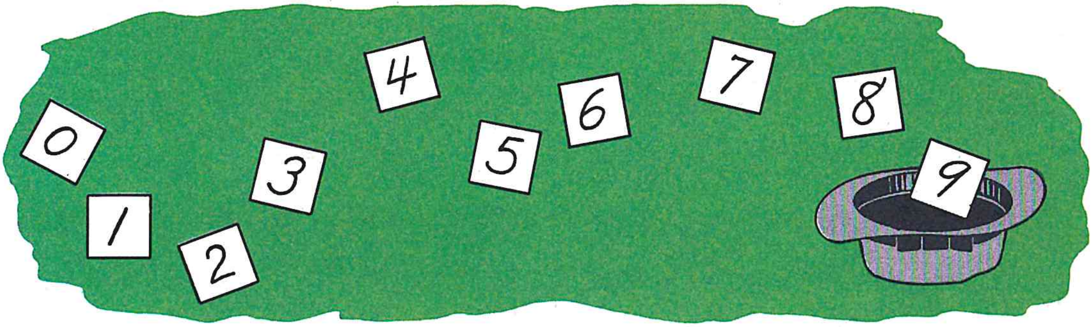
Describe in your own words what a probability of 1 means.

13. How many members in the sample space are neither even numbers nor odd numbers? Then the probability that you will toss neither an even number of dots nor an odd number of dots is 0 out of 6, or $\underline{?}$.

Describe in your own words what a probability of 0 means.

INQUIRY

More probability



1. Suppose you have 10 slips of paper of the same size and shape. On the slips are shown the digits from 0 through 9, one digit on each slip. The slips are placed in a hat and a student is blindfolded and asked to draw out one slip. What do you think is the probability that he will draw a slip showing the digit 5?

▶ The sample space for this situation has 10 members. What are they?

▶ In how many ways can the student be successful?

▶ So the probability of drawing a slip showing the digit 5 is 1 out of 10, or _?_.

2. What is the probability that the student will draw a slip showing the digit 1? the digit 4? the digit 7? the digit 9?

3. When a student draws just one slip, what is the probability that he will draw a slip showing either a 5 or a 9?

4. What is the probability that a student will draw a slip showing either a 2 or a 4?

5. What is the probability that a student will draw a slip showing either a 1, a 3, or a 7?

6. What is the probability that a student will draw a slip showing a numeral for an even number?

7. What is the probability that a student will draw a slip showing a numeral for an odd number?

8. Try an experiment. Make ten slips of paper like those used in Ex. 1. Then perform the drawing operation ten times and record how often you drew a slip showing the digit 5. How many times would you expect to draw a 5?

9. Repeat the experiment in Ex. 8, but perform the drawing fifty times. How many times would you expect to draw a slip showing a 5? How many times did you?

Bar arithmetic

1. Look at the sets shown below.

$$\bar{0} = \{0, 5, 10, 15, \cdots\}$$
$$\bar{1} = \{1, 6, 11, 16, \cdots\}$$
$$\bar{2} = \{2, 7, 12, 17, \cdots\}$$
$$\bar{3} = \{3, 8, 13, 18, \cdots\}$$
$$\bar{4} = \{4, 9, 14, 19, \cdots\}$$

The first set is named by the symbol $\bar{0}$. This symbol is read as *zero bar* and means that the first number in the set is 0.

- In $\bar{0}$, each number is how much more than the number before it?
- In $\bar{0}$, each number is a multiple of what number?
- What is the next member of $\bar{0}$ after 15?
- Is 35 a member of $\bar{0}$?

2. The next set is named by $\bar{1}$, or *one bar*. It begins with 1.

- Each number in $\bar{1}$ is how much more than the number before it?
- What is the next member of $\bar{1}$ after 16?
- Is each member of $\bar{1}$ a multiple of 5 plus 1 more?
- Show that these numbers are members of $\bar{1}$.

$$31 = (5 \times 6) + \underline{?}$$
$$46 = (5 \times \underline{?}) + 1$$

3. Look at $\bar{2}$. Is each member of $\bar{2}$ a multiple of 5 plus 2 more? Is 37 a member of $\bar{2}$? Is 72?

4. Each member of $\bar{3}$ is a multiple of 5 plus what number? Show that 63 is a member of $\bar{3}$.

$$63 = (5 \times \underline{?}) + \underline{?}$$

5. Each member of $\bar{4}$ is a multiple of 5 plus what number? Show that 79 is a member of $\bar{4}$.

$$79 = (5 \times \underline{?}) + \underline{?}$$

The sets of numbers shown in Ex. 1 form a new number system called **the bar-five system.** This system has five numbers which are named by the numerals $\bar{0}$, $\bar{1}$, $\bar{2}$, $\bar{3}$, and $\bar{4}$.

6. Add a number in $\bar{1}$ to a number in $\bar{2}$. Is the answer a number in $\bar{3}$?

- Is this number sentence true?

$$\bar{1} + \bar{2} = \bar{3}$$

7. Add a number in $\bar{2}$ to a number in $\bar{3}$. Is the answer a number in $\bar{0}$?

- Is this number sentence true?

$$\bar{2} + \bar{3} = \bar{0}$$

— Write the bar-five numerals for the sum. To find the sum of $\bar{3}$ and $\bar{4}$, for example, first find the sum of some number in $\bar{3}$ and some number in $\bar{4}$. Then find the set in which the sum can be found.

	a	b
8.	$\bar{3} + \bar{4} = n$	$\bar{0} + \bar{4} = n$
9.	$\bar{2} + \bar{2} = n$	$\bar{4} + \bar{2} = n$
10.	$\bar{4} + \bar{3} = n$	$\bar{4} + \bar{1} = n$

INQUIRY

More bar arithmetic

1. The table shown below is an incomplete addition table for the bar-five number system.

+	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
$\bar{0}$	?	?	?	$\bar{3}$	?
$\bar{1}$	?	$\bar{2}$	?	?	?
$\bar{2}$	?	?	?	?	$\bar{1}$
$\bar{3}$	?	?	$\bar{0}$	?	?
$\bar{4}$	?	?	?	$\bar{2}$	?

- Find the box where the row for $\bar{1}$ and the column for $\bar{1}$ meet. That box should show the sum of $\bar{1}$ and $\bar{1}$. Does it?
- Copy and complete the table.

2. What is the sum of 1 and 3? What is the sum of $\bar{1}$ and $\bar{3}$?

3. What is the sum of 3 and 2? What is the sum of $\bar{3}$ and $\bar{2}$?

— Find the sums in the following pairs of number sentences.

4. $\bar{3}+\bar{4}=n$ $\quad$ $\bar{4}+\bar{3}=n$

5. $\bar{0}+\bar{2}=n$ $\quad$ $\bar{2}+\bar{0}=n$

6. $\bar{1}+\bar{4}=n$ $\quad$ $\bar{4}+\bar{1}=n$

7. $(\bar{1}+\bar{3})+\bar{2}=n$
$\bar{1}+(\bar{3}+\bar{2})=n$

8. $(\bar{4}+\bar{2})+\bar{2}=n$
$\bar{4}+(\bar{2}+\bar{2})=n$

9. Do the commutative and associative properties apply to addition in the bar-five system?

10. Multiply a number in $\bar{3}$ by a number in $\bar{2}$. Is the product in $\bar{1}$?

Multiply another number in $\bar{3}$ by a number in $\bar{2}$. In which set is the product? Is this sentence true?

$$\bar{2}\times\bar{3}=\bar{1}$$

11. Multiply a number in $\bar{4}$ by a number in $\bar{1}$. In which set is the product? Is this sentence true?

$$\bar{1}\times\bar{4}=\bar{4}$$

12. Copy and complete the multiplication table shown below.

×	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
$\bar{0}$	?	?	$\bar{0}$	?	?
$\bar{1}$	?	?	?	?	$\bar{4}$
$\bar{2}$	?	?	?	$\bar{1}$	?
$\bar{3}$	$\bar{0}$	?	?	?	?
$\bar{4}$	?	$\bar{4}$	?	?	?

13. Does the multiplication table show the same product for $\bar{2}\times\bar{3}$ as it does for $\bar{3}\times\bar{2}$? Does it show that $\bar{4}\times\bar{1}$ is equal to $\bar{1}\times\bar{4}$?

Is multiplication commutative in the bar-five system?

14. Find the numbers that make these pairs of sentences true.

$(\bar{2}\times\bar{3})\times\bar{1}=n$ $\quad$ $\bar{2}\times(\bar{3}\times\bar{1})=n$
$(\bar{4}\times\bar{2})\times\bar{2}=n$ $\quad$ $\bar{4}\times(\bar{2}\times\bar{2})=n$

Is multiplication associative in the bar-five system?

— Use the addition table you made for Ex. 1 to find the other addends that make these sentences true.

	a	b
15.	$\bar{4} + n = \bar{2}$	$\bar{0} + n = \bar{3}$
16.	$\bar{2} + n = \bar{1}$	$\bar{2} + n = \bar{4}$
17.	$\bar{1} - \bar{1} = n$	$\bar{1} - \bar{0} = n$
18.	$\bar{0} - \bar{1} = n$	$\bar{1} - \bar{3} = n$

19. When subtracting whole numbers, can you subtract 3 from 1 and get a whole number as an answer?

When subtracting bar-five numbers, can you subtract $\bar{3}$ from $\bar{1}$ and get a bar-five number as an answer?

Can you subtract any bar-five number from any bar-five number and get a bar-five number as an answer?

— Use the multiplication table you made to find the other factors that make these sentences true.

20.	$\bar{3} \times n = \bar{1}$	$\bar{2} \times n = \bar{3}$
21.	$\bar{1} \times n = \bar{1}$	$\bar{3} \times n = \bar{3}$
22.	$\bar{4} \div \bar{2} = \bar{n}$	$\bar{0} \div \bar{4} = n$
23.	$\bar{2} \div \bar{2} = \bar{n}$	$\bar{1} \div \bar{3} = n$

24. When dividing whole numbers, can you divide 1 by 3 and get a whole number as an answer?

Can you divide $\bar{1}$ by $\bar{3}$ and get a bar-five number as an answer?

Can you divide any bar-five number by any bar-five number and get a bar-five number as an answer?

— Solve these sentences.

25. $(\bar{3} + \bar{4}) - 1 = n$

26. $(\bar{2} \times \bar{2}) + \bar{0} = n$

27. $(\bar{0} \times \bar{1}) - \bar{2} = n$

28. $\bar{4} \times (\bar{3} + \bar{3}) = n$

29. $(\bar{2} \div \bar{4}) + (\bar{3} \times \bar{4}) = n$

30. $(\bar{3} \times \bar{4}) \div (\bar{4} \times \bar{3}) = n$

31. $(\bar{2} - \bar{4}) \times (\bar{2} - \bar{4}) = n$

32. $(\bar{1} + \bar{1}) \div (\bar{4} + \bar{4}) = n$

33. Suppose you wish to make a bar-four number system. What four numerals would you use?

- Each number in $\bar{0}$ will be a multiple of what number?
- Each number in $\bar{1}$ will be a multiple of 4 plus what number?
- Each number in $\bar{2}$ will be a multiple of 4 plus what number?
- Each number in $\bar{3}$ will be a multiple of 4 plus what number?

34. Show the numbers missing from these sets in the bar-four system.

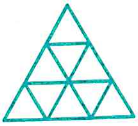
$\bar{0} = \{0, 4, \underline{?}, \underline{?}, \ldots\}$
$\bar{1} = \{1, \underline{?}, \underline{?}, \underline{?}, \ldots\}$
$\bar{2} = \{2, \underline{?}, \underline{?}, \underline{?}, \ldots\}$
$\bar{3} = \{3, \underline{?}, \underline{?}, \underline{?}, \ldots\}$

Each number is how much more than the number named before it?

35. Make an addition table and a multiplication table for the bar-four number system.

INQUIRY

Geometric patterns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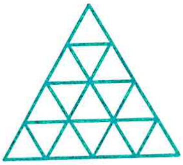
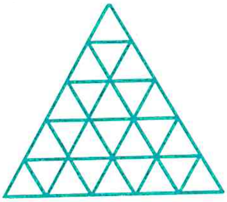
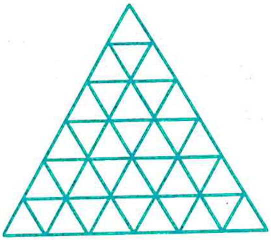

B


C


D

1. Each side of triangle A has been separated into 3 segments of equal length. The interior of triangle A has been separated into how many smaller triangles?

2. Each side of triangle B has been separated into how many segments of equal length? Into how many smaller triangles has the interior of triangle B been separated?

3. Copy and complete the following table.

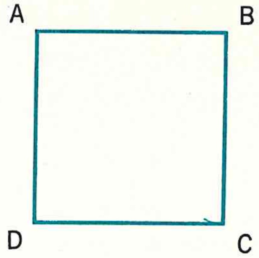
TRIANGLE	NUMBER OF SEGMENTS INTO WHICH EACH SIDE HAS BEEN SEPARATED	NUMBER OF TRIANGLES INTO WHICH THE INTERIOR HAS BEEN SEPARATED
A	3	9
B	4	?
C	5	?
D	6	?

4. Examine the table you completed for Ex. 3 to find a pattern.

- In triangle A, each side has been separated into 3 segments. The number of triangles in the interior is equal to the product of 3 and what number?
- In triangle B, each side has been separated into ? segments. The number of triangles in the interior is equal to the product of 4 and ?.
- Does a similar pattern hold for triangles C and D?

5. If each side of a triangle is separated into 10 segments of equal length, and the points are connected by line segments, how many smaller triangles will be formed in the interior?

More geometric patterns

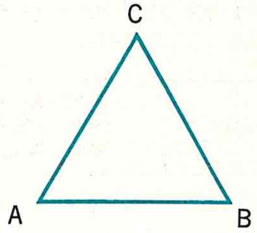


1. What name can be given to figure *ABCD* shown at the right? Make a copy of this figure and then do the following.

▶ Beginning with side *AB*, extend each side of the square $\frac{1}{2}$ inch. That is, extend side *AB* $\frac{1}{2}$ inch to the right, side *BC* $\frac{1}{2}$ inch down, side *CD* $\frac{1}{2}$ inch to the left, and side *DA* $\frac{1}{2}$ inch up.

▶ Connect the ends of the extended line segments with each other. What geometric figure is formed?

2. Using the figure that you drew for Ex. 1, repeat the same procedure. That is, extend each of the sides of the new square $\frac{1}{2}$ inch in order. What figure is formed?



3. Measure the sides of the triangle shown at the right. Are they equal in length?

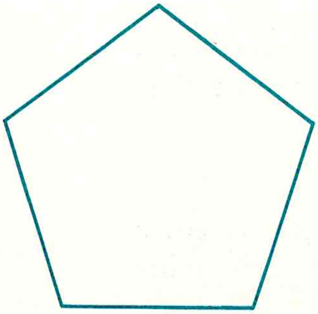
▶ Make a copy of triangle *ABC*.

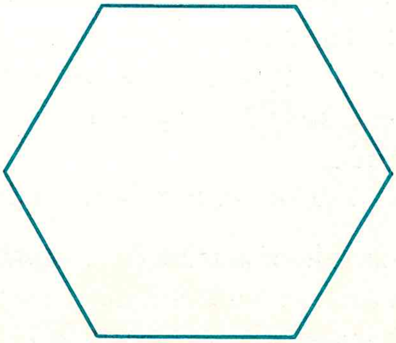
▶ Extend each side $\frac{1}{2}$ inch; side *AB* should be extended $\frac{1}{2}$ inch up, side *BC* $\frac{1}{2}$ inch down, and side *CA* $\frac{1}{2}$ inch to the left.

▶ Connect the ends of the extended segments. What figure have you formed?

▶ Measure the sides of the new figure. Are they equal in length?

4. Make copies of the figures shown below. Then extend the sides as you did in the exercises above and connect the ends of the new segments. What figure is formed in each case?





INQUIRY

Functions

1. What was done to each number named in column x below to get the number named in column y?

x	y
2	5
4	7
5	8
6	9

The number pairs in the table above can be listed like this.

$$\{(2, 5), (4, 7), (5, 8), (6, 9)\}$$

This is called a set of *ordered pairs* because the order of the numbers in each pair is important.

Suppose x stands for the first number in each pair. Then does $x + 3$ stand for the second number?

A set of ordered pairs where the first number in each pair is matched with one and only one second number is called a **function.**

2. Often a rule can be written to show a function. The rule for the set of ordered pairs in Ex. 1 could be written like this.

$$y = x + 3$$

This rule can be read as, "The second number is equal to the first number plus 3."

If x is equal to 7, then the second number, or y, is equal to $7 + 3$, or 10.

What is y when x equals 3? 9?

3. Look at the following set.

$$\{(4, 6), (7, 9), (11, 13), (0, 2)\}$$

- If x stands for the first number in each pair, what would stand for the second number?
- Can the following be used as a rule for this function?

$$y = x + 2$$

- What is y when x is 5? 10?

4. Look for a pattern in this set of ordered pairs.

$$\{(2, 6), (3, 9), (5, 15), (7, 21)\}$$

- Which of the following is the rule for this function?

$$y = x + 4 \qquad y = x \times 3$$

- In the rule, does x stand for the first number and y for the second? What is y when x is 4?

— Write a rule for each function shown below.

5. $\{(4, 6), (7, 9), (11, 13)\}$

6. $\{(7, 2), (11, 6), (10, 5)\}$

7. $\{(2, 4), (3, 6), (4, 8)\}$

8. $\{(1, 6), (2, 7), (5, 10)\}$

9. $\{(0, 0), (3, 12), (4, 16)\}$

10. $\{(3, 3), (5, 5), (7, 7)\}$

11. $\{(2, 5), (3, 7), (4, 9)\}$

12. $\{(2, 4), (3, 9), (4, 16)\}$

Graphing functions

1. If the rule for a function is $y = x + 6$, which of the following ordered pairs is a member of this function?

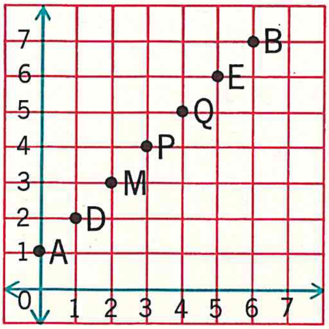
(1, 6) (11, 5) (6, 1) (5, 11)

2. The rule for another function is $y = x + 1$. The ordered pair (0, 1) is a member of this function. What numbers are missing in the following pairs if they also are members of the function?

(3, $\underline{?}$) (5, $\underline{?}$) (2, $\underline{?}$)

3. A graph of the function in Ex. 2 is shown below. Each point shown matches a member of the function.

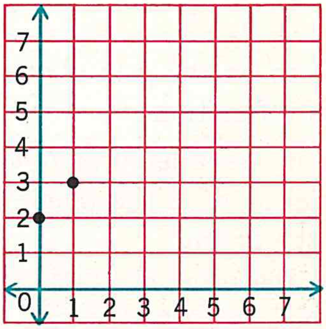
- To find the point for (3, 4), start at 0 and move 3 spaces *to the right* and then *up* 4 spaces. What letter names this point?
- Point *E* is 5 spaces to the right and up 6 spaces. What ordered pair matches this point?
- For each of the other points shown on the graph, name the ordered pair that matches it.



4. Use $y = x + 2$ as the rule for a function and find the following members of the function.

$\{(0, \underline{?}), (1, \underline{?}), (2, \underline{?}),$
$(3, \underline{?}), (4, \underline{?}), (5, \underline{?})\}$

Now copy the grid shown below and draw a graph of this function. The first two points of the graph are already shown for you.



5. Use $y = x$ as the rule for a function and find the following members of the function. Then draw a graph of the function.

$\{(0, \underline{?}), (1, \underline{?}), (2, \underline{?}),$
$(3, \underline{?}), (4, \underline{?}), (5, \underline{?}),$
$(6, \underline{?}), (7, \underline{?})\}$

— Shown below are rules for three different functions. On a 7-by-7 grid, draw a graph of each.

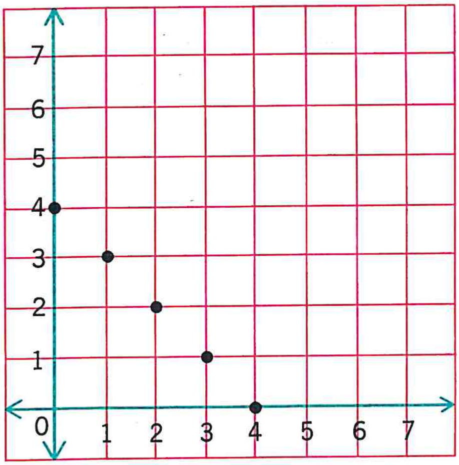
6. $y = x + 3$

7. $y = x + x$

8. $y = 7 - x$

INQUIRY

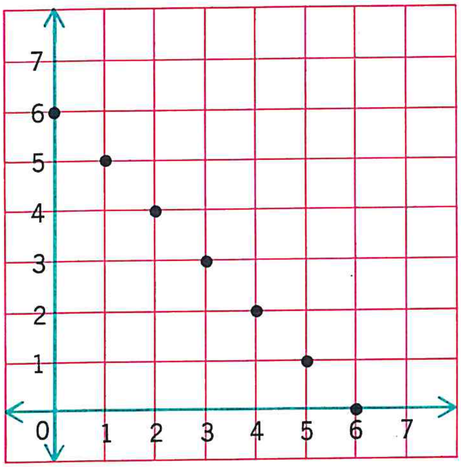
More about graphs of functions



1. Shown on the grid above is the graph of a function.

- Name the ordered pairs that are matched with the points shown on the graph.
- Use the ordered pairs that you named to write the rule for the function graphed above.

2. Shown on the grid below is the graph of a function. Write the rule for this function.



3. Use the edge of your ruler to see if all of the points of the graph in Ex. 1 lie along a line. Do all of the points of the graph in Ex. 2 lie along a line?

4. The rule for another function is $y = 8 - x$. Find the following members of the function.

$\{(2, \underline{?}), (3, \underline{?}), (4, \underline{?}), (5, \underline{?}), (6, \underline{?}), (7, \underline{?})\}$

- Draw a graph of this function.
- Do all of the points that you graphed lie along a line?

5. Use $y = x \times (x - 2)$ as the rule for a function. On a grid, draw a graph of this function.

- Do all of the points of your graph lie along a line?
- Can you always draw a line segment that includes all the members of a function?

— For each of the following rules for functions, draw a graph of the function.

6. $y = x + 4$

7. $y = (2 \times x) - 1$

8. $y = x \times x$

9. $y = (x \times x) + 1$

10. $y = (x \times x) - 1$

11. $y = x \times (x + 1)$

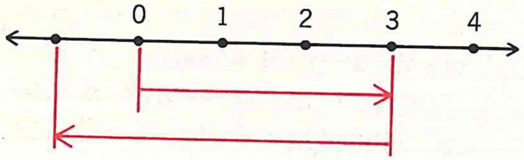
12. $y = x \times (x - 1)$

Positive and negative numbers

1. The fifth-grade class has a fund for parties. The fund at one time contained $3.

The class treasurer spent $4 for a party. To find how much money the fund had then, the teacher said that they had to subtract 4 from 3.

- To show the subtraction of some number from 3 on a number line, do you draw an arrow to the left from the point for 3?
- The teacher drew the following number-line diagram. She said that the arrows show the subtraction of 4 from 3. How many units long is the top arrow? the bottom arrow?



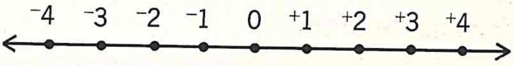
- Does the bottom arrow stop at a point 1 unit to the left of 0? Is there any whole number or fractional number that can be matched with this point?
- The number that can be matched with the point that is 1 unit to the left of 0 is called *negative one.* It is a member of the set of **negative numbers.** The class fund contained negative one dollar. The class owed the treasurer $\underline{?}$ dollar.

2. Since numbers shown at the left of 0 on the number line are called *negative numbers,* then numbers shown at the right of 0 can be called **positive numbers.**

- You can write the numeral for negative one like this: $^{-}1$. How would you write the numeral for negative two? for negative five?
- You can write the numeral for positive one like this: $^{+}1$. How would you write the numeral for positive two? for positive five?

3. Did the class fund contain more money when it had $3 or when it owed $1? Which is the greater number, $^{+}3$ or $^{-}1$?

4. Which point is farther to the right on the number line, the point for $^{+}1$ or the point for $^{-}1$? Then which number is the greater number?

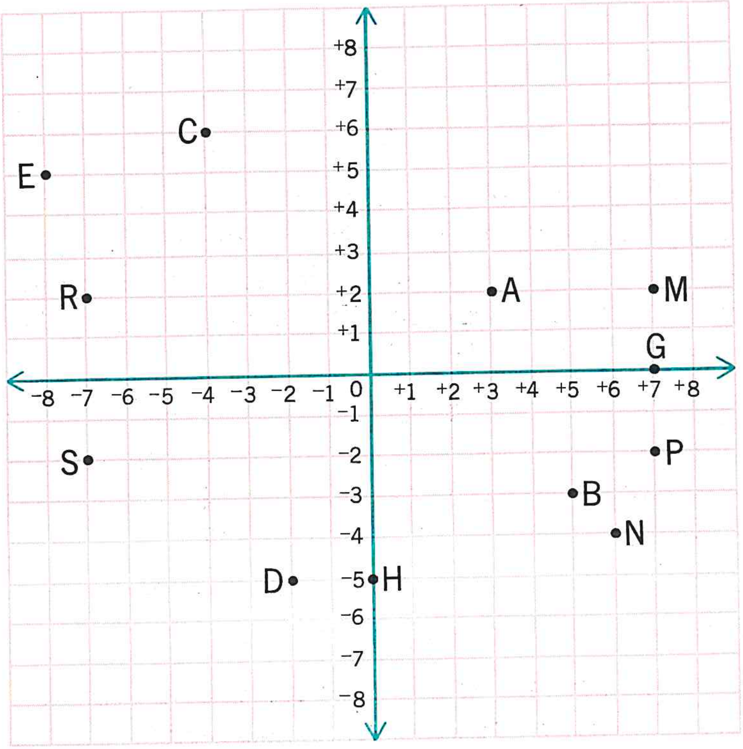


— Copy each of the following sentences and replace ● with > or < to make the sentence true.

	a	**b**	**c**
5.	$^{-}1$ ● $^{+}1$	$^{-}5$ ● $^{-}3$	$^{+}5$ ● $^{+}3$
6.	$^{-}2$ ● $^{+}1$	$^{+}2$ ● $^{+}6$	$^{-}2$ ● $^{-}6$
7.	$^{-}4$ ● 0	$^{-}5$ ● $^{+}1$	$^{-}8$ ● $^{-}1$
8.	$^{+}1$ ● 0	$^{+}8$ ● $^{-}1$	$^{-}7$ ● $^{+}1$

INQUIRY

Graphing ordered pairs of integers



1. Each of the number lines on the grid above shows a set of positive numbers, a set of negative numbers, and zero. Joining these sets forms a new set called the set of **integers.** Points for ordered pairs of integers are shown on this grid.

- To find the point for (⁺3, ⁺2), start at 0 and move 3 spaces *to the right* and then 2 spaces *up.* What letter names this point?
- To find the point for (⁺5, ⁻3), start at 0 and move 5 spaces *to the right* and 3 spaces *down.* What letter names this point?
- To find the point for (⁻4, ⁺6), start at 0 and move 4 spaces *to the left* and then 6 spaces *up.* What letter names this point?
- To find the point for (⁻2, ⁻5), start at 0 and move 2 spaces *to the left* and then 5 spaces *down.* What letter names this point?
- In what direction do you move if the first number is positive? In what direction do you move if the first number is negative?
- In what direction do you move if the second number is positive? In what direction do you move if the second number is negative?

— For each ordered pair of integers, find the matching point on the grid on page 350. Then write the letter that names the point.

	a	b	c	d
2.	$(^+7, ^+2)$	$(^+7, ^-2)$	$(^-7, ^+2)$	$(^-7, ^-2)$
3.	$(^+6, ^-4)$	$(^-8, ^+5)$	$(^+7, 0)$	$(0, ^-5)$

4. On a grid that is 22 spaces wide by 22 spaces high, graph in order from left to right the following ordered pairs of integers. As you draw a dot for each point, connect it to the dot for the point previously located by drawing a line segment. Begin by connecting the point for $(^+5, ^+1)$ to the point for $(^+3, ^+3)$.

$(^+5, ^+1)$	$(^+3, ^+3)$	$(^+1, ^+3)$	$(^-1, ^+2)$	$(^+1, ^+4)$
$(^+1, ^+8)$	$(^-1, ^+9)$	$(^-1, ^+2)$	$(^-7, ^+8)$	$(^-5, ^+8)$
$(^-1, ^+4)$	$(^-1, ^-10)$	$(^-3, ^-9)$	$(^-2, ^-8)$	$(^-1, ^-10)$
$(^+7, ^-10)$	$(^+5, ^-8)$	$(^+4, ^-8)$	$(^+2, ^-4)$	$(^+4, ^-4)$
$(^+6, ^-2)$	$(^+1, ^-2)$	$(^+1, ^-1)$	$(^+3, ^-1)$	$(^+5, ^+1)$

What kind of animal does your finished picture show?

5. Following the same directions as given for Ex. 4, graph the following ordered pairs and join the points with line segments.

$(^+1, ^+1)$	$(^+3, ^+3)$	$(^+5, ^+4)$	$(^+3, ^+4)$	$(^+5, ^+5)$
$(^+2, ^+8)$	$(0, ^+7)$	$(^-2, ^+5)$	$(^-3, ^+4)$	$(^-5, ^+4)$
$(^-7, ^+2)$	$(^-9, ^-2)$	$(^-9, ^-5)$	$(^-6, ^-6)$	$(^+1, ^-7)$
$(^-3, ^-5)$	$(^+1, ^-4)$	$(^-3, ^-4)$	$(^-1, ^-2)$	$(^+3, ^-5)$
$(^+5, ^-5)$	$(^+4, ^-4)$	$(^+5, ^-3)$	$(^+3, ^-3)$	$(^+1, ^+1)$

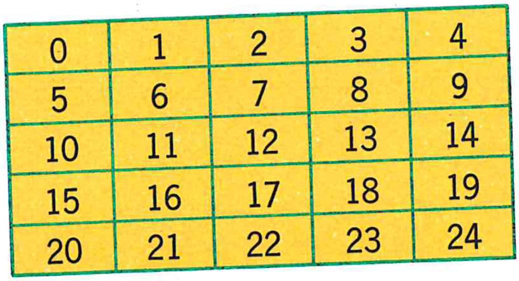
What kind of animal does your finished picture show?

6. Following the same directions as given for Ex. 4, graph the following ordered pairs and join the points with line segments.

$(^-4, ^+4)$	$(^-3, ^+2)$	$(0, ^+5)$	$(0, ^+6)$	$(^-1, ^+7)$
$(^-3, ^+8)$	$(^-4, ^+7)$	$(^-7, ^+8)$	$(^-8, ^+7)$	$(^-10, ^+4)$
$(^-10, 0)$	$(^-8, ^-3)$	$(^-6, ^-4)$	$(^-4, ^-2)$	$(^-4, ^-1)$
$(^-5, ^-1)$	$(^-5, ^-2)$	$(^-6, ^-3)$	$(^-8, ^-2)$	$(^-8, 0)$
$(^-6, ^+2)$	$(^-6, ^+1)$	$(^-4, ^-1)$	$(^-4, ^-3)$	$(^-6, ^-7)$
$(^-1, ^-7)$	$(^-2, ^-5)$	$(^-1, ^-4)$	$(^+4, ^-4)$	$(^+6, ^-5)$
$(^+7, ^-6)$	$(^+6, ^-7)$	$(^+10, ^-7)$	$(^+8, ^-4)$	$(^+9, ^-1)$
$(^+10, ^-2)$	$(^+9, ^+1)$	$(^+7, ^+5)$	$(^+2, ^+7)$	$(^-1, ^+7)$

INQUIRY

Arrow arithmetic



0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24

1. Look at the sentences below.

a. $17 \rightarrow = n$ **b.** $8 \leftarrow \leftarrow = n$

c. $9 \downarrow = n$ **d.** $10 \uparrow \uparrow = n$

- Sentence **a** tells you to start at 17 on the chart above and move one space *to the right.* At what numeral do you stop? Then what number makes sentence **a** true? The symbol → means to add what number?
- Sentence **b** tells you to start at 8 and move two spaces *to the left.* What number makes sentence **b** true? The symbol ← means to subtract what number?
- Sentence **c** tells you to start at 9 and move *down* one space. What number makes sentence **c** true? The symbol ↓ means to add what number?
- Sentence **d** tells you to start at 10 and move *up* two spaces. What number makes sentence **d** true? The symbol ↑ means to subtract what number?

2. Solve each of these sentences.

$12 \rightarrow \rightarrow = n$ $24 \leftarrow \leftarrow \leftarrow = n$

$5 \downarrow \downarrow \downarrow = n$ $23 \uparrow \uparrow \uparrow \uparrow = n$

3. Look at the sentences below.

e. $2 \searrow = n$ **f.** $2 \swarrow = n$

- Sentence **e** tells you to start at 2 and move *diagonally* one space to 8. The symbol ↘ means to add what number?
- Why is 6 the number that makes sentence **f** true? The symbol ↙ means to add what number?

4. Solve these sentences.

g. $11 \nwarrow = n$ **h.** $11 \nearrow = n$

- The symbol ↖ means to subtract what number?
- The symbol ↗ means to subtract what number?

5. Can you explain why this sentence is true? $1 \downarrow \searrow = 12$

— Solve these sentences.

	a	b
6.	$17 \nearrow \nearrow \downarrow = n$	$3 \downarrow \searrow \downarrow \downarrow = n$
7.	$22 \leftarrow \uparrow \uparrow \uparrow = n$	$12 \searrow \searrow \leftarrow = n$
8.	$9 \swarrow \searrow \downarrow = n$	$9 \downarrow \downarrow \downarrow = n$

9. Solve these five sentences.

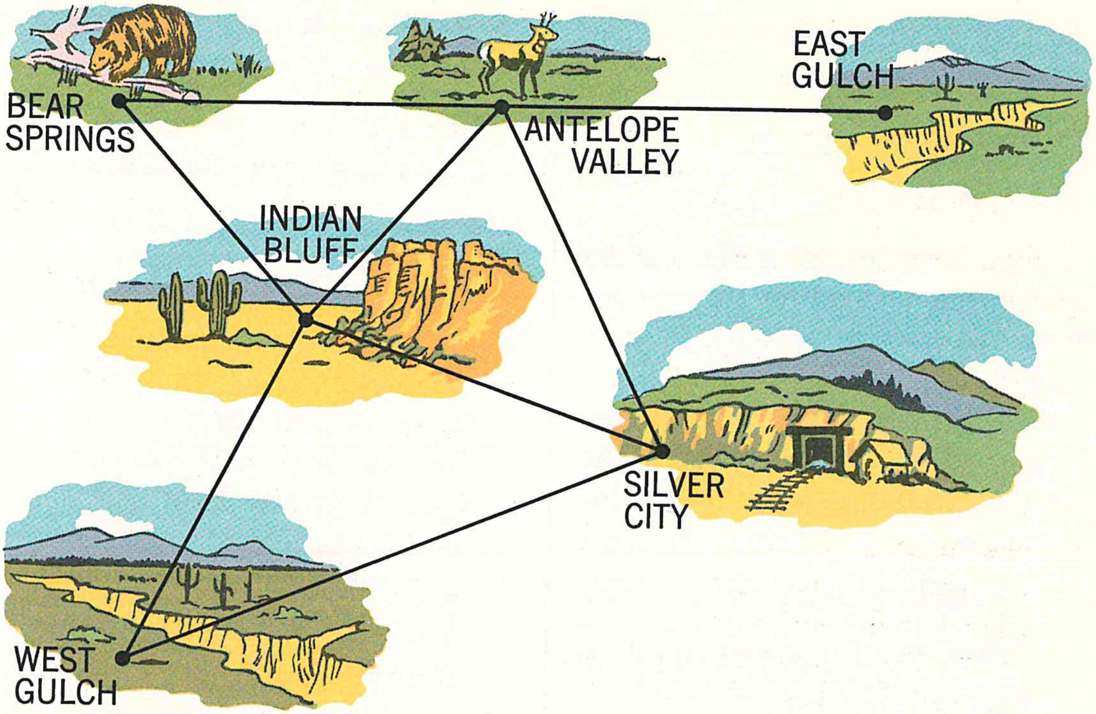
$4 \swarrow \swarrow \leftarrow = n$ $4 \leftarrow \swarrow \swarrow = n$

$4 \leftarrow \leftarrow \swarrow \downarrow = n$ $4 \swarrow \leftarrow \swarrow = n$

$4 \downarrow \downarrow \leftarrow \leftarrow \leftarrow = n$

10. Write six different sentences to show how to move from 3 to 5.

Pony Express



You have read about the Pony Express in the early days of the West. The riders of the Pony Express had to travel fast, so they looked for the shortest possible routes between towns. The map above shows the towns served by Hopalong Digit, one of the first express riders.

1. What route will enable Hopalong to go from East Gulch to West Gulch, passing through each town once and only once?

2. Is the route in Ex. 1 also the shortest route?

3. What is the shortest route from East Gulch to West Gulch and back to East Gulch?

4. What is the shortest route from East Gulch to West Gulch and return that goes through each town twice and only twice?

5. If the scale of the map were $\frac{1}{2}$ inch to 1 mile, what would be the distance traveled on the route in Ex. 1? in Ex. 3? Is the route in Ex. 4 just twice as long as the route in Ex. 1? Explain.

INQUIRY

Fascinating computations

1. Look at these sentences.

11 × 11 = 121

111 × 111 = 12,321

1,111 × 1,111 = 1,234,321

In each sentence, does the middle digit in the numeral for the product name the number of digits in each factor?

Show the next 3 number sentences in the pattern started above. The first of these sentences is started for you below.

11,111 × 11,111 = ?

2. What number times itself is equal to 12,345,678,987,654,321?

3. Look at these sentences.

3 × 37 = 111

6 × 37 = 222

9 × 37 = 333

You can rewrite them like this.

(3 × 1) × 37 = 111

(3 × 2) × 37 = 222

(3 × 3) × 37 = 333

Show the next 3 number sentences in the pattern.

4. Use the pattern to find the products in these sentences.

27 × 37 = ?   21 × 37 = ?

18 × 37 = ?   24 × 37 = ?

5. Look at the pairs of number sentences shown below.

1 × 3 = 3   2 × 2 = 4

2 × 4 = 8   3 × 3 = 9

3 × 5 = 15   4 × 4 = 16

9 × 11 = 99   10 × 10 = 100

- In each pair of sentences, are the factors in the second sentence equal to each other?
- Is each factor in the second sentence 1 greater than one factor and 1 less than the other factor in the first sentence?
- How do the products in the two sentences compare?

6. Complete these sentences.

20 × 20 = 400, so 19 × 21 = ?

25 × 25 = 625, so 24 × 26 = ?

30 × 30 = 900, so 31 × 29 = ?

— Write another number sentence to help you find the product for each sentence below.

	a	b
7.	11 × 13 = n	39 × 41 = n
8.	14 × 16 = n	61 × 59 = n
9.	17 × 19 = n	51 × 49 = n
10.	64 × 66 = n	79 × 81 = n
11.	71 × 69 = n	74 × 76 = n
12.	99 × 101 = n	249 × 251 = n

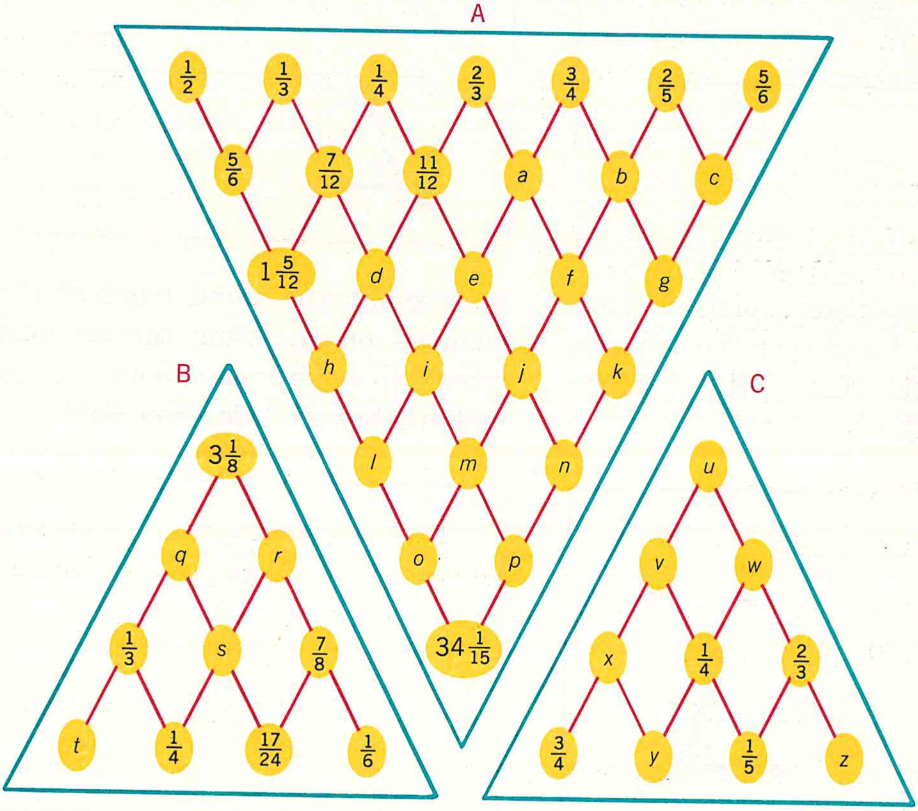
Addition arrays

1. The $\frac{1}{2}$ and $\frac{1}{3}$ shown in the first row of yellow dots in figure A below are joined with red segments to the $\frac{5}{6}$ shown in the second row. Does $\frac{5}{6}$ name the sum of $\frac{1}{2}$ and $\frac{1}{3}$?

2. Does the $\frac{7}{12}$ in the second row of yellow dots name the sum of $\frac{1}{3}$ and $\frac{1}{4}$? The $\frac{11}{12}$ names the sum of __?__ and __?__.

3. The yellow dot containing the letter *a* is joined to $\frac{2}{3}$ and $\frac{3}{4}$. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$? Then *a* stands for the number $1\frac{5}{12}$.

4. Find the numbers that the other letters stand for in figure A.



5. In figure B, the $\frac{7}{8}$ names the sum of $\frac{17}{24}$ and $\frac{1}{6}$. The $\frac{1}{3}$ names the sum of $\frac{1}{4}$ and __?__. Then the letter *t* stands for the number $\frac{1}{12}$. Find the numbers that the other letters stand for in figure B.

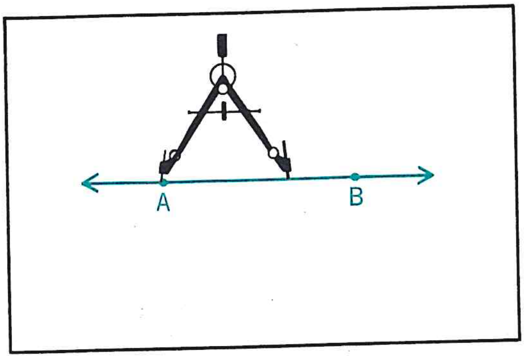
6. Find the numbers that the letters stand for in figure C.

INQUIRY

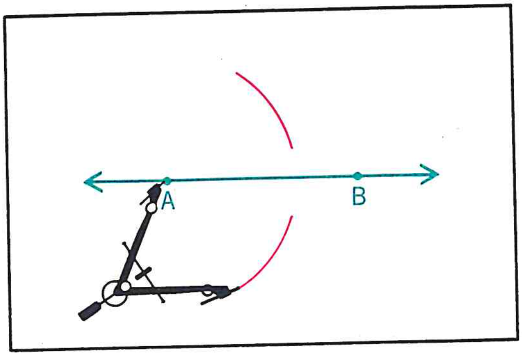
Constructing perpendicular lines

To make geometric constructions, you will need a drawing compass and a straightedge. The edge of a good ruler can be used as a straightedge.

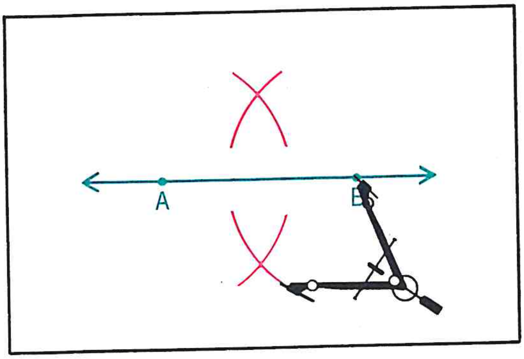
1. As you know, two lines that intersect in a point to form right angles are perpendicular. Suppose you are given line *AB* and are asked to construct another line perpendicular to line *AB*.



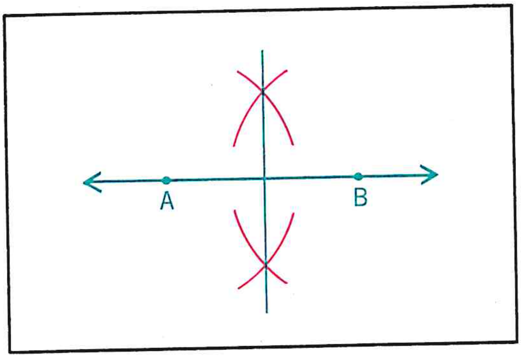
▶ Place the metal point of your compass on *A* and open the compass a little wider than $\frac{1}{2}$ the distance from *A* to *B*.



▶ Keeping the metal point of the compass on *A*, swing the compass around to make an arc above line *AB* and another arc below line *AB*.



▶ Without changing the compass opening, put the metal point on *B*. Make an arc above line *AB* and another arc below line *AB*.



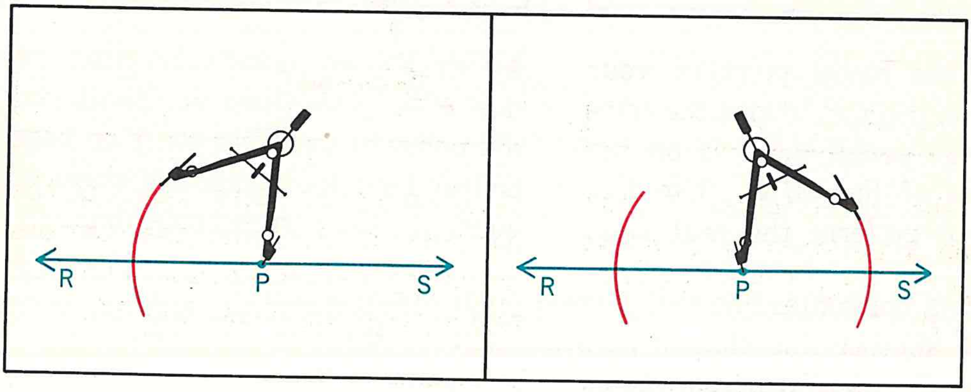
▶ Now use your straightedge and pencil to draw a line through the two points where the arcs that you drew intersect.

2. Draw line segment XY that is 2 inches long. Now, as described in Ex. 1, construct a line perpendicular to segment XY.

- Measure the distance from X to where the line you constructed intersects segment XY. Measure the distance from Y to where the line intersects segment XY. How do these measures compare?
- *To bisect* means to separate into two parts of the same size. Did you bisect line segment XY?

3. Draw a line segment 3 inches long. Now construct a line that is perpendicular to this segment and also bisects it.

4. Suppose you are given line RS and a point P on line RS. You are asked to construct another line that is perpendicular to line RS and passes through point P.



▶ Place the metal point of your compass on P. Make an arc that intersects line RS at the left of P and another arc that intersects line RS at the right of P, as shown in the two diagrams above.

▶ Use the two points where these arcs intersect line RS as you used points A and B in Ex. 1. Follow the remaining steps in Ex. 1 to construct a line perpendicular to line RS.

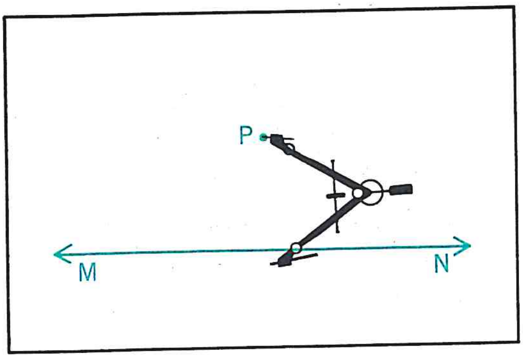
5. Draw a *horizontal* line and mark a point on this line. Now construct another line that is perpendicular to this line and also passes through the point you marked on this line.

6. Draw a *vertical* line and mark a point on this line. Now construct another line that is perpendicular to this line and also passes through the point you marked on this line.

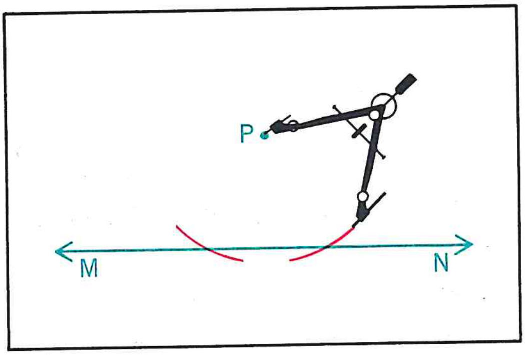
INQUIRY

More on constructing perpendicular lines

1. Suppose you are given line *MN* and a point *P* not on line *MN*. You are asked to construct another line that is perpendicular to line *MN* and passes through point *P*.



▶ Place the metal point of your compass on point *P*. Open the compass so the pencil point is on the other side of line *MN*. Use this opening to perform the next step.



▶ Mark two arcs that intersect line *MN*. Use these points as you did points *A* and *B* in Ex. 1 on page 356 and do the remaining steps to construct the perpendicular line.

2. Draw a horizontal line and mark a point above this line. Now construct another line that is perpendicular to the line that you drew and also passes through the point that you marked.

3. Draw a horizontal line and mark a point below this line. Now construct another line that is perpendicular to the line that you drew and also passes through the point that you marked.

4. Draw a vertical line and mark a point at the right of the line. Construct another line that is perpendicular to the line that you drew and also passes through the point that you marked.

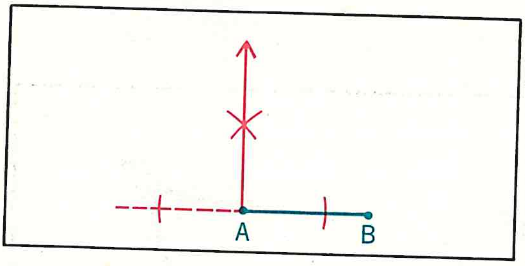
5. Draw a vertical line and mark a point at the left of the line. Construct another line that is perpendicular to the line that you drew and also passes through the point that you marked.

6. Draw line segment *AB* that is 2 inches long.

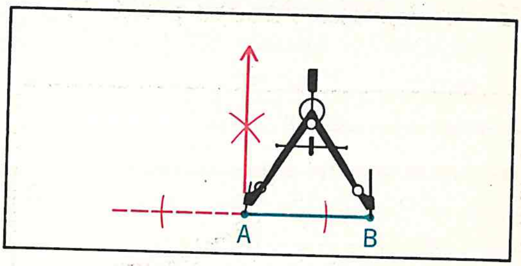
- Construct a line perpendicular to segment *AB* at point *A*.
- Construct a line perpendicular to segment *AB* at point *B*.

Constructing squares

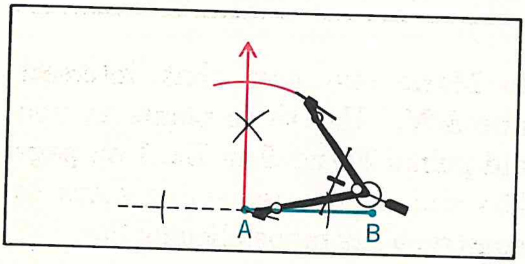
1. Suppose you are given line segment AB and you are asked to construct a square using segment AB as one of the sides.



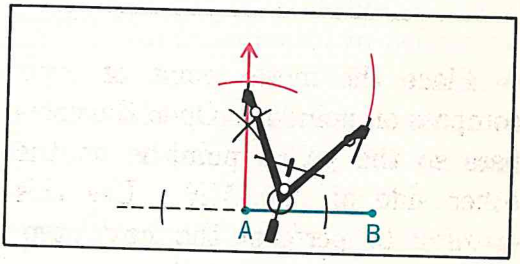
▶ Construct a line perpendicular to segment AB at point A as you learned to do on page 357.



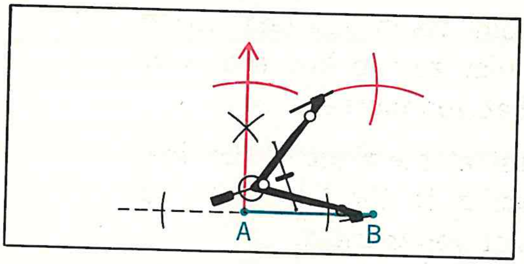
▶ Open your compass until the metal point is on A and the pencil point is on B.



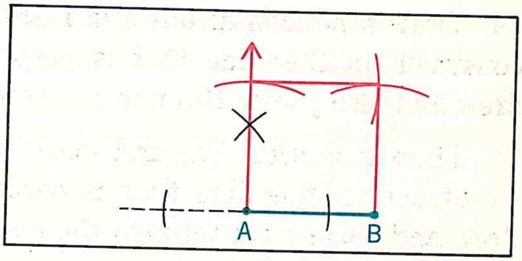
▶ Keep the compass opening the same. Mark an arc on the line that is perpendicular to segment AB.



▶ Keep the compass opening the same. Mark another arc as shown in the diagram above.



▶ Keep the compass opening the same. Mark another arc as shown in the diagram above.



▶ Join the points you determined with line segments as shown in the diagram above.

2. Draw segments 1 inch long, 2 inches long, and 3 inches long. Use each segment to construct a square as described in Ex. 1.

INQUIRY

Find the missing digits

1. The multiplication example shown at the right and those shown below are written in code. You are asked to see if you can break the code.

$$\begin{array}{r} \text{A L} \\ \times\ \text{2 F} \\ \hline \text{A 4 3} \\ \text{5 R O} \\ \hline \text{L 8 B} \end{array}$$

Each letter stands for a digit. Each time it is used in an example, it stands for the same digit. The letter O always stands for *zero*.

▶ Look at the code names for the partial products. What is the sum of 3 and 0? Then what number does B stand for in the code name for the product?

▶ What number added to 4 equals 8? Then is the second partial product equal to 540?

▶ You now know that the second partial product is 540 and is the result of multiplying the first factor by 20. What number when multiplied by 20 equals 540? Then what is the factor that is represented by AL?

▶ Since A stands for 2, the first partial product is 243. The first factor is 27. What number times 27 equals 243? Then what number does F stand for?

▶ Write the multiplication example complete with all the digits that are now represented in code in the example above.

— Break the code in each of the following examples. Then write the multiplication example complete with all the digits.

2.

a
$$\begin{array}{r} \text{Q 1 3} \\ \times\ \text{4 6} \\ \hline \text{1, Q W S} \\ \text{S, 5 Q O} \\ \hline \text{T, W T S} \end{array}$$

b
$$\begin{array}{r} \text{E E O} \\ \times\ \text{7 8} \\ \hline \text{B B O} \\ \text{A, A O O} \\ \hline \text{B, L B O} \end{array}$$

c
$$\begin{array}{r} \text{B O Y} \\ \times\ \text{2 5} \\ \hline \text{A, R B O} \\ \text{6, 1 2 0} \\ \hline \text{G, Y R O} \end{array}$$

d
$$\begin{array}{r} \text{A O B} \\ \times\ \text{C D} \\ \hline \text{D A D} \\ \text{C, 2 4 0} \\ \hline \text{C, 7 D D} \end{array}$$

3.

a
$$\begin{array}{r} \text{C E D} \\ \times\ \text{G J} \\ \hline \text{D D 1} \\ \text{2, 5 7 0} \\ \hline \text{J, J 4 G} \end{array}$$

b
$$\begin{array}{r} \text{Q D N} \\ \times\ \text{C N} \\ \hline \text{A, O E N} \\ \text{D C, A O O} \\ \hline \text{1 5, 3 7 5} \end{array}$$

c
$$\begin{array}{r} \text{A T} \\ \times\ \text{9 R} \\ \hline \text{A O R} \\ \text{2, N F O} \\ \hline \text{A, K F R} \end{array}$$

d
$$\begin{array}{r} \text{9 T} \\ \times\ \text{P S} \\ \hline \text{1 9 P} \\ \text{Q, T T O} \\ \hline \text{P, O N 6} \end{array}$$

4. The division example shown at the right and those shown below are written in code. You are asked to crack the code. Remember that the letter O always stands for *zero*.

$$\begin{array}{r} 4{,}0\,K\,1 \\ 2\,C\overline{)\,8\,X{,}B\,C\,2} \\ \underline{L\,L{,}0\,0\,0} \\ 1{,}B\,C\,C \\ \underline{B{,}B\,0\,0} \\ C\,2 \\ \underline{2\,C} \\ 0 \end{array}$$

▶ What number equals 2 − 0? Then C stands for 2. What is the divisor for the example?

▶ To find the number represented by LL,000, find the product of 4,000 and 22.

▶ In the example, X minus L equals one. Since L stands for 8, then what number does X stand for?

▶ Find the numbers represented by the other letters. Then write the division example complete with all the digits.

— Break the code in each of the following examples. Then write the division example complete with all the digits.

a

5.

$$\begin{array}{r} X\,4\,J \\ P\overline{)\,H{,}\,8\,M\,L} \\ \underline{H{,}\,J\,O\,O} \\ 1\,4\,L \\ \underline{L\,H\,O} \\ H\,1 \\ \underline{H\,L} \\ O \end{array}$$

b

$$\begin{array}{r} 9{,}A\,5\,R \\ R\,R\overline{)\,R\,O\,A{,}L\,C\,R} \\ \underline{A\,A{,}O\,O\,O} \\ R\,O{,}L\,C\,R \\ \underline{A{,}A\,O\,O} \\ 5\,C\,R \\ \underline{Q\,Q\,O} \\ 1\,R \\ \underline{R\,R} \\ O \end{array}$$

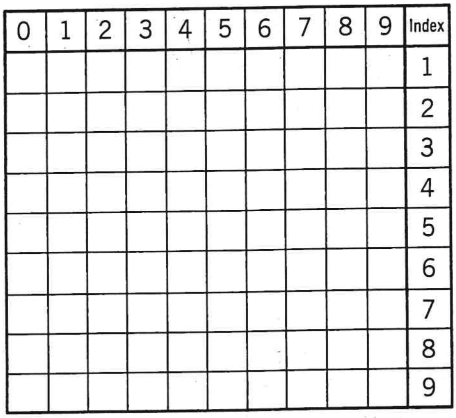
6.

$$\begin{array}{r} 4{,}\,Z\,5\,Z \\ 2\,Y\overline{)\,J\,J{,}\,P\,M\,Y} \\ \underline{J\,6{,}O\,O\,O} \\ 3{,}\,P\,M\,Y \\ \underline{M{,}\,Y\,O\,O} \\ Z{,}M\,M\,Y \\ \underline{Z{,}M\,O\,O} \\ 2\,Y \\ \underline{M\,Y} \\ O \end{array}$$

$$\begin{array}{r} 4{,}\,N\,O\,P \\ N\,5\overline{)\,X\,N{,}X\,3\,T} \\ \underline{X\,O{,}O\,O\,O} \\ 1{,}X\,3\,T \\ \underline{N{,}T\,O\,O} \\ N\,3\,T \\ \underline{N\,R\,T} \\ O \end{array}$$

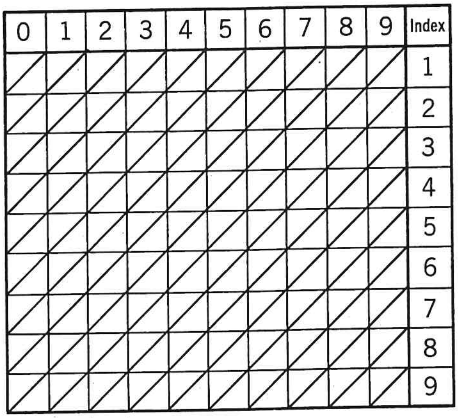
INQUIRY

Napier's Rods



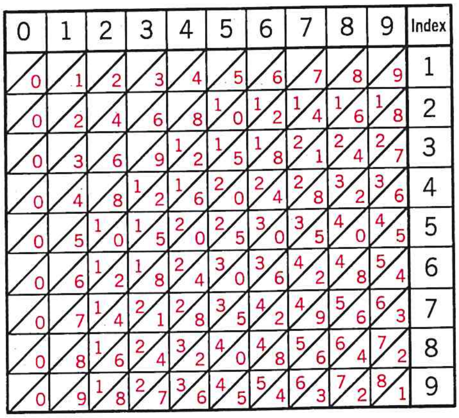
0	1	2	3	4	5	6	7	8	9	Index
										1
										2
										3
										4
										5
										6
										7
										8
										9

I



0	1	2	3	4	5	6	7	8	9	Index
										1
										2
										3
										4
										5
										6
										7
										8
										9

II



0	1	2	3	4	5	6	7	8	9	Index
0	1	2	3	4	5	6	7	8	9	1
0	2	4	6	8	1/0	1/2	1/4	1/6	1/8	2
0	3	6	9	1/2	1/5	1/8	2/1	2/4	2/7	3
0	4	8	1/2	1/6	2/0	2/4	2/8	3/2	3/6	4
0	5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5	5
0	6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4	6
0	7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3	7
0	8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2	8
0	9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1	9

III

About 350 years ago, a Scotsman by the name of John Napier made a computer for multiplication out of a set of bones. Here is how you can make a similar computer out of cardboard strips.

▶ Cut out a piece of cardboard 5 inches long and $5\frac{1}{2}$ inches wide. Mark off half-inch square boxes on the cardboard.

In the top row of boxes, write the digits 0 through 9. In the last box, write *Index*. Then write the digits 1 through 9 down the right column as in figure I.

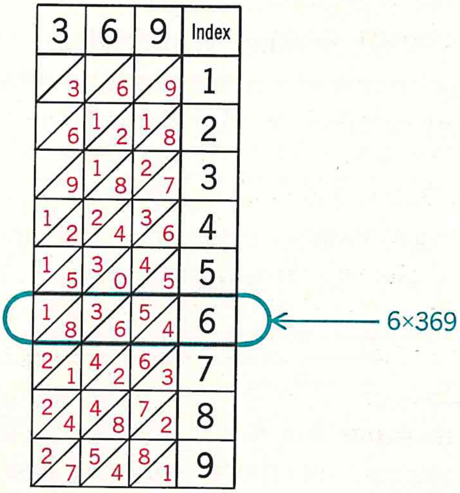
▶ Now draw diagonal line segments through all the empty boxes as in figure II.

▶ Using a red pencil, write the numerals for the first nine multiples of 0 in the first column, the first nine multiples of 1 in the second column, the first nine multiples of 2 in the third column, and so forth. When the numeral for the multiple has two digits, write the tens digit on the left side of the diagonal and the ones digit on the right side of the diagonal. Your card should look like figure III.

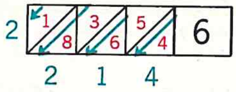
▶ Now cut along the vertical line segments so that each column is a separate card, or rod. You should have 11 rods.

1. Suppose you wish to multiply 369 by 6 using Napier's rods.

▶ Pick out the 3, 6, and 9 rods and place them side by side. Then place the index rod at the right of the three rods as shown below.



▶ Look down the index rod to the numeral 6. To find the product, you must add the numbers named diagonally as shown below. Start at the right and rename as you add.



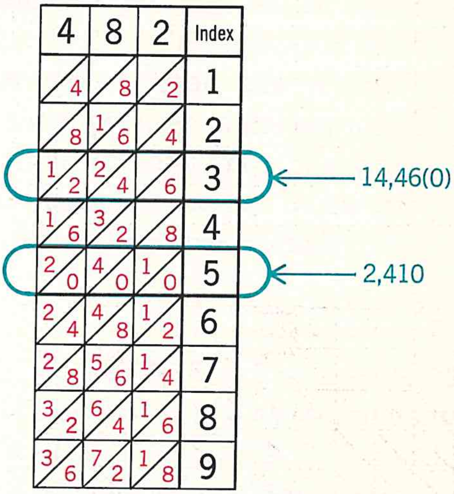
▶ The product is 2,214.

— Find these products using your set of Napier's rods.

	a	b
2.	128 × 6	367 × 8
3.	425 × 7	463 × 4
4.	728 × 5	3,425 × 6
5.	2,683 × 9	9,730 × 8

6. Suppose you wish to multiply 482 by 35 using Napier's rods.

▶ Pick out the 4, 8, and 2 rods and place them side by side. Then place the index rod at the right of the three rods as shown below.



▶ What is the product of 482 and 5? This is the first of your two partial products.

▶ What is the product of 482 and 3? Then what is the product of 482 and 30? This is the second of your two partial products.

▶ Add the two partial products.

▶ Check your work by multiplying in the usual way.

— Find these products using your set of Napier's rods.

	a	b
7.	712 × 36	528 × 67
8.	409 × 53	726 × 18
9.	835 × 44	649 × 28
10.	760 × 52	236 × 14

INQUIRY

Factorial

1. Look at these expressions.

$$4 \times 3 \times 2 \times 1$$

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

- What is the last factor named in each expression?
- What is the next to the last factor named in each expression?
- Copy and complete the number expression below, following the pattern of the expressions above.

$$6 \times 5 \times \underline{?} \times \underline{?} \times \underline{?} \times \underline{?}$$

2. Does $1 \times 2 \times 3 \times 4$ name the same number as $4 \times 3 \times 2 \times 1$?

Does $1 \times 2 \times 3 \times 4 \times 5 \times 6$ name the same number as $6 \times 5 \times 4 \times 3 \times 2 \times 1$?

When the factors named in a multiplication number expression are consecutive whole numbers that either begin or end with 1, the number named by the expression is called **factorial.** The symbol for factorial is **!**. The expression 4! is read "four factorial." It names the number $4 \times 3 \times 2 \times 1$, or the number 24.

3. What number is named by 5!?

4. What number is named by 6!?

5. Copy and complete the following sentence to show the product of 3! and 2!.

$$(3 \times 2 \times 1) \times (2 \times 1) = \underline{?}$$

— Find the products.

	a	b
6.	$4! \times 3!$	$6! \times 2!$
7.	$3! \times 3!$	$5! \times 2!$

8. What number is named by 7!? What number is named by 3!? Then what number is named by $7! \div 3!$?

$$5{,}040 \div 6 = \underline{?}$$

9. Another way to show the division of 7! by 3! is given below.

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

How do the last 3 factors in the numerator compare with the last 3 factors in the denominator? Then does the following name the same number as the expression above?

$$7 \times 6 \times 5 \times 4$$

What number is named by the expression above? How does this number compare with your answer for Ex. 8?

10. Is this sentence true?

$$\frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 5 \times 4$$

What number is named by $5! \div 3!$?

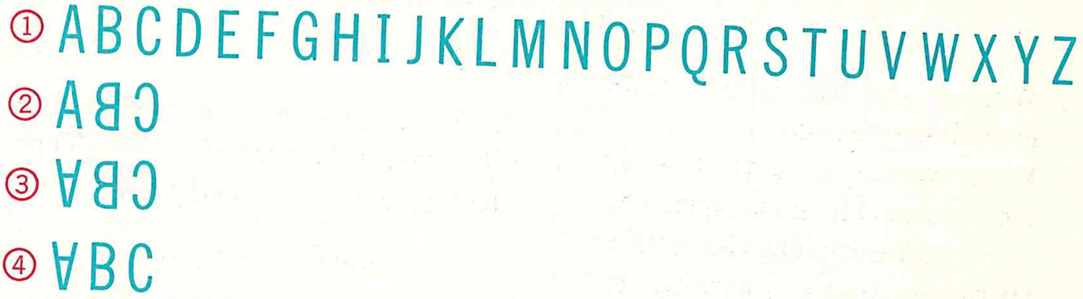
— Find the quotients.

11.	$10! \div 7!$	$7! \div 4!$
12.	$9! \div 5!$	$12! \div 10!$

INQUIRY

Alphabet symmetry

1. In row 1 of the diagram below are shown the letters of the alphabet. Copy the letters in row 1 exactly as shown.



- In row 2, the first three letters of row 1 are shown as they would appear if they were reversed from side to side. Copy and continue row 2 to show how all the letters of the alphabet would look if reversed this way.
- Suppose you took the first three letters shown in row 2 and reversed them from top to bottom. Then they would look like the letters shown in row 3. Copy and continue row 3 to show how all the letters you wrote in row 2 would look if reversed this way.
- Row 4 shows how the letters in row 3 would look if they were reversed from side to side. Copy and continue row 4 to show how the other letters in your row 3 would look if reversed this way.

2. Which letters are different in all four rows?

3. Which letters are identical in rows 1 and 4, and also identical in rows 2 and 3?

4. Which letters are identical in rows 1 and 2, and also identical in rows 3 and 4?

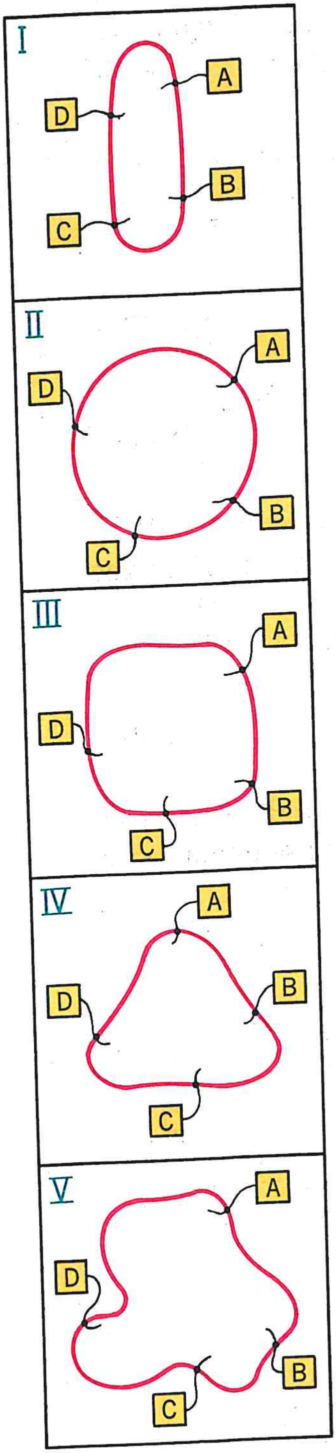
5. Which letters are identical in all four rows?

$100,1031

6. You know that a figure has *line symmetry* if a line can be drawn through it so that the part on one side of the line is the exact opposite, or reflection, of the part on the other side of the line. Do the letters you named in Ex. 2 have line symmetry? Do those you named in Ex. 3 have line symmetry? Do those you named in Ex. 4? Do those you named in Ex. 5?

INQUIRY

Topology



1. Take a rubber band and tie pieces of string tightly around it at four different points. Attach to the strings pieces of paper labeled *A*, *B*, *C*, and *D*. Place the rubber band on your desk as in figure I.

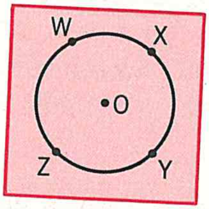
You have now represented a closed path (the rubber band) with four labeled points (*A*, *B*, *C*, and *D*) in a plane (the desk top). The closed path separates the plane into two regions, one region inside the path and one region outside the path.

- Arrange the rubber band to form a circle as in figure II. Do the labels show points on the circle? Are the points in the same order as in figure I? Does the circle separate the plane into one inside region and one outside region?
- Without lifting the rubber band from the desk, arrange it to form a square as in figure III. Are the points still in the same order? Into how many regions does the square separate the plane?
- Arrange the rubber band to form a triangle as in figure IV. Are the points still in the same order? Into how many regions does the triangle separate the plane?
- Stretch the rubber band as shown in figure V. Are the points still in the same order? Into how many regions does this figure separate the plane?
- List two ways in which all five closed paths you have shown are alike. If you think *only* of the order of the points and the number of inside and outside regions, are these five closed paths the same?

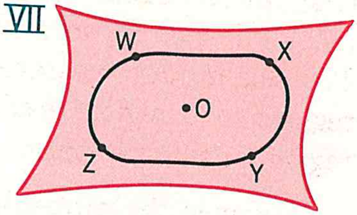
Topology is a special kind of geometry that deals with the properties of geometric figures, without regard to their size or shape. It deals with the properties that remain the same when a figure has been stretched, or distorted, without tearing or joining. Topology is sometimes called *rubber-sheet geometry.*

2. Suppose you had a rubber sheet and drew on it a circle with the center at O. If you stretched and pulled the sheet, the circle might look like figures VII and VIII shown below.

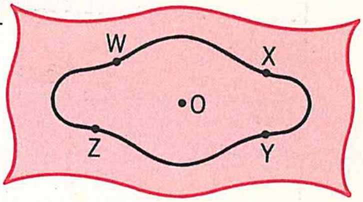
VI



VII

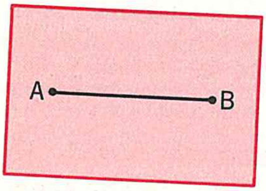


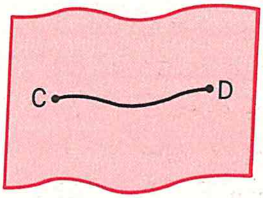
VIII

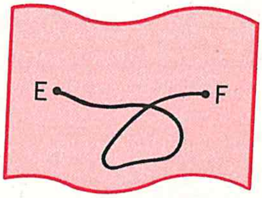


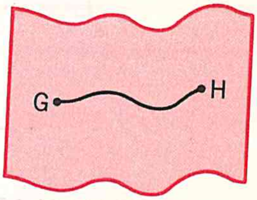
- In each of the three figures above, is point X between points W and Y on the path? Is point Y between points X and Z on the path? Are points W, X, Y, and Z all in the same order?
- In each figure, does the closed path separate the plane into just one inside region and one outside region?
- In each figure, is point O inside or is it outside the closed path? In topology, all three figures are the same.

3. In topology, two of the drawings below are related to line segment AB. Paths CD and GH have one property in common with segment AB. They do not cross themselves.

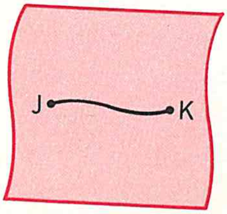


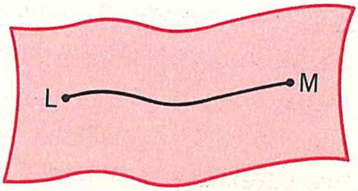


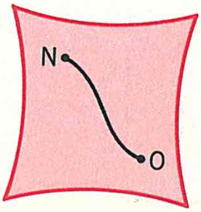


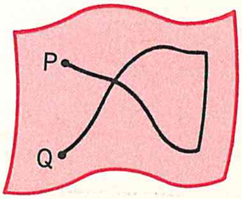


Which paths below have one property in common with segment AB?







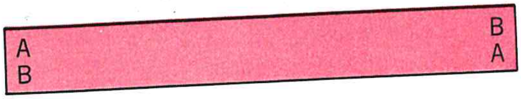


INQUIRY

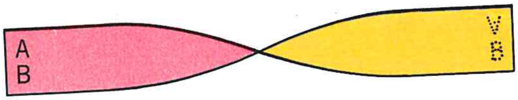
The Möbius strip

1. One of the more famous surfaces in topology is called a **Möbius strip.** This is a strip that has just one surface, or side. How many surfaces does a regular strip of paper have?

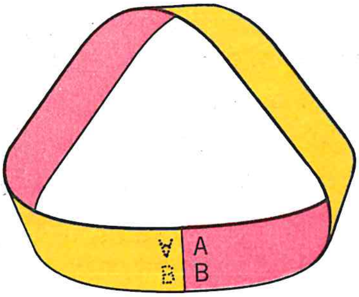
2. To make a Möbius strip, do the following.



▶ Cut out a strip of paper 1 inch wide and 18 inches long. On one side, label one end AB and the other end BA, as shown above.



▶ Hold one end flat and twist the other end so that the letters are facing the other way.



▶ Now fit the ends together so the A on one end is over the A on the other end, and the B on one end is over the B on the other end. Tape or glue the ends together.

3. Take a pencil, and starting halfway between the edges, trace a path around the strip.

- Do you find that you come back to your starting point?
- Is the path you made shown on every surface of the strip?
- Did you trace just one long path? Then does the strip have just one surface?

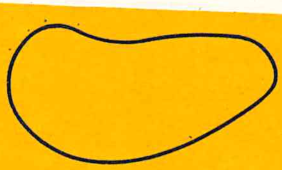
4. Now, using scissors, cut the strip along the path you marked. When you finish, how many strips do you have?

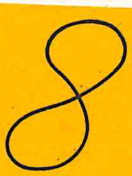
Trace a path halfway between the edges of this new strip. How many surfaces does the strip have? Is the new strip a Möbius strip?

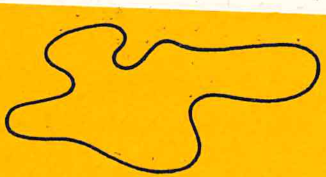
5. Make a Möbius strip $1\frac{1}{2}$ inches wide. Mark the strip into thirds by making a mark $\frac{1}{2}$ inch from each edge. Start at one of the marks and cut the strip always staying $\frac{1}{2}$ inch from an edge. How many strips do you get? How many of them, if any, are Möbius strips?

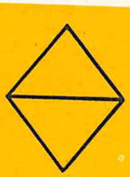
6. Make a Möbius strip 2 inches wide. Mark it into fourths, starting $\frac{1}{2}$ inch from an edge. Cut as you did in Ex. 5. Cut the final loop so that no strip is wider than $\frac{1}{2}$ inch. How many strips do you get? Are any of them Möbius strips?

Simple closed paths


I

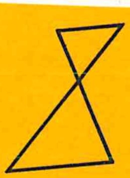

II

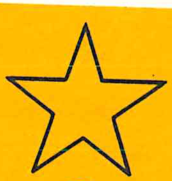

III

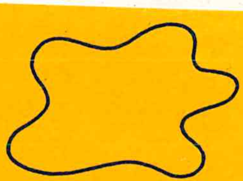

IV

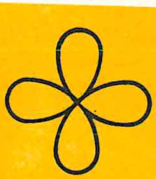
1. Shown above are four closed paths. Which ones do not cross themselves? Closed paths that do not cross themselves are called **simple closed paths.** Every simple closed path has how many inside regions? how many outside regions?

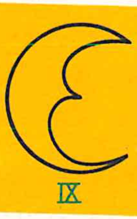
2. How many inside regions does each figure below have? Which of the figures below are simple closed paths?


V

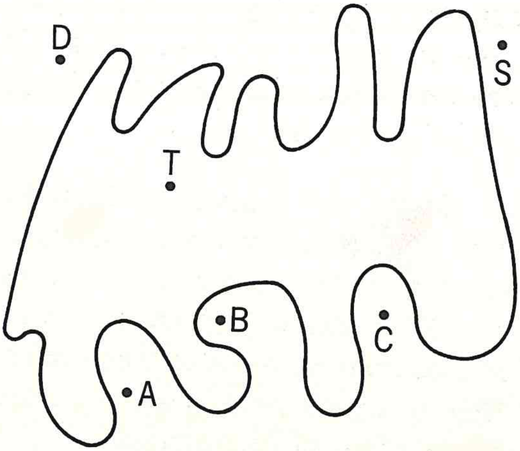

VI


VII


VIII


IX

3. Is the figure below a simple closed path? Is point T inside or is it outside the path? Name the points outside the path.



4. Copy the figure in Ex. 3. Then draw line segments from T to A, T to B, T to C, and T to D.

- Does each segment that joins a point inside the path with a point outside cross the path an even number or does it cross the path an odd number of times?

5. On the figure you drew for Ex. 4, draw segments SA, SB, SC, and SD.

- Does each segment that joins a point outside the path with another point outside cross the path an even number or does it cross the path an odd number of times?

TABLES OF MEASUREMENT

TIME	60 seconds (sec.) make 1 minute (min.)
	60 minutes make 1 hour (hr.)
LENGTH	12 inches (in.) make 1 foot (ft.)
	3 feet make 1 yard (yd.)
	36 inches make 1 yard
	5,280 feet make 1 mile (mi.)
LENGTH Metric System	10 millimeters (mm) make 1 centimeter (cm)
	10 centimeters make 1 decimeter (dm)
	10 decimeters make 1 meter (m)
	10 meters make 1 dekameter (dkm)
	10 dekameters make 1 hectometer (hm)
	10 hectometers make 1 kilometer (km)
AREA	144 square inches (sq. in.) make 1 square foot (sq. ft.)
	9 square feet make 1 square yard (sq. yd.)
	1,296 square inches make 1 square yard
VOLUME	1,728 cubic inches (cu. in.) make 1 cubic foot (cu. ft.)
	27 cubic feet make 1 cubic yard (cu. yd.)
	46,656 cubic inches make 1 cubic yard
WEIGHT	16 ounces (oz.) make 1 pound (lb.)
	2,000 pounds make 1 ton (T.)
	2,240 pounds make 1 long ton
LIQUID MEASURE	8 fluid ounces (fl. oz.) make 1 cup (c.)
	2 cups make 1 pint (pt.)
	2 pints make 1 quart (qt.)
	4 quarts make 1 gallon (gal.)
	32 fluid ounces make 1 quart
	4 cups make 1 quart

Pages 371-378 (Glossary, Index) not required. This is the last page.